Game-Refinement Theory and Its Application to Volleyball

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This paper introduces a recent development of game-refinement theory and shows its application to the game of Volleyball. The game-refinement theory was proposed in 2003 as a new game theory to measure entertainment impact or sophistication of games with focus on the uncertainty of game outcome. The original framework of game-refinement theory was constructed in the domain of board games such as Chess and Go. Later it was extended for various types of games including sports games, while considering a general model of game information progress. We are interested in observing the rule change of sports games and its impact based on the game-refinement theory. We have chosen the Volleyball as a testbed in this study. It is found that the rule change from the side-out system with 15 points to the rally system with 25 points seems better to make the game more fascinating (i.e., higher value of game-refinement measure), but its value seems higher than upper limit of comfortable zone of game-refinement measure.

Keywords: game-refinement theory, Volleyball, side-out system, rally system

1. Introduction

The three master model in games[1)] reveals three distinct master aspects: the master of winning, the master of playing and the master of understanding. They correspond to each of the three important characteristics that games possess: competitiveness, entertainment and interaction. Existing theories that may be related to each are the game theory[9)], game-refinement theory[2)3)], and game information dynamics[4)].

In other words, the optimization in games can be discussed from three different points of view. The first one is strategic optimization for players. In this direction, the works done by von Neumann[9)] and Nash[7)] have been well known and widely recognized as a useful tool in many fields such as economics, political science, psychology, logic and biology. The second aspect is entertainment optimization for game creators. In this direction, no mathematical theory had been discussed until the game-refinement theory was proposed[3)]. The third aspect is optimization of intellectual interaction for observers. In this direction, game information dynamic models has recently been proposed[4)].

In this paper, we focus on the second aspect, i..e, entertainment optimization or game-refinement theory. In the early work, a logistic model of game information progress was proposed in the domain of board games. A measure of game refinement was derived from the model with focus on the uncertainty of game outcome[3)5)]. The measure was used together with tree-space complexity to explain the evolutionary history of rule changes in Chess. The proposed measure seems promising to use as a tool to assess the impact of engagement of Chess-like games.

For example, in the Chess history most variants were outsourced and only a few variants survived to the present[6)]. The surviving variants went through the sophistication of the game rules to maximize the entertainment impact making the depth of lookahead (i.e., intelligent aspect of games) more critical for the outcome of the game. Experienced players often noted that large and complex games were not attractive at all and interesting and enjoyable games are those with more entertainment impacts. The evolutionary process has produced the present version of Chess which seems a well-balanced search-space complexity and entertainment impact. Modern Chess may be considered a highly matured and optimized Chess-like game[2)].

Recently, the game-refinement theory was extended and applied to various games[11)]. In their study a general model of game information progress was proposed to derive a measure of game refinement, and board games and sports games were compared with some data from well-known games such as Chess and Soccer. Interestingly, we observed that such well-refined games were at the same level of entertainment impact quantified by the proposed

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measure.

To establish the game-refinement theory and the general model of game information progress, we need to investigate from various points of view. In the early works (e.g.,[5]), the uncertainty of game outcome have been mainly focused on. It means to assume the existence of the force in mind of observers, which is derived from the second derivative of the game information progress model curve. In the previous study[11], Chess, Soccer and Basketball were compared based on the game-refinement theory. Soccer is a type of game where the time is limited, i.e., 90 mins for one game. In this study, we focus another type of game where the goal is set in advance. We have chosen the domain of Volleyball game, in which one team needs 25 points to win.

In this paper, first we introduce the basic idea of the early works on game-refinement theory as well as the general model of game information progress. We then present an application of game-refinement theory to Volleyball with focus on the rule changes from the side-out system with 15 points to the rally system with 25 points. Then, concluding remarks are given.

2. Game-Refinement Theory

"Game progress" has twofold: one is game speed or scoring rate, while another one is game information progress with focus on the game outcome. In sports games such as Soccer and Basketball, the scoring rate will be calculated by two factors: (1) goal, i.e., total score and (2) time or steps to achieve the goal. For example, in Basketball the total score is given by the average number of successful shoot attempts, whereas the steps to achieve the goal is estimated by the average number total of shoot attempts. Then the game speed of Basketball is given by

$$\frac{\text{average\_number\_of\_successful\_shoots}}{\text{average\_number\_of\_shoots}}$$

We need to consider a reasonable way to obtain the game speed for various type of games. For some sports games such as Basketball and Soccer, we can obtain statistics of average number of shoots and goals per one game. For other sports games such as Volleyball and Table Tennis in which the goal (i.e., score to win) is set in advance, the average number of total points per game may correspond to the time or steps to achieve the goal.

Let G and T be the average number of successful shoots and the average number of shoots per game, respectively. A realistic model of game information progress $x(t)$ was proposed[11], as shown in Equation (1), for $0 \leq t \leq T$ and $0 \leq x(t) \leq G$.

$$x(t) = G(\frac{t}{T})^n \tag{1}$$

Here n stands for a constant parameter which will depend on the perspective of an observer in the game considered.

Meanwhile, we reasonably assume that the parameter would be $n \geq 2$ in many cases like balanced or seesaw games. Thus, we have the second derivative of $x(t)$ in Equation (1), as shown in Equation (2).

$$x''(t) = \frac{Gn(n-1)}{T^n}t^{n-2} \tag{2}$$

Solving the formula at $t = T$, the equation becomes

$$x''(T) = \frac{G}{T^2}n(n-1)$$

In Equation (2), $x''(T)$ stands for the acceleration of game information progress.

Newton mechanics indicates that the force in the physical world is produced from mass and acceleration, which can be witten as $F = ma$. In this study, we assume that the game information progress in any type of games is happening in our minds. We do not know yet about the physics in mind, but it is likely that the acceleration of game information progress is related to the force in mind. The higher force in mind due to game means the higher the game's degrees of excitement.

Suppose that the physical world acceleration a is analogous with game information progress acceleration $x''(T)$ in player's mind, then we can assume that the higher the value $\frac{G}{T^2}$ is, the more exciting the game becomes. Thus, we propose to use the value $\frac{G}{T^2}$ or its root square, as shown in Equation (3), as a game-refinement measure R for the game considered.

$$R = \frac{\sqrt{G}}{T} \tag{3}$$

In the previous study[11], the difference between sports games and board games were discussed to observe that for board games the average number of possible moves B and game length D can be used instead of G and T in case of sports games, respectively. Hence, it is proposed to use the following value as a game-refinement measure for board games.

$$R = \frac{\sqrt{B}}{D}$$

We show, in Table 1, some data of games such as Chess and Go[3] from board games and Basketball and Soccer from sports. For Basketball the data were obtained from the NBA website[8], while the data for Soccer were obtained from the UEFA championship[12]. From Table 1, we suspect that sophisticated games have a common factor (i.e., same degree of acceleration value) to feel engagement or excitement regardless of different type of games. We understand that Chess, Go, Soccer and Basketball are all sophisticated with long history.

Table 1 Measures of game-refinement for various games

Game	B or G	D or T	R
Chess	35	80	0.074
Go	250	208	0.076
Basketball	36.38	82.01	0.073
Soccer	2.64	22	0.073

3. Volleyball

In this section, we show the basic rules of Volleyball and select three variants. For these variants we determine the game progress values to apply game-refinement theory. Then these variants and its rule changes are discussed based on the game-refinement theory.

3.1 Basic rules of Volleyball

Volleyball is a team sport in which two teams of six players are separated by a net. Each team tries to score points by grounding a ball on the other team's court under organized rules[13]. It has been a part of the official program of the Summer Olympic Games since 1964.

The complete rules are extensive. But simply, play proceeds as follows: a player on one of the teams begins a 'rally' by serving the ball (tossing or releasing it and then hitting it with a hand or arm), from behind the back boundary line of the court, over the net, and into the receiving team's court. The receiving team must not let the ball be grounded within their court. The team may touch the ball up to 3 times but individual players may not touch the ball twice consecutively. Typically, the first two touches are used to set up for an attack, an attempt to direct the ball back over the net in such a way that the serving team is unable to prevent it from being grounded in their court.

The game continues in this manner, rallying back and forth, until the ball touches the court within the boundaries or until an error is made. The most frequent errors that are made are either to fail to return the ball over the net within the allowed three touches, or to cause the ball to land outside the court. A ball is "in" if any part of it touches a sideline or end-line, and a strong spike may compress the ball enough when it lands that a ball which at first appears to be going out may actually be in. Players may travel well outside the court to play a ball that has gone over a sideline or end-line in the air. Other common errors include a player touching the ball twice in succession, a player "catching" the ball, a player touching the net while attempting to play the ball, or a player penetrating under the net into the opponent's court. There are a large number of other errors specified in the rules, although most of them are infrequent occurrences. These errors include back-row or libero players spiking the ball or blocking (back-row players may spike the ball if they jump from behind the attack line), players not being in the correct position when the ball is served, attacking the serve in the front court and above the height of the net, using another player as a source of support to reach the ball, stepping over the back boundary line when serving, taking more than 8 seconds to serve[10], or playing the ball when it is above the opponent's court.

When the ball contacts the floor within the court boundaries or an error is made, the team that did not make the error is awarded a point, whether they served the ball or not. If the ball hits the line, the ball is counted as in. The team that won the point serves for the next point. If the team that won the point served in the previous point, the same player serves again. If the team that won the point did not serve the previous point, the players of the team rotate their position on the court in a clockwise manner. The game continues, with the first team to score 25 points by a two-point margin is awarded the set. Matches are best-of-five sets and the fifth set, if necessary, is usually played to 15 points. (Scoring differs between leagues, tournaments, and levels; high schools sometimes play best-of-three to 25; in the NCAA matches are played best-of-five to 25 as of the 2008 season[14].)

Before 1999, points could be scored only when a team had the serve (side-out scoring) and all sets went up to only 15 points. The FIVB changed the rules in 1999 (with the changes being compulsory in 2000) to use the current scoring system (formerly known as rally point system), primarily to make the length of the match more predictable and to make the game more spectator- and television-friendly. The final year of side-out scoring at the NCAA Division I Women's Volleyball Championship was 2000. Rally point scoring debuted in 2001 and games were played to 30 points through 2007. For the 2008 season, games were renamed "sets" and reduced to 25 points to win.

3.2 Three variants

In this study we select three important variants from the history of Volleyball: (1) side-out scoring system with 15 points, (2) rally point system with 30 points, and (3) rally point system with 25 points (see Table 2).

Table 2 Three rule variants of Volleyball

rules	points	set
side-out scoring system	15	best-of-five
rally point system	30	best-of-five
rally point system	25	best-of-five

We first focus on the current rule, i.e., rally point system with 25 points. We show, in Table 3, statistics on the average point per game in Volleyball games from V-league in Japan[15]. The max point and min point are also shown. Since the average point per game is 44, it is expected that the final score on average is $25-19$. Likewise, the score in max point case would be $37-35$ due to deuce and $15-6$ (15 points) in min point case.

Table 3 Statistics on point per game in rally point system Volleyball with 25 points (n=486).

	Points
Max points	72
Min points	21
Ave points	44

The game progress in Volleyball can be given by the average number of goals ($G = 25$) and the average number of all scores ($T = 44$). By applying Equation (3), we obtain as the value as follows:

$$R_{25pts} = \frac{\sqrt{25}}{44} = 0.114$$

We next consider the rally point system with 30 points. Since the data for 30 points rally currently is unavailable, we estimate it based on the statistics of 25 points rally point system Volleyball. For this purpose, we assume the same ratio (25 : 19) of winning points and losing points and obtain 30 : 22.8. It means that the average goals G is 30 while the average total points T is 52.8. By applying Equation (3), we obtain as the value as follows:

$$R_{30pts} = \frac{\sqrt{30}}{52.8} = 0.104$$

We show, in Table 4, some statistics of rally point system which include some other cases.

Table 4 Some statistics for rally point system with various goal points.

G	T	R
25	44	0.114
30	53	0.104
35	62	0.095
40	70	0.090
50	88	0.080
60	106	0.073

For the side-out scoring system, we try to simulate to obtain the data since we have no real data. We assume some different *scoring percentage* γ for the side of serving the ball. From the previous results it is estimated that we have the final score 15 : 11.26. The game progress values for some different parameters can be calculated and shown in Table 5.

Table 5 Some statistics for side-out system with 15 points (simulation).

scoring γ (%)	total score	R
60	43.77	0.088
50	52.52	0.074
40	65.65	0.059
37	72.0	0.054
33	86.66	0.045
25	105.04	0.037

We suspect based on our experience that the scoring percentage $\gamma = 50$ holds in the side-out system with 15 points Volleyball when both opposing teams are well balanced. Hence, we take the $R_{25pts} = 0.074$ for representing the game-refinement measure of 15 points side-out scoring.

We show, in Table 6, the comparison between three Volleyball variants based on the game-refinement measure.

Table 6 Game-refinement measures for three variants of Volleyball.

variants	points	R
side-out scoring system	15	0.074
rally point system	30	0.104
rally point system	25	0.114

The side-out scoring system (with scoring rate roughly $\gamma = 50$) had been played long time (1947-1999). The game-refinement value is similar with other sophisticated games such as Chess and Soccer. However, the rule was changed in 1999 to improve game understandability. At the same time, the rule change has made rise in excitement, depicted in higher value of game-refinement value.

The study using board games suggests that the game-refinement measure of sophisticated games are somewhere in the range $0.07 - 0.08$[3]. The higher R value in the current volleyball rules might mean the game become more exciting, but it is not alligned well with our previous classification of sophisticated game. Thus, it can be assumed that this rule changes might not the optimum method to improve the game attractiveness in the *comfortable range*.

4. Concluding Remarks

We introduced the framework of game-refinement theory, while showing a model of game information progress for various games such as sports games, video games and board games. Then the second derivative, which is the acceleration in the sense of dynamics, was derived from the model to use the value as a game refinement measure. This is because the acceleration of game information progress should relate to the emotional impact such as entertainment and engagement which may correspond to the force in physics. While applying some data from Volleyball variants selected, we considered the evolutionary history of rule changes.

The rule change from "15 points side-out scoring system" to "25 points rally point system" shows the rise of game-refinement measure (0.074 to 0.114) with expectation that the game would be more attractive. However, we observed the existence of appropriate range or zone of game-refinement values (0.07 to 0.08) in the early study using board games. We have not yet reached the conclusion at this moment, but it is assumed that it would not be appropriate or comfortable if the measurement value is out of the range.

Moreover, through this study it is found that the proposed measure of game refinement seems promising in the domain of score-limited sports games as well as time-limited sports games, and that the appropriate zone of game-refinement measure may be effective in any type of sophisticated games including board games and sports games.

5. Future Works

To ensure the accuracy of this study, we need to study real data of the other volleyball matches with different rules, which is currently missing in this study. This paper still used the simulated data from one kind match, therefore the result from each rule quite similar.

References

1) Iida, H. (2000). Three Master's Model, Game Programming Workshop in Japan, Hakone.
2) Iida, H., Takeshita, N., Yoshimura, J. (2002). A Metric for Entertainment of Boardgames: its implication for evolution of chess variants. In: Nakatsu, R., Hoshino, J. (eds.) IWEC 2002 Proceedings, pp. 65–72. Kluwer
3) Iida, H., Takehara, K., Nagashima, J., Kajihara, Y., Hashimoto, T. (2004). An application of game refinement theory to Mah-Jong. In: International Conference on Entertainment Computing (ICEC2004), LNCS 3166, pp. 333–38.
4) Iida, H., Nakagawa, T., Spoerer, K. (2012). Game information dynamic models based on fluid mechanics. Entertainment Computing, 3(3): 89–99
5) Majek, P. and IidaH. (2004). Uncertainty of Game OutcomeProceedings of Inter Academia, 3rd Int. Conf. on Global Research and Education in Intelligent Systems. Budapest, Hungary171-180
6) Murray, H. J. R. (1913) A History of Chess, Oxford University Press
7) Nash, J. (1950). Equilibrium points in n-person games. Proceedings of the National Academy of Sciences 36(1):48-49.
8) NBA. Nba 2012/13 team statistics. URL: http://stats.nba.com. Accessed:30-10-2013.
9) Neumann, J. von and Morgenstern, O. (1944). Theory of Games and Economic Behavior. Second Edition 1947. Princeton University Press,

Princeton.
10) Official Volleyball Rules 2011-2012. FIVB. 2010. Retrieved 2011-10-27.
11) Sutiono, A.P., Purwarianti, A. and Iida, H. (2014). A Mathematical Theory of Game Refinement. submitted.
12) UEFA. Uefa 2012/13 team statistics. URL: http://www.uefa.com/uefachampionsleague/season=2013/statistics Accessed: 8-11-2013.
13) "Volleyball". International Olympic Committee. Retrieved 2007-03-21.
14) "2008 Major Rules-Change Proposals". ncaa.org. National Collegiate Athletic Association. 2008-07-14. Retrieved 2008-10-12.
15) V-League Official Site, http://www.vleague.or.jp/