

## Digital Computer Solution of Power-Flow Problems

TAZUMI DEIDO\*, KAZUO MOGI\*, TATSUKI NORIMATSU\*\*,  
HISAO FUJIKI\*\*\* AND TOKUYA KOBAYASHI\*\*\*

This paper presents the procedures, developed by the authors using a medium-sized digital computer (NEAC-2203), for the calculation of the power-flows necessary to plannings and operations of electric power transmission and distribution systems. The computer program described herein utilizes the characteristics of power systems to save the number of storage required. As the result, the maximum size of power systems acceptable by this program is increased up to, for example, 100 nodes, 210 branches and 14 off-nominal transformers from 28, 64 and 5, respectively, which are the maximum numbers acceptable by the old program organized for the same machine by the authors after the model of the reference [1].

### 1. Power-Flow Problem

The power-flow problem is to seek the node voltages and currents which fill the following two conditions simultaneously—(a) voltages and currents in any part of the circuit satisfy Ohm's law and Kirchoff's law, and (b) the node voltages and currents are subject to the constraints prescribed at every node.

The condition (a) is expressed by the following simultaneous complex equations in nodal method,

$$\dot{I}_k = \sum_{m=1}^N \dot{Y}_{km} \dot{E}_m, \quad k=1, 2, \dots, N, \quad (1)$$

where  $\dot{I}_k$  is the  $k$ -th node current,  $\dot{E}_m$  is the  $m$ -th node voltage,  $\dot{Y}_{km}$  is the  $k$ -th node self-admittance ( $k=m$ ) or mutual admittance between the  $k$ -th and the  $m$ -th node ( $k \neq m$ ), and  $N$  is the total number of nodes in the system.

As the condition (b), usually, the following three kinds are used, and to each node is prescribed one of them:

$$(b-1) \quad P_k + jQ_k (= \dot{E}_k \dot{I}_k^*) \text{ is specified,} \quad (2)$$

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\* Electrotechnical Laboratory, Agency of Industrial Science and Technology, Ministry of International Trade and Industry. \*\* formerly Electrotechnical Laboratory.

\*\*\* Yokogawa Electric Works, Ltd., Tokyo.

$$(b-2) \quad P_k \text{ (real part of } \dot{E}_k \dot{I}_k^* \text{) and } |\dot{E}_k| \text{ are specified,} \quad (3)$$

$$(b-2) \quad \dot{E}_k \text{ is specified,} \quad (4)$$

where  $\dot{I}_k^*$  is the conjugate of  $\dot{I}_k$  and  $j$  is  $\sqrt{-1}$ .

The above equation (1) with the constraints (2)~(4) are usually solved by the iterative procedure.

## 2. Feature of the New Program

(i) The connection table is adopted to input the system configuration, whereas the connection matrix was used in the old program. The connection matrix scheme seems to be elegant but it is too tedious and mistakable to describe the matrix and to punch cards or a tape.

(ii) The scheme of omitting zero-elements is used to store the admittance matrix ( $\dot{Y}_{km}$  in (1)), in which all non-zero elements of the matrix are stored and zero-elements are not. In the old program the triangular storing scheme has been used utilizing the symmetry of the matrix.

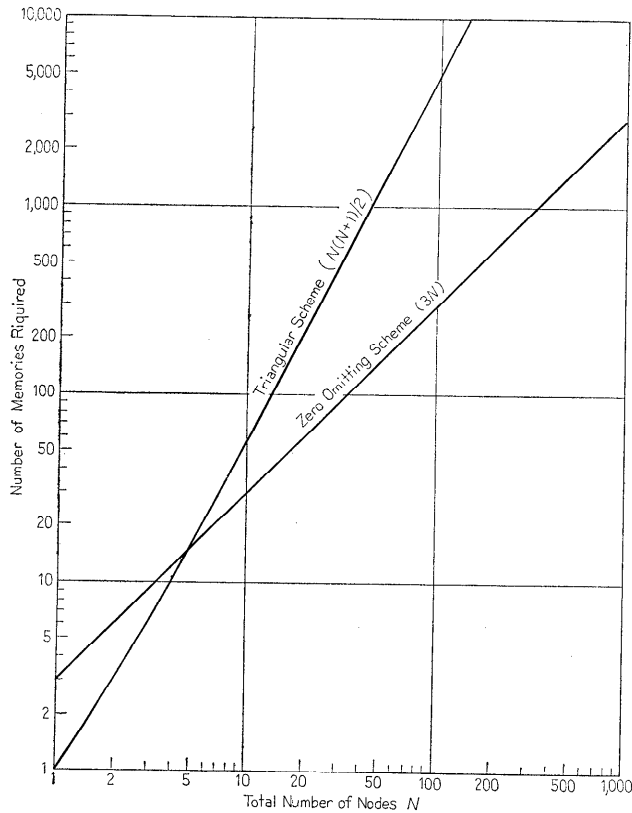


Fig. 1. Comparison of number of memories required for storing the admittance matrix by the both schemes

As the number of branches connected with each node is independent of the system size and is about two on an average in an electric power system, there are about three non-zero elements in each row of the admittance matrix and the other elements are all zero. Consequently, the number of memories required to store the admittance matrix in the zero-element omitting scheme is about three times of the total number of nodes  $N$ , whereas in the triangular scheme it is  $N(N+1)/2$ . Fig. 1 shows the effectiveness of this improvement being remarkable.

(iii) The zero-element omitting scheme for storing the admittance matrix has one more great advantage that a large number of useless computations of multiplication of  $\dot{E}_k$  by zero-elements of  $\dot{Y}_{km}$  in equation (1) are also omitted. And the logical operation for the computation of equation (1) in this scheme is simple and is performable rapidly using the address table, in which the first address numbers of the rows of  $\dot{Y}_{km}$  stored in the order of  $k$  are described and which is tabulated automatically by the computer before the major computation.

The computing time required is much reduced by adopting the zero-element omitting scheme as shown in Fig. 2.

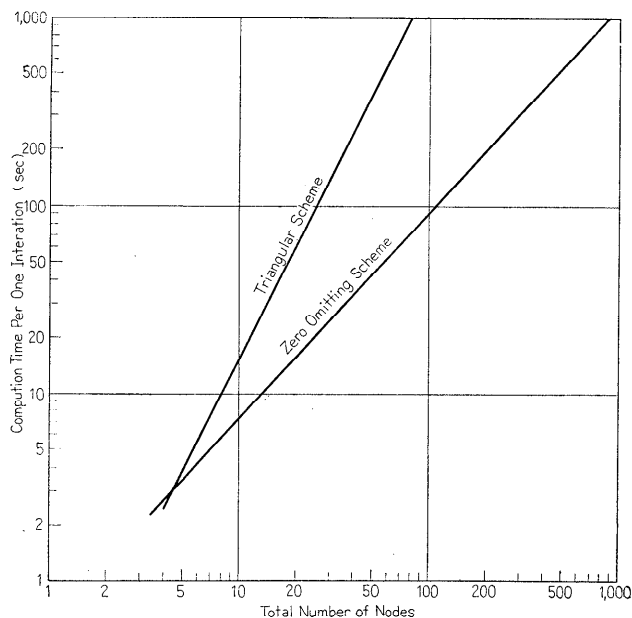


Fig. 2. Comparison of computing time by the both schemes for storing the admittance matrix

### 3. Conclusion

(i) The medium sized digital computer (2,000 words memory) can

solve the power-flow problems containing up to about one hundred nodes using this program and this capability is comparable with that of a super large scale analogue machine (A.C. Network Analyser).

(ii) As the computing time is also reduced by this program, the digital solution of power-flow problems can be performed economically even though a medium sized and low speed computer is used.

(iii) The zero-element omitting scheme for storing the matrix may be used for general matrix computations in case of the matrix having a considerably large number of zero-elements and will be effective to save memories and computing time required.

#### *References*

- [1] WARD, J. B., AND H. W. HALE, Digital computer solution of power-flow problems. *A.I.E.E. Transactions, III, 75* (1956), 398-404.