A Curve Fitting by Use of Tchebycheff's q-functions

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Abstract

This paper describes an algorithm of a curve fitting by use of Tchebycheff's q-functions and discusses the accuracy of the fitting in comparison with other methods.

In electronic engineering, V-I characteristics of diodes are often required to be represented in mathematical forms in order to analyze the performance of the circuit incorporating the diode. For this purpose, least squares approximation or so is commonly used, but the accuracy of it is not necessarily good. Therefore, the method using Tchebycheff's q-functions is presented. The features of this method are the sufficient accuracy of the fitting and the capability of making the estimated value at the origin coincide with the actual one so that the physical property of the diode is satisfied. A few examples of the curve fitting are illustrated in the paper.

1. Introduction

Some new semiconductors such as a tunnel diode, a transferred electron diode and so on have found extensive application in many branches of electronics since their appearances. However, an exact analysis of a circuit employing such a semiconductor has not been made, owing to difficulty in representing an extremely nonlinear characteristic of a diode in mathematical terms. Some years ago, it was required to mathematically describe a characteristic of a tunnel diode in order to investigate the behaviours of the diode in an oscillation circuit. Therefore, we made an approximation for it with very good accuracy by use of Tchebycheff's q-functions[(1)] in a manually-operated desk calculator.[(2)]

The purpose of this paper is to describe an algorithm of the curve fitting by a series of Tchebycheff's q-functions on a computer instead of a desk calculator. The procedure of determining the coefficients of the functions and the accuracy of the method, in addition to the property of the functions, are discussed in detail.

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2. Principle of the curve fitting

2.1 Equations for the curve fitting

The Tchebycheff's q-functions are represented by the following equation

$$\overset{*}{q}_\nu(n,\xi) = \frac{1}{\overline{M_\nu(n)}} \sum_{m=0}^{\nu} \binom{\nu+m}{m}\binom{\nu-n}{\nu-m}\binom{\xi}{m} \tag{1}$$

where

$$\overline{M_\nu(n)} = G.C.M.\left(\binom{n-1}{\nu}, \binom{\nu+1}{1}\binom{n-2}{\nu-1}, \ldots, \binom{2\nu-1}{\nu-1}\binom{n-\nu}{1}, \binom{2\nu}{\nu}\right) \tag{2}$$

n: total number of data

ξ: order of uni-interval measuring points

ν: degree of Tchebycheff's q-functions

$\overset{*}{q}_\nu(n,\xi)$: Tchebycheff's q-functions for data n and degree ν at order ξ.

These functions are orthogonal functions with a symmetrical property, as shown in the following expressions

$$\sum_{\xi=0}^{n-1} \overset{*}{q}_s(n,\xi)\cdot\overset{*}{q}_k(n,\xi) = 0 \quad (k \neq s) \tag{3}$$

$$\overset{*}{q}_\nu(n,\xi) = (-1)^\nu \cdot \overset{*}{q}_\nu(n,n-1-\xi) \tag{4}$$

Fig. 1 shows how the curve of Tchebycheff's q-functions changes according to the degree ν of the functions. It is clear that Tchebycheff's q-functions are almost similar to a periodic function with a symmetrical property.

For the first step towards approximation, the data Y_ξ are read out from an observed curve at regular intervals of 1, where the suffix ξ indicates the order of data y. And then, these data are approximated in mathematical terms so that the sum of squared deviations between the observed values and the estimated values by Tchebycheff's q-functions may be as small as possible. The expression is given by eq. (5) and it can be, if necessary, expanded into the polynomial (6)

$$y(x) = \sum_{\nu=0}^{m} a_\nu \cdot \overset{*}{q}_\nu(n,x) \tag{5}$$

$$= \sum_{\nu=0}^{m} c_\nu \cdot x^\nu \tag{6}$$

where

$$a_\nu = \frac{\sum_{\xi=0}^{n-1} y_\xi \cdot \overset{*}{q}_\nu(n,\xi)}{\overset{*}{\Sigma}_\nu(n)} \tag{7}$$

$$\overset{*}{\Sigma}_\nu(n) = \frac{\sum_{\xi=0}^{n-1} [\overset{*}{q}_\nu(n,\xi)]^2}{\left[\overline{M}_\nu(n)\cdot\frac{\nu!}{2^\nu}\right]^2} \tag{8}$$

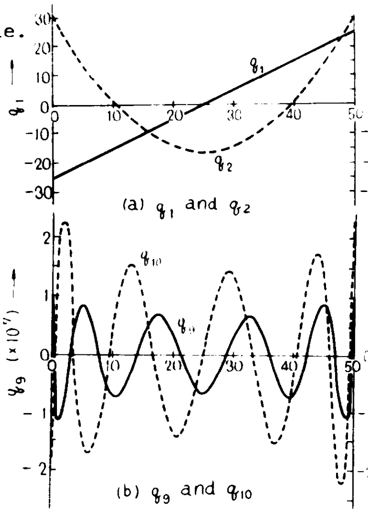

Fig.1 Tchebycheff's q-functions where n=51

The symbol a_ν and m are the coefficient and the degree of the q-functions respectively and c_ν is the coefficient of the νth degree polynomial. The coefficients a_ν are easily determined by use of eq.(7) when the values of $\overset{*}{q}_\nu(n,\xi)$ and $\overset{*}{\Sigma}_\nu(n)$ are previously tabulated.

2.2 Modification to make the estimated value at the origin coincide with the observed one

It is frequently required for a diode to make the estimated value at the origin coincide with the observed value. The procedure of modification is to compensate the differences between the observed and the estimated values in the first-order approximation. The series of y', which are differences between them, are taken as

$$y'_0,\ y'_1,\ .\ .\ .\ .\ ,\ y'_\ell$$

where, the number of the differences to be taken is not necessarily equal to the total number of the data. When the modified value at the origin (by which the coincidence at the origin is obtained) is named x_0, the differences to be approximated by the q-functions become the series of

$$x_0,\ y'_1,\ .\ .\ .\ .\ ,\ y'_\ell$$

The coefficients a'_ν of Tchebycheff's q-functions for the above data are easily determined by the following equation by means of a similar procedure to the first order approximation.

$$a'_\nu=\{x_0\cdot\overset{*}{q}_\nu(\ell,0)+\sum_{\xi=1}^{\ell} y'_\xi\cdot\overset{*}{q}_\nu(\ell,\xi)\}\ /\ \overset{*}{\Sigma}_\nu(\ell) \tag{9}$$

The value at the origin calculated by the q-functions with above coefficients should be equal to y'_0, in order to satisfy the requirement. Therefore, the following equation holds

$$\begin{aligned} y'_0 &= \sum_{\nu=0}^{m} a'_\nu\cdot\overset{*}{q}_\nu(\ell,0) \\ &= x_0\sum_{\nu=0}^{m}\frac{[\overset{*}{q}_\nu(\ell,0)]^2}{\overset{*}{\Sigma}_\nu(\ell)} + \sum_{\nu=0}^{m}\frac{\sum_{\xi=1}^{\ell} y'_\xi\cdot\overset{*}{q}_\nu(\ell,\xi)\cdot\overset{*}{q}_\nu(\ell,0)}{\overset{*}{\Sigma}_\nu(\ell)} \end{aligned} \tag{10}$$

$$x_0=\{y'_0-\sum_{\nu=0}^{m}\frac{\sum_{\xi=1}^{\ell} y'_\xi\cdot\overset{*}{q}_\nu(\ell,\xi)\cdot\overset{*}{q}_\nu(\ell,0)}{\overset{*}{\Sigma}_\nu(\ell)}\}\ /\ \sum_{\nu=0}^{m}\frac{[\overset{*}{q}_\nu(\ell,0)]^2}{\overset{*}{\Sigma}_\nu(\ell)} \tag{11}$$

Using x_0 determined by eq.(11), we can modify the first order approximation so as to make the estimated value at the origin coincide with the observed one. The real coefficients of Tchebycheff's q-functions to represent the data are the sum of the coefficients in the first and the second order approximations.

3. Computer programming

If the amount of the data is too large, or if the estimated equations are of a high degree, then it may be necessary to previously tabulate the q-functions to save a computer memory. Because of the small capacity of the computer available to us, Tchebycheff's q-functions of n=51 and ν=10 were previously tabulated. Fig.2 shows a flow chart of the program, which is composed of a number of subroutine programs.

For the purpose of obtaining better approximations, the modifications for the differences between estimated and observed data were repeated several times. However, even when the modifications were repeated more than three times, the higher accuracy for the approximation was not achieved. Therefore, it seems effective for a good approximation to repeat the modification two times, or at least no more than three times.

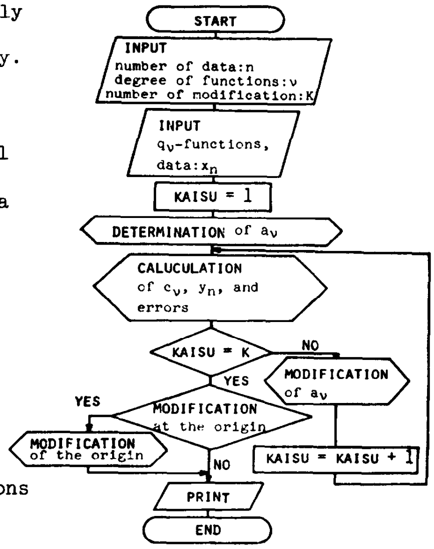

Fig.2 Flow chart of the curve fitting by use of Tchebycheff's q-functions

4. Computer experiment

For evaluation of accuracy and effectiveness of a curve fitting by Tchebycheff's q-functions, a static characteristic of a tunnel diode and a pulse waveform were approximated by the q-functions, and then the results were compared with those by two other common methods: least squares approximation and Fourier series approximation.

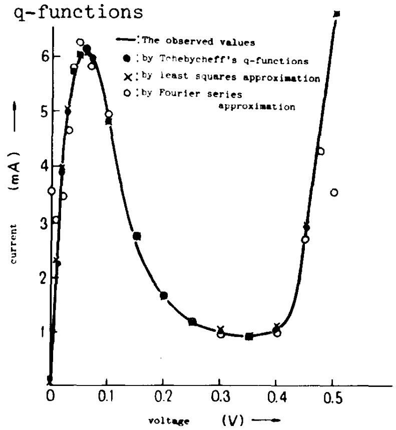

Fig.3 Curve fitting of a characteristic of tunnel diode

Example 1. Approximation of a static characteristic of a tunnel diode

A static V-I characteristic of a tunnel diode as shown in Fig.3, was approximated by the three methods. For the approximations, the characteristic was sampled at regular intervals of 51. The results of these approximations are also shown by the plotted symbols in Fig.3. In order to make the accuracy more clear, the percentage errors for three approximations are shown in Fig.4. It is seen from these results that

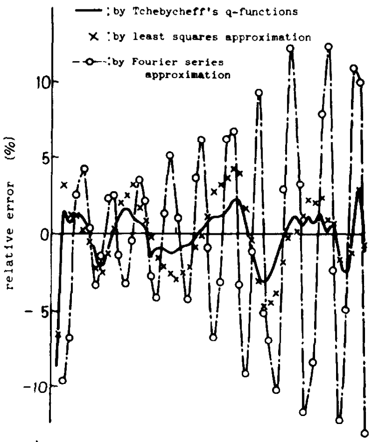

Fig.4 Comparison of the accuracy of the fitting among ones by other methods

the method by Tchebycheff's q-functions is the most accurate method among them.

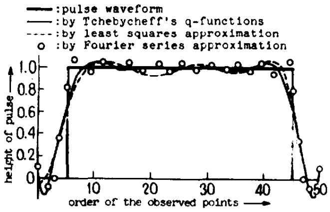

Fig.5 Curve fitting of a pulse waveform

Example 2. Approximation of a pulse waveform

As the next example, an approximation was performed for a pulse waveform as shown in Fig.5. From the original waveform, the data of 51 was read out at regular intervals by the same procedure for the previous example, and then for this data, the approximation was carried out by the three methods. Fig.5 shows the results of these approximations. It is apparent that the best accuracy is also obtained by Tchebycheff's q-functions. Furthermore, the modification at the origin was performed by the q-functions with a satisfactory agreement.

Not only will this method be useful for a pulse waveform, but it also has good approximation for different waveforms. Hence, Tchebycheff's q-functions seem to have an advantage in curve fitting over the other methods.

5. Conclusion

A new method is presented to describe an observed nonlinear voltage-current characteristic of a semiconductor in mathematical terms by use of Tchebycheff's q-functions. This methos is not only useful to approximate such a characteristic but also to represent other data in a polynomial. Main features of this method are summarized as follows

(1) The observed data can be more closely represented in mathematical terms by Tchebycheff's q-functions than other common methods. The estimated Tchebycheff's q-functions are, if necessary, arranged in a polynomial expression with the same degree.

(2) It is easily performed to make an estimated value coincide with an observed datum at any point, for instance at the origin.

We would like to thank Dr. Ziro Yamauti for kindly suggesting to us the possibility of curve fitting by Tchebycheff's q-functions.

Reference

(1) Ziro Yamauti: "On Tchebycheff's q-function" Report of ETL (Japan), No. 454, pp.1 - 29 (1941)

(2) M. Goto, Y. Komamiya, H. Tajima and M. Morisue: "Theoretical analysis of non-linear oscillation in distributed constant circuit connected with an Esaki-diode" Proc. of IECE of Japan, vol. 28, No.4, pp.599 - 607 (1964)