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Coupled Switching Linear Model を利用した複雑なジェスチャー認識

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本論文では複数のスイチング線形モデルを Coupling させる手法を述べる。 Coupling は各 switching linear model の離散状態変数の間の条件付確率を導入して行う。EM アルゴリズムでモデルのパラメータを学習し、追跡は、あのパラメータを使って coupled-forward アルゴリズムで行われる。いくつのモデルを用意して，毎時間尤度を計算して一番高い値を持つモデルを選ぶことで，認識を行う。この手法を両手ジェスチャーの追跡と認識に適用することを示す。

Complex gesture recognition using coupled switching linear model

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Abstract

We present a method coupling multiple switching linear models. The coupled switching linear model is an interactive process of two switching linear models. Coupling is given through causal influence between their hidden discrete states. The parameters of this model are learned via EM algorithm. Tracking is performed through the coupled-forward algorithm based on Kalman filtering and a collapsing method. A model with maximum likelihood is selected out of a few learned models during tracking. We demonstrate the application of the proposed model to tracking and recognizing two-hand gestures.

1. Introduction

Gesture recognition plays an important role in a host of man-machine interaction applications. A well-known method in gesture recognition is HMM(Hidden Markov Model), which is essentially a quantization of time series (observation sequence) into a small number of discrete states with transition probabilities between states. In HMM-based gesture recognition schemes, there are two bottlenecks: First, it has a limitation to treat with trajectory information in time, since HMM are based on distributions of statistically independent observations,

Many interesting man-machine interfaces are composed of multiple interacting processes. This is typically the case for systems that have structure both in time and space [Brand,1996]. The second bottle- neck is that HMM is ill-suited to this problem because it has a single state variable (hidden discrete states).

Switching linear dynamic model has been developed in fields ranging from econometrics to engineering. It combines the discrete transition structure of HMM with the stochastically linear dynamic model of state-space model. Therefore, it may be able to overcome the first bottleneck of HMM.

Our goal is to track and recognize two-hand gestures simultaneously. Assuming the process of two-hand gestures is an interacting process of one-hand gestures, as noted, we cannot depend on HMM. CHMM(coupled hidden Markov model) [Brand] gives a method to capture these interactions. However, since CHMM inherits HMM it also meets a ceiling in case considering trajectory information in time.

To overcome the two bottlenecks in HMM-based gesture recognition, it is worth to make an interacting process of some switching linear models. We present algorithms for coupling switchi- ng linear dynamic models. Two switching linear dynamic models are coupled through causal influences between their hidden discrete states. We introduce a collapsing method approximately solving the state-estimation problem to avoid the explosive increase in computational cost[Harrison and Stevens,1976] [Gordon and Smith,1988] [Kim,1994].

We use active contour model [Blake and Isard,1998] for the representation of motion and shape of hands and demonstrate an application of the coupled switching dynamic model to tracking and recognizing two-hand complex gestures.

The paper is organized as follows. In the following section we address the coupled switching linear model. In section 3, we explain an active contour model shortly to be introduced to repre- sent motion and shapes of hands. In section 4, we describe the EM learning algorithm for the coupled switching line- ar model. In section 5 and 6, we demon- strate the application of the coupled switching linear model to tracking and recognizing two-hand gestures. Finally, we conclude with section 7.

2. Coupled Switching Linear Dynamics

Human describes any meaning by changing shapes of each hand besides moving positions of two hands. To model these two-hand gestures, we have to consider shapes and motions of hands and interactions between two hands. Assuming that a two-hand gesture is an interacting process of two hands whose shapes and motions are described by the switching linear dynamics, it deserves to couple switching linear dynamic models to capture interactions between two hands.

2.1 Model specification

Switching linear model can be seen as a hybrid model of the linear state-space model and HMM. It is described using the following set of state-space equations:

$$
\begin{aligned}
& x_t = F_{m_t} x_{t-1} + u_t,\ u_t \sim N(0, Q_{m_t}) \\
& z_t = H_{m_t} x_t + v_t,\ v_t \sim N(0, R_{m_t}) \qquad (2.1) \\
& \Pi_{m_t, m_{t+1}} = p(m_{t+1} \mid m_t) \\
& \pi_{m_1} = p(m_1)
\end{aligned}
$$

In the above equation, z_t means an observation at time t and is statistically independent from all other observation vectors. x_t is a hidden continuous state variable. m_t denotes hidden discrete states obeying the first order Markov process. π_{m_1}, F_{m_t} and H_{m_t}, which are typical parameters of linear dynamic model, denote the prior probability of a discrete state, the continuous state tran- sition matrix and the observation matrix, respectively. The noise u_t and v_t are independently distributed on each Gaussian distribution. m_t-subscripted parameters of the above model are dependent on discrete state variable m_t. And the switching process is defined with the discrete state transition matrix Π. This model can be expressed graphically in the form of Figure 2.1.

Coupled switching linear model is an interactive process of two switching linear models. Coupling is given through causal influence between their hidden discrete states. The complex state space representation is equivalently depicted by dependency graph in Figure 2.2. To accommodate another interacting process, it seems good to consider a single lumped system joined into multiple state variables. However there are a few ceilings: The computational cost is prohibitive, a surfeit of parameters leads to overfitting, and there is often insufficient data for a large number of states, usually resulting in undersampling and numerical under- flow errors [Brand,1996]. Consequently the suggested coupling scheme, as shown in Figure 2.2, offers conceptual advantages of parsimony and clarity with computational benefits in efficiency and accuracy. That is revealed in the following sections.

In the coupled switching linear model, since coupled transitions of discrete states have Markov process, it follows that

$$p(m_t,n_t \mid m_1,\Lambda,m_{t-1},n_1,\Lambda,n_{t-1}) = p(m_t,n_t \mid m_{t-1},n_{t-1})$$

Assuming

$$p(m_t,n_t \mid m_{t-1},n_{t-1}) \propto p(m_t \mid m_{t-1}) \cdot p(m_t \mid n_{t-1}) p(n_t \mid n_{t-1}) p(n_t \mid m_{t-1}) \quad (2.2)$$

referred to in [Brand,1996], coupling transition probability of discrete states can be parameterized as

$$p(m_t,n_t \mid m_{t-1},n_{t-1}) = k_c \Pi_{m_{t-1}m_t} \Gamma_{n_{t-1}m_t} \hat{\Pi}_{n_{t-1}n_t} \hat{\Gamma}_{m_{t-1}n_t} \quad (2.3)$$

where k_c is a normalizing constant, Γ is the state transition matrix representing causal influences between two switching linear system, and superscript ^ denotes the lower switching linear system in Figure 2.2.

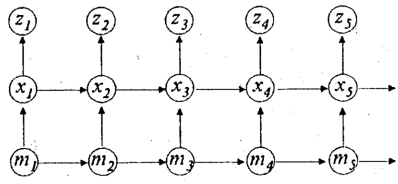


Figure 2.1 Switching linear model

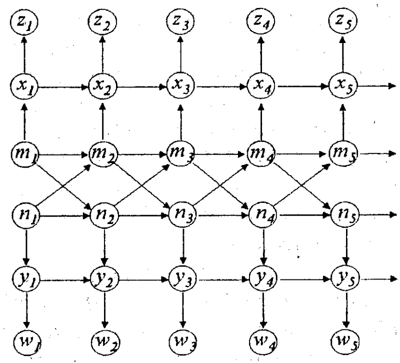


Figure 2.2 Coupled switching linear model
n_t, y_t and w_t denote a discrete state variable, a continuous state variable and an observation vector, respectively

2.2 Coupled-forward Algorithm

Given known parameters of the coupled switching linear model, $\{F,H,Q,R,\pi,\Pi,\Gamma\}$, $\{\hat{F},\hat{H},\hat{Q},\hat{R},\hat{\pi},\hat{\Pi},\hat{\Gamma}\}$, we can perform tracking or filtering, which means estimations of continuous states and coupled-jointed probabilities of hidden discrete states. In this section we describe a filtering method called as the coupled-forward algorithm.

Following [Kim,1994], given the known parameters of switching linear dynamics, the predicted jointed-continuous state variable and the corresponding covariance are defined dependently on m_t and m_{t-1}:

$$x^{(i,j)}_{t|t-1} = F_j x^{(i)}_{t-1|t-1}$$
$$P^{(i,j)}_{t|t-1} = F_j P^{(i)}_{t-1|t-1} F_j' + Q_j \quad (2.4)$$

where $x^{(i)}_{t-1|t-1}$ and $P^{(i)}_{t-1|t-1}$ are estimations at time $t-1$ based on information up to time

$t-1$, given $m_{t-1}=i$. Now the filtered jointed-continuous state $x_{t|t}^{(i,j)}$ and its covariance $P_{t|t}^{(i,j)}$ are estimated by the conventional Kalman updating algorithm. In particular, we follow Kalman filtering application of [Blake,Isard and Rubin, 1995] and [Blake and Isard, 1998] to active contour model.

From the above fact, as noted by [Gordon and Smith,1988], switching linear dynamic model requires computing a Gaussian mixture with M^t components at time t for M switching states. If coupled with a N-switching linear system, typically $(MN)^t$ computations are required (M^t+N^t in the case of the presented coupled switching linear model) which is clearly intractable for moderate sequence length. It is necessary to introduce some approximations to solve the intractable computation problem.

We collapse M^2+N^2 jointed continuous state variables into $M+N$ state variables at each time, and can avoid prohibitive increase of computational cost. Building upon ideas introduced by [Harrison and Stevens,1976], [Gordon and Smith,1988] and [Kim,1994], we present the following collapsing method:

Through the paper, expediently we proceed by evolving equations only in terms of the upper system in Figure 2.2.

$$x_{t|t}^{(j)}=\frac{\sum_{i=1}^{M}\left(\sum_{ii=1,jj=1}^{N}p\begin{pmatrix}m_{t-1}=i,n_{t-1}=ii,\\ m_t=j,n_t=jj\mid O_t\end{pmatrix}\cdot x_{t|t}^{(i,j)}\right)}{p(m_t=j\mid O_t)} \quad (2.5)$$

$$P_{t|t}^{(j)}=\frac{\sum_{i=1}^{M}\begin{pmatrix}\sum_{ii=1,jj=1}^{N}p(m_{t-1}=i,n_{t-1}=ii,m_t=j,n_t=jj\mid O_t)\\ \cdot\left(P_{t|t}^{(i,j)}+(x_{t|t}^{(j)}-x_{t|t}^{(i,j)})(x_{t|t}^{(j)}-x_{t|t}^{(i,j)})'\right)\end{pmatrix}}{p(m_t=j\mid O_t)}$$

where O_t is a sequence (o_1,o_2,Λ,o_t) and o_t is an observation vector (z_t,w_t). In the above collapsing, the coupled-jointed probability of discrete states plays a role of weighting factor of jointed- -continuous state variables. To complete the collapsing, we have only to calculate the weighting factor. Now we present the coupled-forward algorithm:

The filtered coupled-jointed distribution of discrete states is defined by

$$\begin{aligned}&p(m_{t-1},n_{t-1},m_t,n_t\mid O_t) &(2.6)\\ &=k_t p(o_t\mid m_{t-1},n_{t-1},m_t,n_t,O_{t-1})\,p(m_{t-1},n_{t-1},m_t,n_t\mid O_{t-1})\\ &=k_t p(z_t\mid x_{t|t-1}^{(m_{t-1},m_t)})p(w_t\mid y_{t|t-1}^{(n_{t-1},n_t)})\,p(m_{t-1},n_{t-1},m_t,n_t\mid O_{t-1})\end{aligned}$$

where k_t is a normalizing constant, Z_{t-1} and W_{t-1} are the observation sequences up to time $t-1$ in the upper and lower system, respectively in Figure 2.2. From (2.3) the prediction step given sequence up to time t gives

$$p(m_t,n_t,m_{t+1},n_{t+1}\mid O_t)=k_p\Pi_{m_tm_{t+1}}\Gamma_{n_tm_{t+1}}\hat{\Pi}_{n_tn_{t+1}}\hat{\Gamma}_{m_tn_{t+1}}\cdot\sum_{m_{t-1}=1,n_{t-1}=1}^{M,N}p(m_{t-1},n_{t-1},m_t,n_t\mid O_t) \quad (2.7)$$

where k_p is a normalizing constant. And then the probability of predicted coupled-discrete states is calculated by marginalizing the predicted coupled-jointed distribution of discrete states

$$p(m_{t+1},n_{t+1}\mid O_t)=\sum_{m_t=1,n_t=1}^{M,N}p(m_t,n_t,m_{t+1},n_{t+1}\mid O_t) \quad (2.8)$$

Since the coupled-forward algorithm is iterative process as shown in (2.6) and (2.7), it is necessary to define the predicted distribution $p(m_1,n_1,m_2,n_2\mid O_1)$ at initial time in (2.6). It follows that

$$\begin{aligned}&p(m_1,n_1,m_2,n_2\mid O_1)=\\ &\quad p(m_1,n_1\mid O_1)p(m_2,n_2\mid m_1,n_1) \quad (2.9)\end{aligned}$$

By Bayesian rule, the initial distribution of discrete states, $p(m_1,n_1\mid O_1)$ gives

$$\begin{aligned}p(m_1,n_1\mid O_1)&\propto p(O_1\mid m_1,n_1)p(m_1,n_1)\\ &=p(z_1\mid m_1)p(w_1\mid n_1)p(m_1)p(n_1) \quad (2.10)\end{aligned}$$

Substituting (2.3) and (2.10) into (2.9) gives

$$\begin{aligned}p(m_1,n_1,m_2,n_2\mid O_1)&=k_i\pi_{m_1}\hat{\pi}_{n_1}p_{m_1}(z_1)\hat{p}_{n_1}(w_1)\\ &\cdot\Pi_{m_1m_2}\Gamma_{n_1m_2}\hat{\Pi}_{n_1n_2}\hat{\Gamma}_{m_1n_2} \quad (2.11)\end{aligned}$$

where k_i is a normalizing constant.

The above algorithm can be extended up to more coupled models easily. However, since the coupled-forward algorithm has the complexity $O(TC_1{}^2\Lambda C_n{}^2)$ where T is the length of an observation sequence and C_n is the number of states of each switching linear model participating in the n-coupled switching linear model, the computation cost increases sharply as the number of coupling increases. In the case of $n=2$ considered in this paper, we have no problem in terms of computational cost only if M and N have reasonable length.

2.3 Coupled -backward Algorithm

While the coupled-forward algorithm is a filtering process given sequence up to current time, the coupled-backward algorithm is a smoothing process given sequence of full length. Like the conventional Kalman smoothing method, jointed-continuous state variable and its covariance based on full sequence can be smoothed as follows:

Given $m_t = j$ and $m_{t+1} = k$,

$$x_{t|T}^{(j,k)} = x_{t|t}^{(j)} + \tilde{P}_t^{(j,k)}(x_{k+1|T}^{(k)} - x_{k+1|t}^{(j,k)}) \quad (2.12)$$

$$P_{t|T}^{(j,k)} = P_{t|t}^{(j)} + \tilde{P}_t^{(j,k)}(P_{k+1|T}^{(k)} - P_{k+1|t}^{(j,k)})\tilde{P}_t'^{(j,k)}$$

where $\tilde{P}_t^{(j,k)} = P_{t|t}^{(j)} F_k'(P_{t+1|t}^{(j,k)})^{-1}$

To calculate the smoothed continuous state variable and its covariance, given that $m_t = j$, collapsing is performed similarly to (2.5):

$$x_{t|T}^{(j)} = \frac{\sum_{k=1}^{M}\left(\sum_{jj=1,kk=1}^{M,N} p(m_t = j, n_t = jj, m_{t+1} = k, n_{t+1} = kk \mid O_T) \cdot x_{t|T}^{(j,k)}\right)}{p(m_t = j \mid O_T)} \quad (2.13)$$

$$P_{t|T}^{(j)} = \frac{\sum_{k=1}^{M}\left(\sum_{jj=1,kk=1}^{M,N} p(m_t = j, n_t = jj, m_{t+1} = k, n_{t+1} = kk \mid O_T) \cdot \left(P_{t|T}^{(j,k)} + (x_{t|T}^{(j)} - x_{t/T}^{(j,k)})(x_{t|T}^{(j)} - x_{t|T}^{(j,k)})'\right)\right)}{p(m_t = j \mid O_T)}$$

Then we can estimate the smoothed state by taking a weighted average over the discrete states at time t from

$$x_{t|T} = \sum_{j=1}^{M} p(m_t = j \mid O_T) x_{t|T}^{(j)} \quad (2.14)$$

To complete (2.13) and (2.14), we turn to derivation of the smoothed coupled-jointed distribution of discrete states, which is given by

$$p(m_t, n_t, m_{t+1}, n_{t+1} \mid O_T) = p(m_t, n_t, m_{t+1}, n_{t+1} \mid O_t) \cdot \frac{p(m_{t+1}, n_{t+1} \mid O_T)}{p(m_{t+1}, n_{t+1} \mid O_t)} \quad (2.15)$$

From (2.15) the smoothed coupled distribution of discrete states is given by

$$p(m_t, n_t \mid O_T) = \sum_{m_{t+1}, n_{t+1}} p(m_t, n_t, m_{t+1}, n_{t+1} \mid O_T) \quad (2.16)$$

From (2.16) the smoothed distribution of discrete states is obtained by

$$p(m_t \mid O_T) = \sum_{n_t} p(m_t, n_t \mid O_T) \quad (2.17)$$

$p(m_{t+1}, n_{t+1} \mid O_T)_{t=T-1}$ and $p(m_{t+1}, n_{t+1} \mid O_t)$ already has been computed from (2.15) in the coupled-forward algorithm.

2.4 Likelihood of the Coupled switching linear model

The coupled switching linear model can be represented by the parameter set, λ, which consists of $\{F, H, Q, R, \pi, \Pi, \Gamma\}$ and $\{\hat{F}, \hat{H}, \hat{Q}, \hat{R}, \hat{\pi}, \hat{\Pi}, \hat{\Gamma}\}$. Likelihood of λ given an observation sequence can be calculated by

$$L(\lambda \mid O_\tau) = p(O_\tau \mid \lambda) = \prod_{t=1}^{\tau} p(o_t \mid O_{t-1}, \lambda) \quad (2.18)$$

Abbreviating λ,

$$p(o_t \mid O_{t-1}) = \sum_{\substack{m_t, m_{t-1}, \\ n_t, n_{t-1}}} \left(\begin{array}{l} p(m_{t-1}, n_{t-1}, m_t, n_t \mid O_{t-1}) \\ \cdot\, p(o_t \mid m_{t-1}, n_{t-1}, m_t, n_t, O_{t-1}) \end{array} \right) \quad (2.19)$$

Substituting (2.6) and (2.19) into (2.18), log-likelihood $\tilde{L}$ is obtained by

$$\tilde{L} = \sum_{t=1}^{\tau} \log\left(\sum_{\substack{m_t, m_{t-1}, \\ n_t, n_{t-1}}} \frac{p(m_{t-1}, n_{t-1}, m_t, n_t \mid O_t)}{k_t} \right) \quad (2.20)$$

where both the denominator and the numerator have been computed in the coupled-forward algorithm.

3. Active Contour Model

We intend to track and recognize two-hand gestures and apply active contour model to represent a variety of shapes of hands. Active contour model using B-spline parameterization was well established by [Blake and Isard,1998]. A curve is parameterized into a control vector, which is composed of B-spline control point coordinates. Control vectors may be described on an specific shape space, however, typically, it takes high degrees of freedom to represent complex shapes of hands. Since it is undesirable for robust contour tracking or fitting, we perform principal component analysis to derive a small shape space of contours, which is constructed as follows:

Given a training set, $\{\hat{Q}_1, \hat{Q}_2, \Lambda\ \hat{Q}_M\}$, of control vectors representing outlines of hands, it follows that

$$Q = WX + \overline{Q}, \quad \overline{Q} = \sum_{i=1}^{M} \hat{Q}_i \qquad (3.1)$$

where columns of W are eigenvectors of the covariance matrix of a given training sequence, X with a reduced dimension is called as a shape vector of shape space and correspond to an approximated control vector, Q. We use shape vectors as the continuous state variables of the coupled switching linear dynamics.

In applying active contour model to linear dynamics, we need not estimate an observation matrix, H in (2.1) and have only to compute observation probability in (2.6). It is assumed that sampled points on the contour line of a predicted state vector (shape vector), have normal distributions along normal lines at each point on the contour line. Then the observation probability is computed by normal displacement between sample points and observed edges on normal lines [Blake and Isard,1998].

4. Learning via EM

EM algorithm is a general iterative technique for finding maximum likelihood parameter estimates in problems where some variables were unobserved [Dempster,1977] [Neal and Hinton,1998]. It is natural to use EM algorithm for our problem, in which unobserved variables are continuous state variables and discrete state variables. Assume that the probability density for observation sequence is parameterized using λ, $p(O_T \mid \lambda)$.
Then log-likelihood is

$$L(\lambda \mid O_T) = \log p(O_T \mid \lambda) \qquad (4.1)$$
$$= \log \sum_{M_T, N_T} \int_{X_T, Y_T} p(M_T, N_T, X_T, Y_T, O_T \mid \lambda) dX_T dY_T$$

where M_T, N_T and X_T, Y_T are sequences of discrete states and continuous states, respectively. [Dempster, 1977] shows the auxiliary log-likelihood is given by

$$\tilde{L} = \sum_{M_T, N_T} \int_{X_T, Y_T} \begin{array}{l} p(M_T, N_T, X_T, Y_T \mid O_T, \overline{\lambda}) \\ \cdot \log p(M_T, N_T, X_T, Y_T, O_T \mid \lambda) dX_T dY_T \end{array} \qquad (4.2)$$
$$= E_{p(M_T, N_T, X_T, Y_T \mid O_T, \overline{\lambda})}[\log p(M_T, N_T, X_T, Y_T, O_T \mid \lambda)]$$

where $\overline{\lambda}$ is previously estimated parameter set. EM algorithm starts with some initial guess and proceed by repeatedly applying the following two steps:

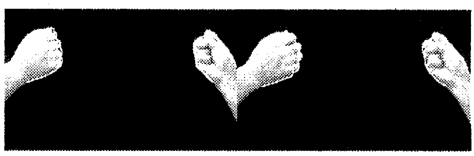
E step : Estimate hidden continuous states and coupled-jointed probabilities of hidden discrete states given observation sequence of full length. These estimations are performed through the filtering and smoothing process described in section 2.2 and 2.3

M step : Estimate λ maximizing $E_{\overline{p}}[\log p(M_T, N_T, X_T, Y_T, O_T \mid \lambda)]$, which is approximately expressed by λ and the estimations from E step. Through M step parameters are updated iteratively until likelihood value converges.

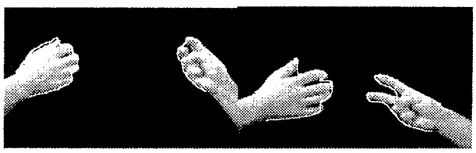
5. Application to tracking two-hand gestures

We applied the presented coupled switching linear model to tracking two-hand gestures. An outline of hands is expressed as a shape vector of the shape space, which is constructed by PCA using sequence of hand contours as training set. We get the parameter set dominating a two-hand gesture through EM algorithm.

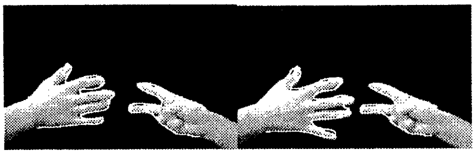
Tracking is performed through the coupled-forward algorithm. Figure 5.1 shows tracked hands corresponding to each frame. We calculated the probability of each discrete state with respect to frame numbers, so transitions between discrete states is described in Figure 5.2.



(a) (b)



(c) (d)



(e) (f)

Figure 5.1 Tracked contours of two hands (a)1st frame (b)11th frame (c)21th frame (d)51st frame (e)61st frame (f)91st frame

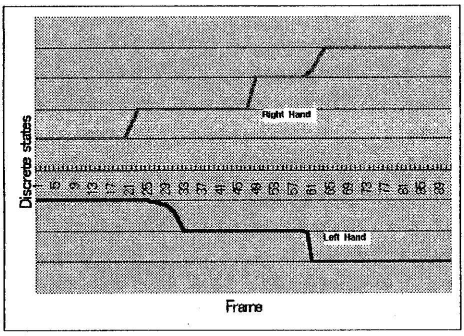


Figure 5.2 Transition of discrete states

6. Application to recognizing two-hand gestures

Recognition of two-hand gestures means the problem to determine which model tracks a two-hand gesture well. Therefore, the likelihood of a model presented in section 2.4 can be a good tool to recognize two-hand gestures. Given some models learned beforehand by EM algorithm (section 4), the likelihood of each model is calculated at each time by (2.18) based on the coupled-forward algorithm. Of course, an observed sequence is recognized as the model of the maximum likelihood. To reduce a computation effort, we may neglect models with the likelihood less than any value by suspending the computation of likelihood and the coupled-forward algorithm.

7. Conclusion and future work

We have presented a coupled switching linear model that is an interacting process between two switching linear models and also presented the coupled-forward and coupled-backward algorithm in the coupled switching linear model. Explosive increase in computation cost, resulting from (2.4), could be avoided through the collapsing method [Kim,1994] [Hamilton, 1989].

The scheme was applied to tracking and recognizing two-hand gestures. Tracking is performed through the coupled-forward algorithm whose parameters are estimated by EM algorithm. Two-hand gestures are recognized by comparison of the likelihood values of learned models. The likelihood can be computed easily through the coupled-forward algorithm as presented section 2.4.

We will show the results of recognition of two-hand gestures and try to track occluded hands under complicated backgrounds.

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