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ヒルベルト曲線による点照合ための新しい類似度計算法

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あらまし 本研究では、ヒルベルト曲線を利用した点照合ための新しい類似度計算法について述べる. これは、ヒルベルト曲線を利用して、二次元の点情報を一次元点情報に変換し、一次元上で高速に類似度を計算するものである. 点照合ための従来手法と比較して、計算量が少なく、雑音の影響を受けにくいことを確認した.

キーワード 点照合, 類似度計算法, ヒルベルト曲線, ヒルベルト走査距離

A Novel Similartiy Measure for Point Matching using Hilbert Curve

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Abstract In this report, we present a novel similarity measure using Hilbert curve for point pattern matching. In our method, the similarity measure is computed in one-dimensional (1-D) sequence in stead of in two-dimensional (2-D) space by using Hilbert curve. The experimental results show that our measure is fast and robust to noise than conventional similarity measures.

Key words point pattern matching, similarity measure, Hilbert curve, Hilbert scanning distance.

1. Introduction

Any image registration problem can be classified into four factors[1] including feature space, search space, search strategy and similarity measure. Points are the most desired features. Knowing the coordinates of a number of corresponding points in two images, a transformation can be determined to resample on image to the geometry of the other. Point features are also known as interest points, point landmarks, corner points and control point. Point pattern matching is a primary and essential approach for establishing a correspondence within two related patterns. The conventioal similarity measures for point patter matching including normal Hausdorff distance(HD), partial Hausdorff distance(PHD) [2] and modified Hausdorff distance(MHD) [3].

In fact, conventional similarity measures such as HD and PHD are very sensitive to outlier points or noise. A very small number of outlier points or noise can lead to significant errors. MHD works well on the matching task,however, as we will discuss in the experiment, it suffers from certain noise. In addition, for hausdorff similarity measures, computing in the 2-D space for each candidate costs too much computational time, especially in dense feature points set or high noisy images. In order to solve these possible problems, a novel HSD is proposed here to extend the measure for matching images. HSD can be computed much faster than Hausdorff distance measures. On the other hand, it is an accurate measure overcoming the mismatch problem caused by outlier points or noise.

The rest of paper is organized as follows: First we present our HSD in detail in section 2. Then, in section 3, we analyze the performance of HSD for object matching. Section 4 is about the experimental results using HSD and comparisons with Hausdorff distance measures. We conclude this paper in the last section.

2. Hilbert Scanning Distance

We now present HSD for grid points. For more details, we recommend readers to refx Assume that we are given two finite point sets $A = \{a_1, \ldots, a_I\}$ and $B = \{b_1, \ldots b_J\}$ such that each point $a \in A$ and $b \in B$ has integer coordinates in the 2-D space. We firstly use Hilbert scanning to convert them to new sets $S = \{s_1, \ldots, s_I\}$ and $T = \{t_1, \ldots t_J\}$ in the 1-D sequence, respectively. Then, the directed HSD from A to B $h_{hsd}(A, B)$ is computed by

$$h_{hsd}(A, B) = \frac{1}{I} \sum_{i=1}^{I} \rho(\min_{j} \| s_i - t_j \|) \quad (1)$$

where $\| \cdot \|$ is the Euclidean norm distance in the 1-D space and function ρ is defined as:

$$\rho(x) = \begin{cases} x & (x \leq \tau) \\ \tau & (x > \tau) \end{cases} \quad (2)$$

where ρ is called threshold elimination function and τ is a threshold predefined. We also can obtain the directed HSD from B to A $h_{hsd}(B, A)$ similarly and HSD is defined by

$$H_{hsd}(A, B) = \max(h_{hsd}(A, B), h_{hsd}(B, A)) \quad (3)$$

This definition of HSD is similar to the definition of MHD. Without the threshold function, it will be a special MHD in the 1-D space.

The detailed algorithm of HSD for comparing images is given in here. If two binary images A and B including feature points have been given, we first construct a new image C by combining them under a certain translation. Note that there must be 3 types of feature point in the combined image C: feature point from A, feature point from B and overlapped feature point from both A and B. In order to distinguish the 3 types of point, we give values 1, 2 and 3 to them, respectively. Then, the process of computing $h_{hsd}(A, B)$ can be summarized as the following steps:

Step1 Using a prepared lookup table to convert the combined image to a 1-D sequence C'.

Step2 For any feature point c'_i in C': if (value of c'_i) = 2, next feature point; else if (value of c'_i) = 1, find the nearest feature point c''_i whose value is 2 or 3, and the distance $d_i = \| c'_i - c''_i \|$; else (value of c'_i) = 3, $d_i = 0$.

Step3 If $d_i > \tau$, then $d_i = \tau$.

Step4 Repeat Step2 and Step3 until all the feature points have been processed.

Step5 Obtain $h_{hsd}(A, B)$ by calculating the mean value of all d_i.

Step6 Stop.

The process of computing $h_{hsd}(B, A)$ is with the same steps, excepting the value of feature point in A is 2 and in B is 1. Finally, HSD can be easily obtained as the larger one of $h_{hsd}(A, B)$ and $h_{hsd}(B, A)$.

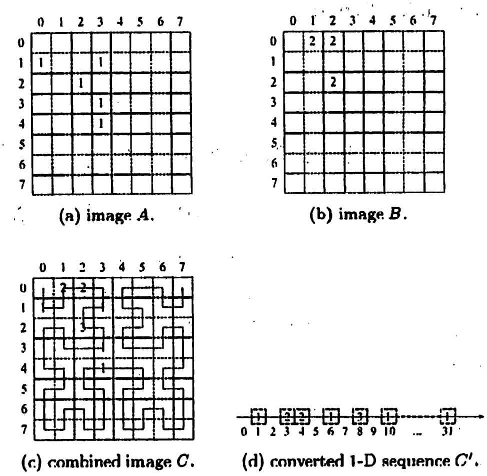


図 1 An example of computing Hilbert scanning distance in images.

For example, in Figure 1(a) and in Figure 1(b), there are two images A and B. The coordinates of feature points are $\{(0,1), (3,1), (2,2), (3,3), (3,4)\}$ and $\{(1,0), (2,0), (2,2)\}$ in A and B, respectively. We combined them into a new image C as shown in Figure 1(c). Then, by using Hilbert scanning, image C is converted to a 1-D sequence C' as shown in Figure 1(d). For all points whose value are not 2, we compute all d_i which are $\{2, 2, 0, 2, 23\}$, and when $\tau = 10$, then all d_i will be $\{2, 2, 0, 2, 10\}$. Finally, $h_{hsd}(A, B)$ is $(2 + 2 + 0 + 2 + 10)/5 \doteq 3.2$ in this example.

3. Analysis of HSD for Matching Images

In this chapter we analyze the computational complexity of HSD and the reason why HSD is accurate for matching images in presence of noise.

3.1 Computational Complexity of HSD

It is reported in [2] that the directed Hausdorff distance measures can be trivially computed in complexity $O(IJ)$ for two point sets of size I and J. That is, for each point in A, we compute the distance from it to every point in B. This is quite computational expensive. It can be improve to $O((I + J)\log(I + J))$ by using some other strategies [5] (In fact, this will be a Frechet distance, however, can be viewed as a lightly modified version of Hausdorff distance). In order to efficiently compute the Hausdorff distance, it is useful to adopt the distance transformation of binary image. Distance transform converts a binary image, consisting of feature (value 1) and non-feature pixels (value 0), into an image where all non-feature pixels have a value corresponding to the distance to nearest feature pixels. On the other hand, HSD is computed in the 1-D space, we do not use a distance transform here. For justice, we do not use any distance transformation in either.

For comparing images using HSD in practice, we combined the two sets into one new set(which means combining a new image). As shown in Figure 1(c), there are 3 different types of point in the new set: "1" denotes points belonging to A but not belonging to B; "2" denotes points belonging to B but not belonging to A; "3" denotes overlapped points belonging to both A and B. As a result, when computing $h_{hsd}(A, B)$, for each point from A, if the point is a "3" point, we set the distance d_i to 0 directly; if the point is a "1" point, we only need to consider two nearest points which are not "1" points in front and back of the current position in the 1-D sequence. If we denote δ as the average number of searched points to find the appropriate two points in 1-D sequence and suppose there are r percentage "3" points of I points, the computational complexity of HSD is $O(\delta(1-r)J))$. Notice that δ is often much smaller than I, most of its value is between 2 and 10 in practice. In addition, I and J are always large numbers exceeding 1000 in images, therefore $O(\delta(1-r)J))$ is much smaller than $O(IJ)$ and also smaller than $O((I+J)\log(I+J))$ even without using any strategy. Moreover, when computing HD or PHD, we should to rank all distances to select the appropriate value, however, there is no need to rank all distances when computing HSD as well as MHD. Hence, our HSD can be computed faster that the traditional distances, especially in point matching problem with a large number of points.

3.2 Accuracy of HSD

In our method, because we use threshold elimination function ρ when computing HSD, extreme distance values caused by 3 major negative factors can be reduced. These three factors are:

- First, outlier points can affect final distance result greatly, since they always have large values where we are computing the distance;
- Second, when we use Hilbert scanning, a part of the distances between two points in the 2-D image may change greatly in the 1-D sequence. It is also shown in the example that two points (the two 1 point in the center of the Figure 1(a)) which are close in 2-D image, become far away after Hilbert scanning in the 1-D sequence. The distance between them is only 1 in the 2-D image, however, it became 21 in the 1-D sequence as shown in Figure 1(d);
- Finally, noises such as Multiplicative and Gaussian noise can also affect the result of distance.

In order to solve the possible problem, threshold elimination function ρ is proposed here. Because the distance errors caused by the above negative factors always tends to be large values, these large values can be discarded by setting τ in threshold elimination function to be a smaller value. For instance, as described in the second item above, the distance between two points which is 1 in the 2-D image, become 21 after Hilbert scanning. If we set $\tau = 10$, the distance values between two points can be restricted in a small value range less than 10. Consequently, the final result which is computed as the mean of all distance values should also be small and can get rid of the above negative factors. Figure 2 shows a directed HSD value surface around the best matching position in our experiment. We can find that the values become larger and larger while the position being away from the best matching position (global minimum) farther and farther. We also observed in other cases, all of them have similar value surfaces. Therefore, it is an accurate matching measure in presence of noise.

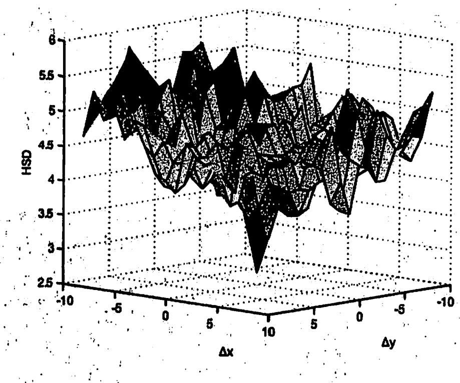


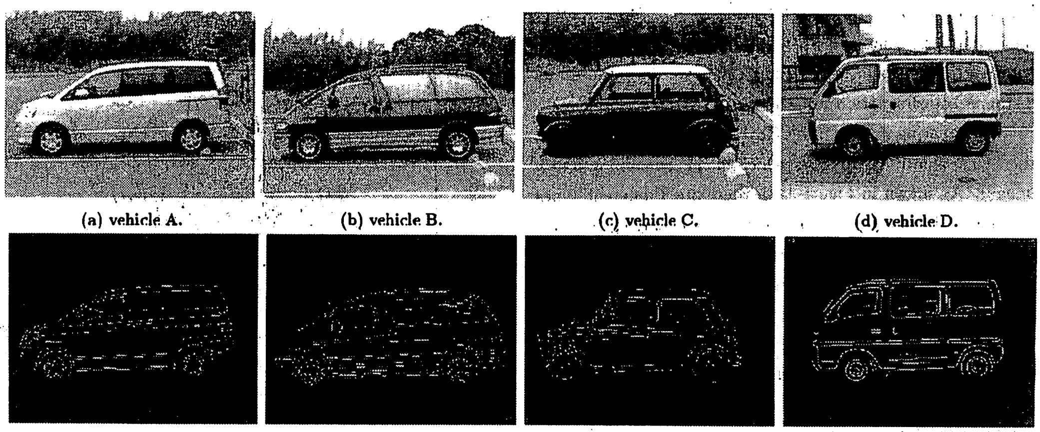
図 2 A value surface of HSD near global minimum

4. Experimental Results and Discussions

In this chapter, we make three experiments to verify the efficiency and robustness of our HSD. The first experiment is about the discriminatory of different distance measures. The second one is about the matching result and cost time of our HSD comparing with other measures when doing image matching. The last one is about an experiment to show the role of threshold elimination function in our HSD.

4.1 Discriminatory of Distance Measures

First, we apply the proposed HSD and Hausdorff distance measures to real edge images of different vehicles. Figure 3 shows four vehicles and their edge images. We only retain the edges belonging to the vehicles of interest. We compute the four distance measures HD, PHD($50\%^{th}$ ranked value),



(a) vehicle A. (b) vehicle B. (c) vehicle C. (d) vehicle D.

(e) edge of (a). (f) edge of (b). (g) edge of (c). (h) edge of (d).

図 3 Different vehicles and their edges.

MHD and HSD for pairs of vehicles and get the values which are shown in Table 1.

We know that distance measures present the similarities of two vehicles here. From the values of similarity in Table 5.1, we can see that HD is not reliable. For instance, the HD value is 33 between A and B, however only 32 between A and C. Note that the two vehicles A and B are most similar in shapes than other pairs, whereas A is a little different from C. That means the similarity measure value between A and B should be smaller than it between A and C. Now, we also evaluate the ratio of distance values for the best matching and second best matching. If the best distance value is v_1, and the second best value is v_2, then the *ratio* is computed by

$$ratio = \frac{v_1}{v_2} \tag{4}$$

This ratio can represent the discriminatory of a distance measure: the smaller this ratio is, the more discriminatory the distance measure has [3]. This ratio is about 1 for HD, PHD and MHD, however only about 0.43 for HSD. Hence, it is easily understood that HSD is more discriminatory than Hausdorff distance measures.

4.2 Matching Images Using Different Distance Measures

We are now considering an experiment to find a vehicle model from an image including many vehicles. When we are considering the distance between a model and an image, the image is often considerable larger than the model. In this experiment, we choose the model which is smaller than the image, that we can use the directed distance from model to image instead of normal definitions of all distance measures. We recommend readers to [2] to read more about the matching portions of the model and image for details. Our HSD is

表 1 The values of different distance measures for image pairs. (a)HSD. (b)HD. (c)PHD. (d)MHD.

	A	B	C	D
A	0	13	30	41
B	13	0	35	32
C	30	35	0	37
D	41	32	37	0

(a)

	A	B	C	D
A	0	13	30	41
B	13	0	35	32
C	30	35	0	37
D	41	32	37	0

(b)

	A	B	C	D
A	0	13	30	41
B	13	0	35	32
C	30	35	0	37
D	41	32	37	0

(c)

	A	B	C	D
A	0	13	30	41
B	13	0	35	32
C	30	35	0	37
D	41	32	37	0

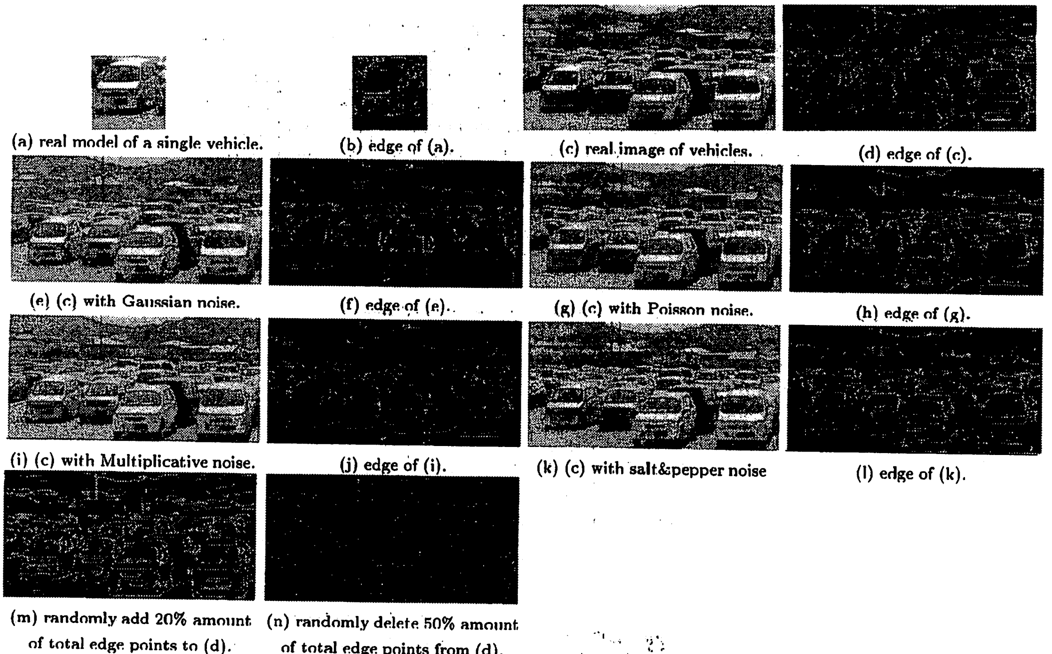
(d)

a similarity measure designed for finding the spatial transformation between two point sets, so it is no differences between different transformation forms (such as affine or only translation) since we concentrate on nothing but the measure itself. In this paper, we only consider the translation in the experiments for simplicity. Without loss of generality, we fix the image and allow only model to translate. Thus, the transformation equation used in our experiment is simply as follow:

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \tag{5}$$

We let model translate in interval [0, 200] along x-coordinate and [0, 200] along y-coordinate. We only consider the integer in our case, because Hilbert scanning is grid scanning. We don't use any strategy for searching in this experiment that means we do a full search.

$$M_{\Omega}(A, B) = \min_{\omega} h(A, B \oplus \omega) \tag{6}$$



(a) real model of a single vehicle. (b) edge of (a). (c) real image of vehicles. (d) edge of (c).
(e) (c) with Gaussian noise. (f) edge of (e). (g) (c) with Poisson noise. (h) edge of (g).
(i) (c) with Multiplicative noise. (j) edge of (i). (k) (c) with salt&pepper noise (l) edge of (k).
(m) randomly add 20% amount of total edge points to (d). (n) randomly delete 50% amount of total edge points from (d).

図 4 Vehicles and their edges with different noise level.

where $\oplus$ is the standard Minkowski sum notation (i.e., $X \oplus \omega = x + \omega \mid x \in X$).

In this experiment, PHD1, PHD2 and PHD3 present the PHD of $50\%^{th}$, $75\%^{th}$, $90\%^{th}$ ranked distance of all distances, respectively. We compute the matching errors in this experiment by the root-mean-square value (RMS) defined as

$$RMS = \sqrt{(\omega_x - \omega_{x_0})^2 + (\omega_y - \omega_{y_0})^2} \quad (7)$$

where $(\omega_{x_0}, \omega_{y_0})$ is the best matching position.

Our test sets are a vehicle model of 128 × 128 pixels and a 512 × 256 pixels image including some similar vehicles. All images are shown in Figure 4. Figure 4(a)-Figure 4(d) are the original images and binary edge images without noise. Then, we add some different types of noise [6] such as Gaussian (Figure 4(e), $\sigma = 10$, σ means the variance), Poisson(Figure 4(g)), Multiplicative(Figure 4(i), $v = 10$, v means the variance) and salt&pepper noise(Figure 4(k)) to Figure 4(c), respectively. Gaussian (also called normal) noise models are used frequently in image processing. Multiplicative noise is also called speckle noise. Poisson and salt&pepper noise are unipolar. Figure 4(f), Figure 4(h), Figure 4(j) and Figure 4(l) are the edge images extracted from those noisy images using same parameters. Figure 4(m) and Figure 4(n) are the edge images by randomly adding and deleting edge points in Figure 4(d). In this experiment, we set $\tau = 10$. The best matching position is (28, 119) here (we obtain the best matching position by observing the translation in Photoshop in practice). Table 2 and Table 3 present the matching results and RMS position errors.

On the other hand, HSD can be computed faster than Hausdorff distance measures. Figure 5 shows the running time in the experiment. Since MHD is the fastest one of all Hausdorff distance measures, we choose the running time of it for comparing with the time of HSD. This time is a CPU time of a normal PC with 3Ghz CPU and 1G memories using C++ language. The computational time using MHD increases gradually when edge points increasing. Contrarily, the computational time using HSD keeps in a low level even edge points increasing. The average running time using HSD is about 1/49 of the time using Hausdorff distance measures in the experiment.

5. Conclusions

A fast and accurate similarity measure for point pattern matching algorithm using Hilbert Curve has been presented in this paper. This similarity measure computes the distance measure in the one-dimensional (1-D) sequence rather than in the two-dimensional (2-D) space, hence, we only need to consider front and back neighborhood instead of multi-

表 2 The position results of experiment

Figure	Position result					
	HD	PHD1	PHD2	PHD3	MHD	HSD
図 4(d)	(27, 119)	(27, 119)	(28, 119)	((33, 117)	(28, 119)	(28, 119)
図 4(f)	(233, 117)	(28, 115)	(199, 113)	(27, 118)	(208, 112)	(28, 119)
図 4(h)	(219, 118)	(28, 119)	(26, 115)	(28, 119)	(28, 119)	(28,119)
図 4(j)	(203, 114)	(205, 110)	(212, 112)	(27, 119)	(207, 114)	(28, 119)
図 4(l)	(23, .129)	(27, 119)	(28, 117)	((28, 119)	(28, 119)	(28,119)
図 4(m)	(24,118)	(28,119)	(28, 119)	(27, 119)	(28, 119)	(28,119)
図 4(n)	(31,121)	(28,119)	(27,118)	(26,119)	(28,119)	(28,119)

表 3 The RMS results of experiment

Figure	Position result					
	HD	PHD1	PHD2	PHD3	MHD	HSD
図 4(d)	1.0	1.0	0.0	5.4	0.0	0.0
図 4(f)	205.0	4.0	171.1	1.4	180.1	0.0
図 4(h)	191.0	0.0	4.47	0.0	0.0	0.0
図 4(j)	175.1	177.2	184.1	1.0	179.1.0	0.0
図 4(l)	11.2	1.0	2.0	0.0	0.0	0.0
図 4(m)	4.1	0.0	0.0	1.0	0.0	0.0
図 4(n)	3.6	0.0	1.4	2.0	0.0	0.0

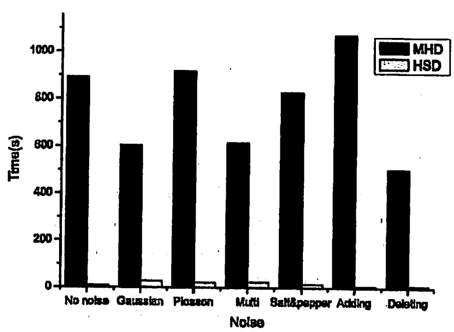


図 5 Running time comparison.

neighborhood in the 2-D space which greatly reduces the computational complexity. By applying a threshold elimination function, large distance values caused by noise and position errors are also removed. The major contribution of our work is that it convents the conventional similarity measure in the 2-D space to a new similarity measure in the 1-D space.

The future studies will aim at extending the application fields, not just in point pattern matching. More efficient search strategy for HSD and mathematical prove for the properties of HSD should also be considered.

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