

YAGLR¹アルゴリズムの実現と評価

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あらまし

本研究では、YAGLR法と呼ぶ新しい一般化LR法を提案する。YAGLR法に基づくパーザを計算機にインプリメントし、その性能を評価する。YAGLR法は、実際にグラフ構造化スタックを用いる。現在YAGLRアルゴリズムは、木構造化スタックを用いて重複計算を回避する。YAGLR法では、スタックのマージをトップ節点よりも深く進める。解析中に統語解析木は作らず、統語解析木を構成する部品(逆ドット項)を作成する。YAGLR法は、スタックのマージを深く進めることと、逆ドット項を作成することによって従来のパーザと比較して統語解析時間が高速になる。さらに、メモリ空間の圧縮も可能になる。YAGLRパーザは論理型言語 Prolog を用いて、計算機上に実現する。木構造化スタックが消費するメモリ空間は、Prologのshared-structureの機構によりスタックが消費するメモリ空間はグラフ構造化スタックと同じである。本論文では、YAGLR法のインプリメンテーションと実験による性能評価の結果を示す。また、実験結果に基づいて、YAGLRの解析時間が n^3 のオーダーであるという事実を示す。

A Practical Realization of YAGLR Algorithm on Prolog Using Tree-Structure Stacks

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Abstract

In this paper we describe an implementation and evaluation of our new generalized LR parsing algorithm called YAGLR. In our implementation we used tree-structure stack to realize YAGLR algorithm, whereas in its original version we use graph-structure stack. The merge operations of YAGLR proceeds deeper than top nodes effectively. Through reduce actions YAGLR creates items called *drit* which are symmetrically different from Earley's item. Through our implementation we show the advantages of creating *drits* instead of Earley's items. We also show through our implementation that, the parsing time of YAGLR is in the order of n^3 , where n is the length of an input sentence. Because of our merge algorithm and due to the nature of shared-structure of Prolog, even though we use tree-structure stack, we retain the packed nature of GSS. This reduces the memory space used by YAGLR to a greater extent.

¹YAGLR : Yet Another Generalized LR parser

1 Introduction

In recent times, ambiguous context-free grammars (CFG) are used for the syntactic and semantic processing of natural language. Efficient syntactic and semantic parsing for context-free languages are generally characterized as complex, specialized, highly formal algorithms. There are two major parsing algorithms that are used for the efficient parsing. The first is Earley's algorithm (Earley, 1970), which produce the parsing result in the form of a *parse list* consisting of set of items. This reduces the computational dependence on input sentence length from exponential to cubic cost. An attractive feature of Earley's algorithm is that it can easily be modified to parse co-ordinate structures of unlimited breadth. Such structures exist in the logical form of natural sentences. Numerous variations on Earley's method have developed into a family of chart parsing algorithms (Winograd, 1983).

The second is Tomita's algorithm (Tomita, 1986), which generalizes Kunth's (Kunth, 1965) and DeRemer's (DeRemer, 1971) computer language LR parsing techniques. Tomita's algorithm constructs parse forests, as the parsing result, which are nothing but a set of items. Tomita's algorithm uses the data structure called graph-structure stacks (GSS). Empirical results of Tomita's and Earley's algorithm reveal that the Earley/Tomita ratio of parsing time is larger when the length of an input sentence is shorter or when an input sentence is less ambiguous (Tomita, 1986).

Even though in (Tomita, 1986) it was claimed that Tomita's algorithm produces all the parsing trees during the parsing process, they are nothing but a set of items with pointers. A method for disambiguation of trees from the parse forest is proposed by (Tomita, 1986), in which the disambiguation is done by asking the user. In (Johnson, 1989) and (Kipps, 1989) it was stated that any algorithm which uses packed forest representation will take exponential time for certain sets of CFGs. In Earley's algorithm, parse trees are formed from the parse list created during parsing. Earley's algorithm works with the time complexity, in the order of n^3 for any CFG (Aho, Ulman 1972). In our new generalized LR parsing algorithm we stick on to the formation of items as the result of parsing. But our items are symmetrically different from Earley's.

In this paper we present an implementation and evaluation of our new generalized LR parsing algorithm called YAGLR (Tanaka, 1991). In the implementation of this algorithm we used tree-structure stacks (*trss*). In its original version we used graph-structure stacks (GSS). In this paper we explain all the actions of YAGLR on a set of *trss*, which we call as TRSS. Because of our merge algorithm, we find that even using *trss*, the parsing time and reduction in memory space are remarkable. Instead of packed forest as in Tomita's algorithm, YAGLR creates items called *dot reverse item (drit)*. These *drits* not only make effective merge operations possible, but also ease the removal of duplicated items.

In the following sections we state briefly about the *drits* and we give the implementation details of our algorithm along with the experimental results. We also prove experimentally that, the parsing time of YAGLR is in the order of n^3 for a grammar with reasonable size and time complexity in practical natural language processing. We

conclude this paper with a brief discussion on our future research directions.

2 A Brief Introduction to Generalized LR Parsing

The generalized LR parsing algorithm uses stacks and an LR parsing table generated from given grammar rules. An English grammar and its LR parsing table are shown in figure 2-1 (Tomita, 1987).

- (1) S → NP,VP
- (2) S → S,PP
- (3) NP → n
- (4) NP → det,n
- (5) NP → NP,PP
- (6) PP → p,NP
- (7) VP → v,NP

Figure 2-1 : A simple and ambiguous English grammar

State	Action field					Goto field			
	det	n	v	p	\$	NP	PP	VP	S
0	sh3	sh4				2			1
1				sh6	acc		5		
2			sh7	sh6			9	8	
3		sh10							
4			re3	re3	re3				
5				re2	re2				
6	sh3	sh4				11			
7	sh3	sh4				12			
8				re1	re1				
9			re5	re5	re5				
10			re4	re4	re4				
11			re6	re6/sh6	re6		9		
12				re7/sh6	re7		9		

Figure 2-2 : LR table for the grammar in fig-2-1

The parsing table consists of two fields, a parsing action field and a goto field. The parsing actions are determined by state (the row of the table) and a look-ahead preterminal (the column of the table), which is the grammatical category of an input sentence. Here, \$ represents end of the sentence. There are two kinds of stack operations: shift and reduce. Some entries in the LR table contain more than two operations and are thus in conflict. In such cases, a parser must conduct more than two operations simultaneously.

The 'shN' in some entries of the LR table indicates that the generalized LR parser has to push a look-ahead preterminal on to the LR stack and shift to 'state N'. The symbol 'reN' indicates that the parser has to pop the number of elements, corresponding to right hand side of the rule numbered 'N', from the top of the stack and then goto the new state determined by goto field. The symbol 'acc' means that the parser has successfully completed parsing. If an entry contains no operation, the parser will detect an error.

The LR table in figure 2-2 has conflicts in state 11 and 12 for column 'p'. Each of the two conflicts contain both a shift and a reduce actions, which is called a shift/reduce conflict. When our parser encounters the conflict, all reduce actions should be carried out before the shift action.

No.	Stack	Input	Actions
(1)	0	I saw a girl with the tel\$	shift to 4
(2)	0 n 4	saw a girl with the tel\$	reduce by NP → n
(3)	0 NP 2	saw a girl with the tel\$	shift to 7
(4)	0 NP 2 v 7	a girl with the tel\$	shift to 3
(5)	0 NP 2 v 7 det 3	girl with the tel\$	shift to 10
(6)	0 NP 2 v 7 det 3 n 10	with the tel\$	reduce by NP → det, n
(7)	0 NP 2 v 7 NP 12	with the tel\$	shift to 6/ reduce by VP → v, NP
(8)	0 NP 2 √ 7 NP 12 VP 8	with the tel\$	*shift to 6 reduce by S → NP, VP
(9)	0 NP 2 v 7 NP 12 S 1	with the tel\$	*shift to 6 shift to 6
(10)	0 NP 2 v 7 NP 12 p 6 S 1 p 6	the tel\$	

Figure 2-3. An Example of Generalized LR Parsing

On input "I saw a girl with the telescope", the sequence of stack and input contents is shown in fig.2-3. For example, at line (1) the parser is in state 0 with "I" the first input symbol. As the action field of fig.2-2 in row 0 and column 'n' (the preterminal of "I") contains 'sh4', it pushes 'n' and cover the stack with state 4. That is what was happened in line (2).

Then, "saw" becomes the current input symbol. As the action of state 4 on 'v' (the preterminal of "saw") is 're3', it carries out a reduce action by using the rule $NP \rightarrow n$. One state symbol and one grammar symbol are popped from the stack and 0 again becomes the top of the stack. Since the goto field of state 0 on NP is 2, NP and 2 are pushed onto the stack. We now have the configuration in line (3). Each of the remaining moves are determined similarly until the shift of "girl".

In line (7), we get a conflict with 'sh6/re7', where we carry out 're7' at first and 'sh6' is waited until all the other remaining stacks experiences shift actions. At line (10) both the stacks with shift action 'sh,6' are shifted to state 6. The remaining parsing proceeds in this way.

3 Dot Reverse Item

During reduce actions, YAGLR creates *drits* which are symmetrically different from Earley's items. Since a state always accompanied with position number, we call the pair as a node in the rest of this paper (see section 4). From the *trss* we can create either Earley's items or *drits*.

One of the important factor in creating *drits* is that, it enable us to do deep merge on TRSS and makes the structure of TRSS much simpler. In other case, if we create Earley's items, the deep merge is not possible and we have to restrict only to the merge of top nodes, and if we do deep merge then it leads to the creation of unwanted items. The reason why the creation of proper *drits* is possible comes from the fact that LR parsing is based on right-most derivation. Another important factor in using *drits* is the localization of duplication checks. The position number j inside Earley's item $I_i \ni [A \rightarrow B \cdot C, j]$ will change within the processing of a single input word w_k . On the other hand, the position number j inside *drits* will remain the same throughout the processing of the input word w_k , ($w_k = w_j$) and thus it enables us to limit the duplication check within the processing of a single input word. Therefore we can localize the range of duplica-

tion check of *drits*. The *drits* are elaborately discussed in (Tanaka, 1991).

The following is a formal definition of a *drit*.

Let $G = (N, T, P, S)$ be a CFG and let $w = w_1 w_2 \dots w_n \in T^*$ be an input sentence in T^* which is a set of a sequence of terminal symbols. For a CFG rule $A \rightarrow X_1 \dots X_m$ and $0 \leq j \leq n$, $[A \rightarrow X_1 X_2 \dots X_k \cdot X_{k+1} \dots X_m, j]$ is called a *drit* for w . The dot between X_k and X_{k+1} is a metasympol not in N and T . The position number '0' represents the left hand side position of w_1 .

I_i , a set of *drit* is defined as follows. For i and j ($0 \leq i \leq j \leq n$), $[A \rightarrow \alpha \cdot \beta, j] \in I_i$ iff $S \xRightarrow{*} \gamma A \delta$, $\beta \xRightarrow{*} w_{i+1} w_{i+2} \dots w_j$, and $\delta \xRightarrow{*} w_{j+1} w_{j+2} \dots w_n$ where the dot position is a suffix i of an item set I_i .

The difference of a *drit* with an Earley's item lies in the interpretation of j . It is evident from the above definition that, in the *drit*, the analysis has been completed for β which is on the right hand side of the dot symbol. On the contrary, in case of Earley's item, the analysis has been completed for α which is on the left hand side of the dot symbol.

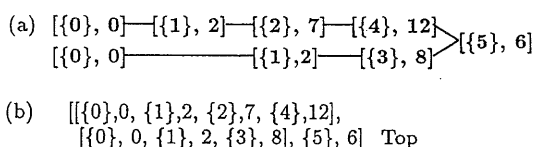
4 Realization of tree-structure stack

In YAGLR method, we do not use grammatical categories along with a state number as in generalized LR parsing algorithm shown in fig.2-3. In Tomita's method, packed forest representation is used instead of grammatical symbols. On the other hand, we use position numbers along with a state number. The position number indicates the position upto which the shift of an input sentence has been completed. Note that a sequence of state numbers in the stack completely determines the basic parsing process. In other words, whether or not we use grammatical symbols or packed forests or position numbers along with a state in the stack, they do not affect our basic parsing process.

Each node of a *trss* used in YAGLR has the following structure :

$[< \text{a set of position numbers } >, < \text{state } >]$.

The set of position numbers are used to create *drits* during reduce actions. In general, there will be several top nodes in TRSS, but after merging, the remaining top nodes will be at most no more than the number of distinct states. Even though we use *trss* in our implementation, because of our merge operations and due to the shared-structure of Prolog, we retain the packed nature of GSS. An example of *trss* is given in (a) and its list structure in (b). In the *trss* in (a), $[\{5\}, 6]$ is the top node and other nodes below top nodes such as $\{4\}, 12]$, $\{3\}, 8]$, $\{2\}, 7]$, $\{1\}, 2]$ and $\{0\}, 0]$ are all called parent nodes of the top node.



(M2). Finally, with the resultant TRSS after merge, the parser will continue its next action.

5.4 Merge Algorithm of TRSS

Since we defined the merge operations considering two nodes, we now give the merge algorithm of TRSS as follows. Our merge is performed in depth-first method by considering two *trss* at a time.

```

procedure merge (TRSS);
begin
  Initialize TmpStk to [ ];
  while TRSS  $\neq$  empty do
    repeat
      pickup and retract a trss (call target_trss) from TRSS;
      if at least one trss with target_trss's same top node
        exists in TRSS
      then
        begin
          repeat
            pickup and retract a trss (call s_trss) from
              TRSS having same top node that of target_trss;
            apply (M1) to the top node of target_trss and
              s_trss to get a merged top node;
            for the parent nodes of merged top node
              apply (M2);
            name the result of the merges (M1) and (M2)
              of target_trss and s_trss as m_trss;
            target_trss := m_trss;
          until no more trss with same top nodes as
            target_trss in TRSS exist;
        end
        put the target_trss into the TmpStk;
      until TRSS becomes empty;
      TRSS := TmpStk;
    end
  end

```

In applying (M2), if (M21) is applied then our merge proceeds one level down towards the parent nodes by calling (M2) recursively. However in case of applying (M22), we do not need to proceed our merge further.

5.5 YAGLR Algorithm

Let us give a complete algorithm of YAGLR.

1. Set the initial state of a set of *trsses* (TRSS) as :
(Bottom) [{0}, 0] (Top)
2. Initialize the TempStack to []
3. **If** TRSS is empty **then goto** 5;
Pick up and retract one *trss* from TRSS (TRSS := TRSS - *trss*);
for this *trss*
 Assign the *actions* determined by LR table;
 case actions of
 'accept': end with 'success' for the *trss* and **goto** 3;
 'error' : end with 'failure' for the *trss* and **goto** 3;
 'shift' : push the *trss* into TempStack and **goto** 3;
 'reduce' : **goto** 4;
 'shift/reduce' : push the *trss* with the shift action
 into TempStack and **goto** 4 carrying
 the *trss* with the reduce action(s)
 end;
4. **do** the reduce action(s) and push the newly formed *trss(es)*
 into TRSS and merge the TRSS;
 goto 3.

5. **If** TempStack := [] **then return**;
 Perform shift action for every *trss* in TempStack and push the
 resultant into TRSS ;
 merge the TRSS;
 goto 2.

6 Evaluation of YAGLR

In this section we describe the evaluation of YAGLR based on the preliminary experimental results comparing with SAX (Matsumoto, 1988) and SGLR (Numazaki, 1991). SAX is based on the bottom up version of Chart algorithm and SGLR is based on Tomita's algorithm using tree-structured stacks.

6.1 Experimental Environment

The experiments were done on Sun 3/260 machine and using Quintus Prolog. We used different sets of grammars in our experiment ranging from the grammars with 3 rules to 550 rules to study the parsing efficiency of our algorithm. In this paper we concentrate on three different type of grammars. Gram-1 is a grammar in (Johnson 1989) which is a highly densely ambiguous grammar. This grammar and its input pattern are given in the appendix. Gram-2 is a grammar with 44 rules, and gram-3 with 400 rules. Gram-2 and Gram-3 are taken from (Tomita, 1986). However, original version of Gram-3 is from our lab in Tokyo Institute of Technology. Gram-3 becomes highly ambiguous and could therefore be considered as one of the toughest natural language grammars in practice. In this paper we center all our experimental results mainly around gram-3. The evaluation of other types of grammars along with gram-1 are give in (Tanaka, 1991), (Suresh, 1991).

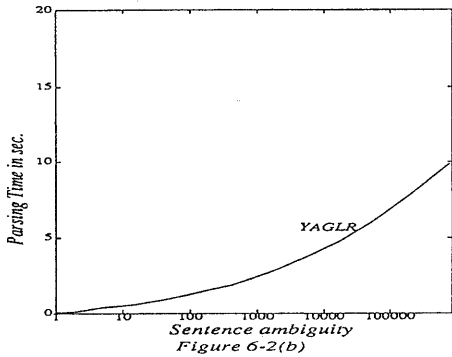
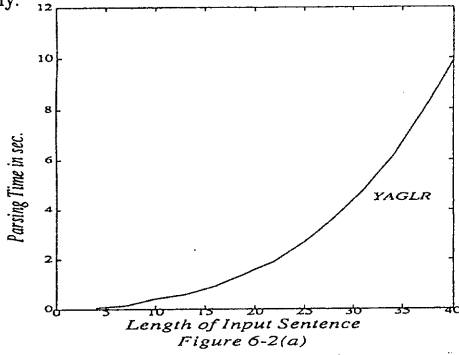
The input for the grammars 2 and 3 is made more systematically. The *n*-th sentence in the set is obtained by the schema, *noun verb det noun (prep det noun)ⁿ⁻¹* (Tomita, 1986). The example sentence with this structure is: *I saw a girl on the bed in the apartment with a telescope* The ambiguity of this type of sentences grows enormously. This type of sentences are necessary to find the parsing efficiency against sentence ambiguity.

All our program are written in Prolog and are compiled using Quintus Prolog. Since we are interested in the ratio of parsing time, it will be the same either interpreted or compiled. The parsing time is determined by CPU time minus the time consumed for garbage collection (gc). We find that the gc consumed during the execution of our algorithm is very less (even though we use *trss*). If we include the gc time, then the ratio between YAGLR and other parsers will vary a large extent - in a positive way to YAGLR. The parsing time in our implementations are without forming trees for SAX, SGLR parsing while YAGLR parsing creates *drits*.

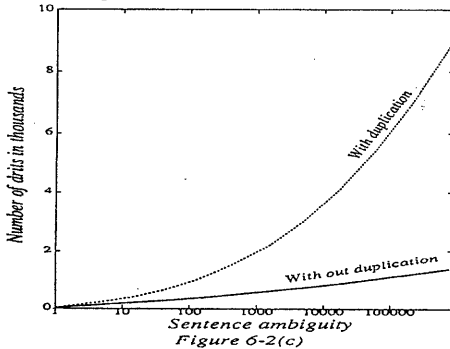
6.2 Experimental Evaluation of YAGLR

Here, we give our preliminary results on the implementation of YAGLR. The figure 6-2(a) and 6-2(b) shows the parsing time of YAGLR for gram-3 against length of

the input sentence and against sentence ambiguity respectively.



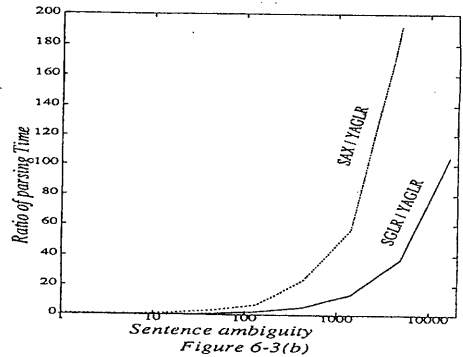
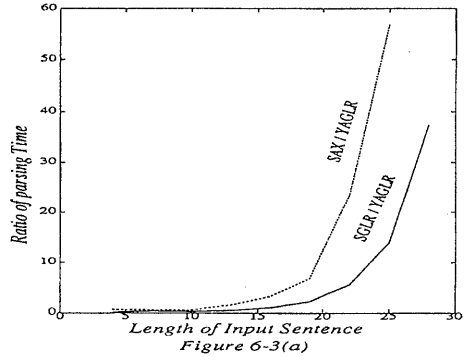
We find that YAGLR parses the sentence more faster, as the ambiguity of the sentence increases. In other words, as the ambiguity increases, the parsing time of YAGLR decreases rapidly regardless to the size of the grammar or length of the input sentence.



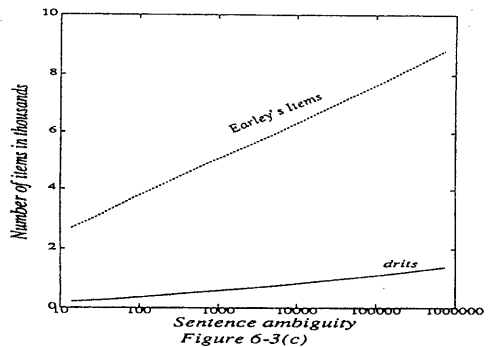
The figure 6-2(c) shows the number of *drits* created by YAGLR against the ambiguity. Here, the total *drits* produced during parsing is indicated by a dashed curve, which includes duplicated *drits*. After the shift of an input word w_i , our parser makes duplication check to the *drits* produced in between w_{i-1} and w_i and removes all the duplicated items. The other curve shows the number of non-duplicated items created among duplicated items. The parsing time shown in fig.6-2(a) and (b) includes the time consumed for removing the duplicated items. If the sentence is ambiguous, the creation of duplicated *drits* are unavoidable. It should be noted that, if we do not do the duplication check, YAGLR parser will run much faster.

6.3 Comparison of YAGLR

In this subsection, we would like to compare the performance of YAGLR with other parsers. In figure 6-3(a) and 6-3(b), we give the ratio of parsing time of SGLR/YAGLR and SAX/YAGLR against sentence length and sentence ambiguity respectively, for gram-2. The ratio will be the same by taking it either against sentence length or ambiguity. The high, the ratio of parsing time of SGLR/YAGLR or SAX/YAGLR, the low, the parsing time of YAGLR. Here, we see that, SGLR/YAGLR ratio and SAX/YAGLR ratio is high for a sentence with considerable length, as the ambiguity increases.



The figure 6-3(c) shows a comparison of Earley's items created using Earley's algorithm and *drits* created by YAGLR for gram-3. All the duplicated items are removed during parsing and the graph in fig.6-3(c) shows that of non-duplicated items. From the fig.6-3(c) we can realize the advantages of creating *drits* over Earley's items.



There are some grammars, for which the number of non-duplicate Earley's items created using YAGLR algorithm are less than that of *drits*. But, the total number of items created including duplicated items are far less in case of *drits*. The parsing time includes the creation of total number of items which includes duplicated items. The more the duplicated items, the more the time consumed for creating and removing. Also, as we discussed briefly in section 2, on creating Earley's items using our algorithm, leads to the creation of unwanted items. Hence it is safe to conclude that *drits* are better than Earley's items.

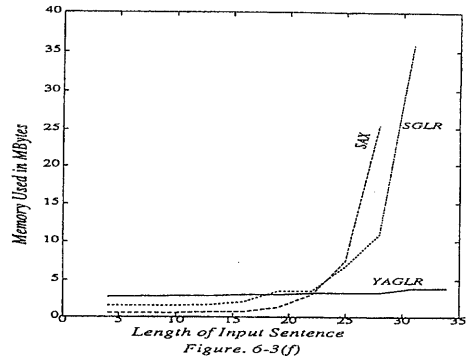
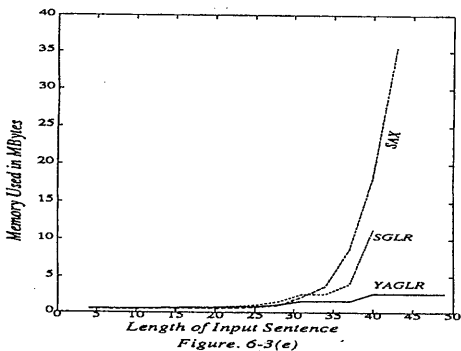
I/P	TIME in msec.			Trees
	SAX	SGLR	YAGLR	
5	34	50	67	20
6	67	83	117	70
7	233	250	183	256
8	800	833	367	969
9	2867	3117	517	3762
10	10750	12650	866	14894
11	41616	49716	1383	59904
12	262250	222235	2017	244088

Figure 6-3(d)

n	TIME in msec.			Trees
	SAX	SGLR	YAGLR	
1	50	17	67	1
2	117	84	167	2
3	267	150	400	5
4	967	350	600	14
5	3067	1000	934	42
6	9700	3200	1417	132
7	32217	10683	1917	429
8	113135	37800	2700	1430
9	398832	137000	3667	4862
10	-	498731	4750	16796

Figure 6-3(e)

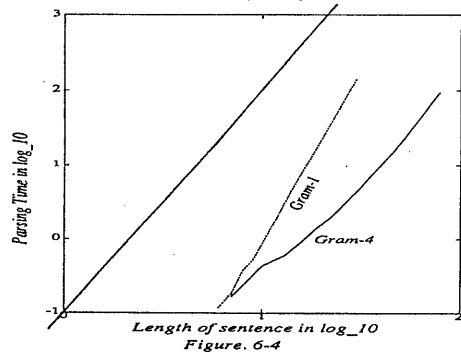
Some raw empirical datas got from the experimental results using gram-1 and gram-2 are given in the table in fig. 6-3(d) and 6-3(e) respectively. In the table, I/P denotes length of the input sentence, n denotes sentence number according to the schema described in 6.1 and *Trees* denotes the number of ambiguities. These datas entirely depends on the machine system and the programming language used. But we hope that, the ratio of parsing time will be the same for any system under a particular programming environment.



The figure 6-3(f) and 6-3(g) shows results of memory space consumed by YAGLR for the parsing of gram-2 and gram-3 respectively. YAGLR consumes very less memory space due to its effective merge operations. Note that in YAGLR we produce *drits*, whereas in our experiments, SAX and SGLR are not producing any form of partially parsed informations. It is the reason why YAGLR needs more space up to the sentence of length 18. The amount of memory space needed depends on the size and ambiguity of the grammar and the input we use. However, when the length of input sentence gets long, the reduction in memory space is remarkable regardless to the size and ambiguity of the grammar.

6.4 Experimental Computational Complexity of YAGLR

For gram-1 we proved theoretically, the complexity of YAGLR as in the order of n^3 (Tanaka, 1991 [in Japanese]). But we are yet to prove in case of general CFG. In this subsection we give our experimental proof for the complexity of YAGLR. The figure 6-4 shows the order of parsing time of YAGLR for gram-1 and gram-3.



On taking log scale for both X and Y axis we find that for the parsing time to be in the order of n^3 , the slope of the time curve must be ≤ 3 . Thus the line passing through X and Y axis in fig. 6-4 shows the sample line with slope 3. In the fig.6-4 we find the time curve of gram-3 is in parallel with the sample line and so we can conclude that, the time complexity of gram-3 is in the order of n^3 . In case of the time curve of gram-1, we find that it is not in parallel with the sample line and it is nearly in the order of

n^4 . However, we proved theoretically that the complexity of YAGLR for gram-1 is in the order of n^3 .

7 Conclusion

We have shown the basic idea of YAGLR parsing algorithm and its implementation with evaluation. It should be noted that after completing parse, the syntactic trees are formed from *drits* obtained during the parsing process. Even though we used TRSS in our implementation, we find that the parsing speed of YAGLR increases and the memory space consumed by YAGLR is very less. There are certain grammars for which if we use TRSS the copying of stacks creates inefficiency. For this reason we would like to implement our algorithm in GSS as in our original version. Through our implementation, we practically proved that for a CFG with reasonable size and complexity, YAGLR's parsing time is in the order of n^3 . Our future works includes showing the time complexity of YAGLR for any CFG, developing a parallel algorithm for YAGLR method and also for the tree generation from *drits*.

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Appendix

The CFG given in (Johnson, 1989) is shown below.

- (1) S → a
- (2) S → SS
- (3) S → S^{m+2}

The input pattern for this type of grammar is aⁿ⁺², where n > m.