

## 局所支配と局所交配による多目的進化型アルゴリズムの強化： 多目的0/1 ナップザック問題を用いた性能検証

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この論文では、局所支配と局所交配に基づく分散探索の実行によって多目的進化型アルゴリズム (MOEA) の性能を強化する方法を提案する。この方法では、まず、すべての評価値ベクトルを目的関数空間において局座標ベクトルに変換する。解集団は、得られた偏角情報を用いていくつかのサブ集団に再帰的に分割される。結果として、各サブ集団は同様な探索方向の周辺に位置する個体によって多目的な目的関数空間の部分領域を被覆する。次に、各サブ集団について局所支配が計算され、選択、交叉および突然変異を各サブ集団内の個体に対して行う。提案法は支配に基づく選択を利用する MOEA の性能を改善するとともに、全体の解の支配関係算出コストも低減する。この論文では、提案法のパレート最適解導出における有効性を、2つの代表的 MOEA すなわち NSGA-II と SPEA2 に多目的0/1 ナップザック問題を用いて性能検証している。

## Enhancing Multiobjective Evolutionary Algorithms by Local Dominance and Local Recombination: Performance Verification in Multiobjective 0/1 Knapsack Problems

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This paper proposes a method to enhance multiobjective evolutionary algorithms (MOEAs) by performing a distributed search based on local dominance and local recombination. In this method, first, all fitness vectors of individuals are transformed to polar coordinate vectors in objective function space. Then, the population is recursively divided into several subpopulations by using declination angles. As a result, each sub-population covers a sub-region in the multiobjective objective space with its individuals located around the same search direction. Next, local dominance is calculated separately for each sub-population and selection, recombination, and mutation are applied to individuals within each sub-population. The proposed method can improve the performance of MOEAs that use dominance based selection, and can reduce the entire computational cost to calculate dominance among solutions as well. In this paper we verify the effectiveness of the proposed method obtaining Pareto optimal solutions in two representative MOEAs, i.e. NSGA-II and SPEA2, with Multiobjective 0/1 Knapsack Problems.

### 1. Introduction

Recently, multiobjective evolutionary algorithms (MOEAs) [1, 2] are being increasingly investigated to solve multiobjective problems. MOEAs evolve simultaneously a population of potential solutions to the problem in hand and are able to find a set of Pareto optimal solutions (POS) in a single run of the algorithm. Two important goals of a MOEA are to achieve POS converging to the true Pareto front and keep a good distribution in objective space of the solutions found. Among the various methods proposed so far [1, 2], approaches that use elitism based on dominance are becoming the state of the art. In general, these algorithms are quite effective obtaining POS when the search space is relatively small. However, when the search space becomes large and/or the number of objectives

increase, it becomes gradually difficult for them to obtain POS with sufficient diversity in objective space, i.e. solutions tend to be distributed in a relatively narrow region of the Pareto optimal front.

In this work we propose a method to enhance MOEAs by performing a distributed search based on local dominance and local recombination to obtain POS satisfying diversity conditions. The proposed method can be easily applied to MOEAs that use dominance based selection. An additional and important advantage of the proposed method is that it can reduce the entire computational cost to calculate dominance among solutions. We chose NSGA-II [3] and SPEA2 [4] as two representatives of the latest generation of elitist MOEAs and enhance them with our method. We verify the effectiveness of the proposed method obtaining POS satisfying diversity conditions by comparing the search performance between the conventional algorithms and their enhanced versions.

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## 2. Proposed Method

### 2.1 Concept

Dominance offers important advantages to multiobjective algorithms. However, some global non-dominated solutions may have a too strong influence and may undermine the contribution of other solutions that, although globally dominated, have the potential to make the entire population diverse in objective space.

In order to introduce necessary diversity and accomplish efficient search for POS, we divide the entire population into several sub-populations generation by generation. Each sub-population consists of individuals having similar search directions. Then, we calculate local dominance among individuals in each sub-population after rotating the the sub-population's search direction of towards  $\pi/4$ . Next, we apply parent selection and genetic operations to individuals within each sub-population by reflecting local dominance. Population division allows us to reduce the entire computational cost while obtaining dispersed POS. After calculation of local dominance, we can simply apply conventional MOEAs [3, 4] in each sub-population. In the following we detail the procedures for population division and local dominance.

### 2.2 Population Division in Objective Space

The objective of the population division is to group individuals with similar search direction in the  $m$ -dimensional objective space. To achieve this efficiently, the  $m$ -dimensional fitness vector  $\mathbf{f}(\mathbf{x})$  for each individual is expressed in polar coordinates by a norm  $r$  and  $m - 1$  declination angles  $\theta_j (j = 1, 2, \dots, m - 1)$ . The joined population of parents and offspring  $P(t)$  at  $t$ -th generation is split iteratively according to declination angles into  $d^{m-1}$  sub-populations  $P_k(t) (k = 1, 2, \dots, d^{m-1})$ , where  $d$  is a parameter indicating the division factor at each one of the  $m - 1$  iterations. In other words,  $P(t)$  is split into  $d$  sub-populations according to  $\theta_1$ , then each one of these  $d$  sub-populations is split again into other  $d$  sub-populations according to  $\theta_2$ , and so on. The population dividing scheme slightly varies the size of sub-populations. This fluctuates the regions covered by the sub-populations avoiding the appearance of gaps among sub-populations in objective space.

### 2.3 Calculation of Local Dominance in Sub-population

Local dominance among individuals in each sub-population  $P_k (k = 1, 2, \dots, d^{m-1})$  is calculated after rotating the principle search direction of  $P_k$ . The main steps of this procedure are as follows. First, find maximum and minimum declination angles,  $\theta_{kj}^{\max}$

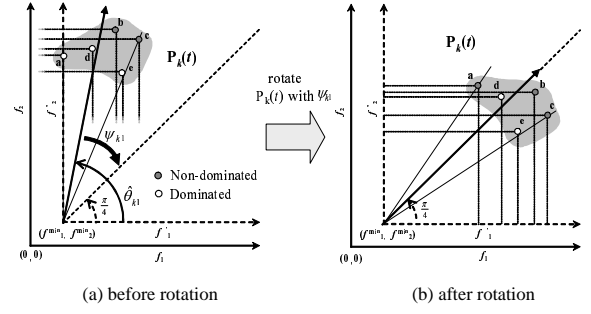


Figure 1: Rotation of sub-population  $P_k(t)$  and its affection to dominance among solutions

and  $\theta_{kj}^{\min} (j = 1, 2, \dots, m - 1)$ , in  $P_k(t)$ , and determine the principle search direction by  $\hat{\theta}_{kj} = \frac{\theta_{kj}^{\max} - \theta_{kj}^{\min}}{2} + \theta_{kj}^{\min} (j = 1, 2, \dots, m - 1)$ . Second, calculate  $m - 1$  rotation angles  $\psi_{kj} = \hat{\theta}_{kj} - \frac{\pi}{4} (j = 1, 2, \dots, m - 1)$ . Third, rotate all declination angles of the polar coordinate vectors of individuals in  $P_k(t)$  by  $\mathbf{p}'(\mathbf{x}) = (r_k(\mathbf{x}), \theta_{k1}(\mathbf{x}) - \psi_{k1}, \theta_{k2}(\mathbf{x}) - \psi_{k2}, \dots, \theta_{km-1}(\mathbf{x}) - \psi_{km-1})$  as shown in **Fig.1**. Fourth, transform all rotated polar coordinates vectors  $\mathbf{p}'(\mathbf{x})$  in  $P_k(t)$  to modified temporal fitness vectors  $\mathbf{f}''(\mathbf{x}) = (f''_{k1}(\mathbf{x}), f''_{k2}(\mathbf{x}), \dots, f''_{km}(\mathbf{x}))$ . Fifth, calculate local dominance using the modified fitness vectors  $\mathbf{f}''(\mathbf{x})$  in  $P_k(t)$ .

### 2.4 Local Dominance and Local Recombination

Recalculation of fitness vectors for individuals in each sub-population after rotation changes dominance among solutions in objective function space. This brings more chance to potential solutions to be selected as parent individuals rather than conventional schemes. As shown in **Fig.1(a)**, if we calculate dominance among solutions with a conventional scheme, say NSGA-II [3], individuals  $a$ ,  $d$  and  $e$  would be dismissed with high probability in the parent selection process since they are dominated by  $b$  and  $c$ . On the other hand, if we take into account the principle search direction of  $P_k$  and properly rotate declination angles, as shown in **Fig.1(b)**, the individual  $a$  becomes a non-dominated solution, which is expected to make the entire population spread. In this example,  $a$  has the potential to disperse the distribution of  $P_k$  to the direction of objective function  $f_2$ .

Local dominance is reflected in parent selection within the current sub-population. We apply crossover and mutation operators to parent individuals selected within each sub-population based on local dominance. Because the individuals included in each sub-population have similar search direction, the enhanced algorithm locally achieves recombination between in-

dividuals having similar fitness vector. This effectively works to avoid inefficient recombination in MOEAs.

### 3. Computational Cost Reduction

Recent MOEAs generally require the computation of order  $O(mN^2)$  to calculate dominance, where  $m$  and  $N$  denote number of objectives and population size, respectively. Since we divide the entire population into  $d^{m-1}$  sub-populations, the proposed method can reduce substantially the overall computational cost to calculate dominance to  $O\left(\frac{mN^2}{d^{m-1}}\right) + O(mN \log_2 N) < O(mN^2)$  for  $m \geq 2$  and  $d \geq 2$ , assuming that population division takes  $O(mN \log_2 N)$ , where sorting by angle information is used.

### 4. Results and Discussion

In this paper we use multiobjective 0/1 knapsack problems (KP $n$ - $m$ ) to verify the search performance of the proposed method. The problems consist of  $n = 500$  objects and  $m = \{2, 3\}$  objectives, with known true POS only in the case of  $m = 2$  objectives. We adopt two-point crossover with probability  $p_c = 1.0$  and bit-flipping mutation with probability  $p_m = 1/n$ . In the following experiments, we show the average performance with 30 runs, each of which spent 2,000 generations. Population sizes are set to  $|P| = \{200, 600\}$  for  $m = \{2, 3\}$  objectives, respectively.

We use the hyper-volume ( $HV$ ), generational distance ( $GD$ ), inverse generational distance ( $IGD$ ), and spread ( $SP$ ) as performance measures [1, 2] to evaluate MOEAs performance.  $HV$  measures the  $m$ -dimensional volume covered by POS in objective function space and POS showing higher  $HV$  can be considered as better POS from both convergence and diversity viewpoints.  $GD$  measures the degree of convergence to the true POS by taking the average distance from all members in the obtained POS to their nearest solutions in the true POS.  $IGD$  measures the average distance from all members in the true POS to their nearest solutions in the obtained POS. Note that  $IGD$  gives a small value only if all members of the obtained POS dispersively converges to all members of the true POS, while  $GD$  becomes small even if they converge to some of the members in the true POS.  $SP$  measures the degree of dispersion on the distribution of POS. POS showing smaller  $SP$  can be considered as better POS satisfying diversity condition.

First, we show in **Fig.2** (a) and **Fig.3** (a) the normalized  $HV$  obtained by the enhanced MOEAs over the parameter  $d$  used for population division. Note that the size of sub-population is given by  $|P_k| \sim N/d^{m-1}$ . The two parallel dashed lines are the results by conventional NSGA-II and SPEA2. Vertical bars over-

laying the mean represent 95% confidence intervals. From these figures we can see that the  $HV$  achieved by the proposed method is remarkably better than the  $HV$  achieved by conventional algorithms. Also, note that there is an optimum parameter  $d^*$  to maximize  $HV$  depending on the algorithm to be used. If we increase  $d$  excessively, the performance is gradually deteriorated because the algorithm searches with many but very small sub-populations, which leads to unstable performance with larger variance. Larger improvement by our method can be observed in case of SPEA2 rather than NSGA-II, while conventional NSGA-II always outperforms SPEA2 in these problems. **Fig.2** (b), (c) and **Fig.3** (b), (c) show the Pareto front of the obtained POS. We can see that enhanced NSGA-II and SPEA2 implementing our method achieve robust performance obtaining fully dispersed POS close to the true POS for  $m = 2$  and  $m = 3$  objectives. On the other hand, the range of the obtained POS by conventional NSGA-II and SPEA2 is narrow for the entire distribution of the true POS.

Second, we observe the performance separately on convergence and diversity by using optimum parameter  $d^*$  for test problem KP500-2 for which we know the true POS. We show in **Fig.4** (a) and (b) the transition of  $GD$  and  $IGD$  over the generations as indicators of convergence of POS, respectively. Conventional NSGA-II and SPEA2 achieve smaller  $GD$  than their enhanced versions because conventional methods tend to incline the search to the direction of a part of the true POS, which advantageously works to reduce  $GD$ . On the other hand, enhanced NSGA-II and SPEA2 achieve clearly smaller  $IGD$  than conventional ones. This is because the enhanced methods evolve the search dispersively inducing a necessary diversity in the entire population. Population division and calculation of local dominance within sub-populations are quite effective to keep the search dispersive in the objective space. Furthermore, we show the transition of  $SP$  over the generations as an indicator of diversity of POS in **Fig.4** (c). From this figure, we can see that initially  $SP$  increases substantially in conventional NSGA-II and SPEA2 for all problems indicating that these algorithms remarkably lose diversity in an early stage of evolution. On the other hand, the enhanced methods continuously induce diversity into the entire population from the beginning of evolution. Precisely, enhanced SPEA2 always achieved smaller  $SP$  than enhanced NSGA-II, which support the result that the former algorithm shows larger improvement on  $HV$ .

These results illustrate the difficulty a single population algorithm faces to cover widely spread POS in the objective space and also show the effectiveness of the proposed method based on local dominance and local recombination.

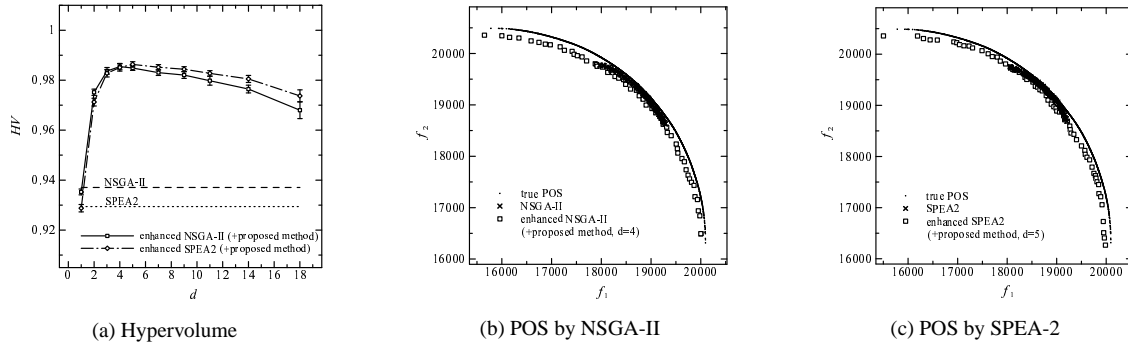


Figure 2: Hypervolume and obtained POS for KP500-2 ( $m = 2$ )

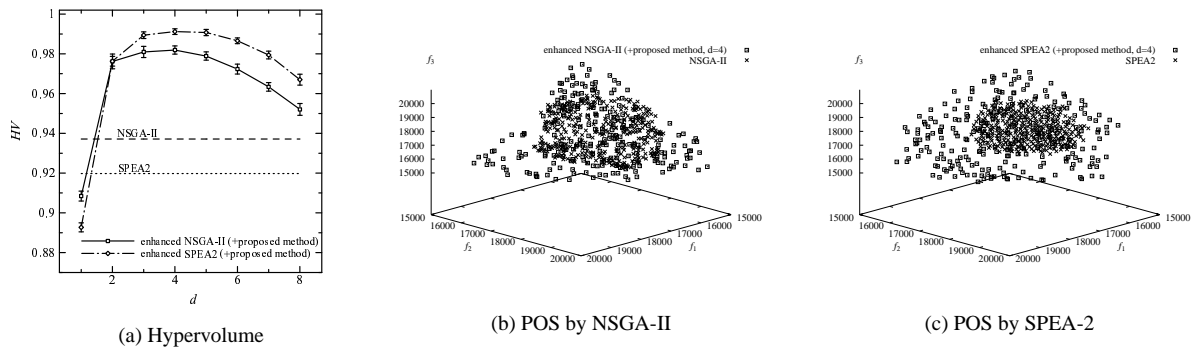


Figure 3: Hypervolume and obtained POS for KP500-3 ( $m = 3$ )

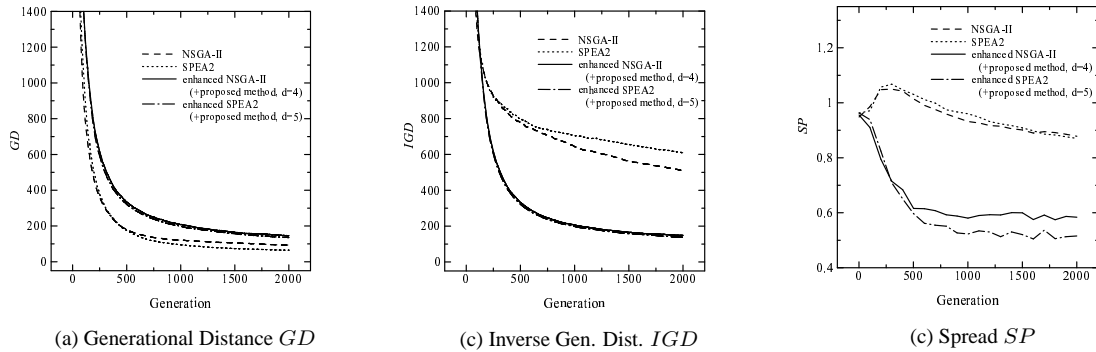


Figure 4: Transition of  $GD$ ,  $IGD$ , and  $SP$  over generations (KP500-2)

## 5. Conclusions

In this paper, we have proposed a method to enhance MOEAs by performing a distributed search based on local dominance and local recombination. We verified that enhanced NSGA-II and SPEA2 implemented with our method show better search performance to obtain fully spread POS than the conventional versions of the same algorithms. Also, we showed that another important advantage of our method is a reduction in the entire computational cost. As future works, we would like to improve this method to achieve higher convergence of POS while keeping diversity as it is. We would also like to introduce a more flexible and adaptive mechanism for population division by opti-

mally controlling the degree of local dominance and local recombination.

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