ランダムスペースフィリングカーブに基づくディジタルハーフトーニングのアルゴリズム

浅野哲夫
北陸先端科学技術大学院大学 (石川県辰口町)

本文では、ランダムスペースフィリングカーブに基づく新たなディジタルハーフトーニングの技法を提案する。ランダム性を導入するのは、明るさの一様な部分において一定方向の縞模様が発生するのを抑制するためである。実行時間の短縮化を考えると、画面分割によって誤差拡散の依存性を一定値で打ち切るのがよいが、分割パターンが目立つという欠点もある。その改善策として、各画素に伝搬されてくる誤差の量に応じて受け取る誤差量を加減するという適応型の方法を提案する。実験結果についても述べる。

Digital Halftoning Algorithm Based on Random Space-Filling Curve

Tetsuo Asano
JAIST (Tatsunokuchi-cho, Ishikawa)

This paper introduces a new digital halftoning technique based on error diffusion along a random space-filling curve. The purpose of introducing randomness is to erase regular patterns which tend to arise in an image area of uniform intensity. For efficient implementation we need to remove long dependency by partitioning the entire image into small square regions. Due to this partition, however, region boundaries may become clearer. For this reason we introduce an adaptive method to distribute error to neighboring pixels based on how many neighboring pixels affect them. Some experimental results are also given.

1 Introduction

Digital halftoning is a well-known technique in image processing to convert an image having several bits for brightness levels into a binary image consisting of black and white dots. Up to now, a large number of methods and algorithms for digital halftoning have been proposed (see e.g., [2, 3, 4, 5, 7, 4]). The error-diffusion method [3] among them is known to be good enough in quality of its output images and efficiency in ordinary serial machines. It scans each pixel in a raster manner and determines its output level by comparing its gray level with 0.5. The difference of its level from 1 is distributed as error over its neighboring pixels which have not been examined yet. Unfortunately, the regularity of error distribution is sometimes recognized as regular patterning in large uniform regions. Therefore, to have better quality we need to incorporate some randomness for error distribution.

An idea here is to use random space-filling curve instead of raster scan and also to utilize a graph-theoretic concept for weighting error distribution. Recently, it has been observed that error diffusion along some space-filling curves such as Peano curve [2] and Hilbert curve [9] sometimes achieve better quality compared to the traditional error diffusion based on a raster scan. In [8] parallel implementation of cluster-dot technique using a space-filling curve is proposed. One drawback of the methods comes from the fact that such space-filling curves are usually defined recursively on a square grid plane. Thus, there is some difficulty when they are applied to rectangular images. Another drawback is found in its quality of an output image due to its recursice structure. Since it is recursively defined, each quarter of an image is completely separated and their boundaries are often recognized in the resulting binary image. This kind of problems arises in a different situation. For efficient implementaiton of error diffusion, an entire image is sometimes partitioned into small square regions to process them in parallel. Then, the region boundaries may become clearer in a large uniform region.

We have two main ideas. One is to use a space-filling curve based on random choices to avoid regular patterning. We have studied space-filling curves for dufferent standpoints [1, 6]. The other is an adaptive method for error diffusion based on the spacefilling curve to calculate how errors are to be distributed over adjacent pixels. Some experimental results are included to show the effectiveness of those ideas.

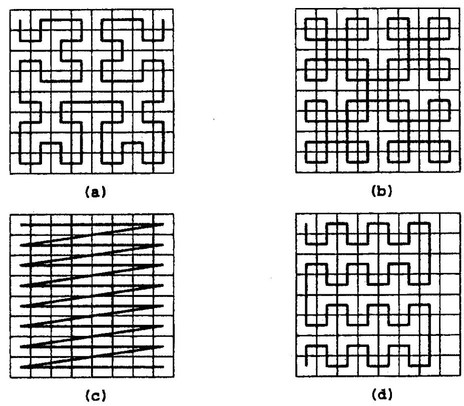


Figure 1: Representative space-filling curves. (a) Hilbert curve, (b) Sierpinski curve, (c) Raster scan, and (d) Serpentine rack.

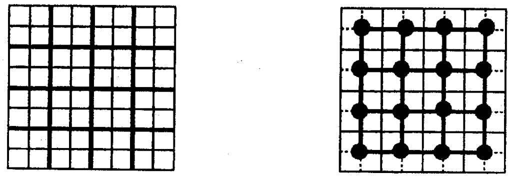
2 Random Space Filling Curve

Given a lattice plane G, a space-filling curve on G is a curve which visits every lattice point on G exactly once. Since the shape of the curve itself is not important, it is sometimes represented as a permutation of lattice points of G. Many space-filling curves such as Hilbert and Peano curves are non-selfcrossing although this property is not a necessary condition for a curve to be space-filling. A number of space-filling curves are defined in addition to those famous curves (see for example, [6]).

The idea of using space-filling curves for digital halftoning is not new. Velho and Gomes [9] use space-filling curves for cluster-dot dithering. Zhang and Webber [8] give a parallel halftoning algorithm based on space-filling curves. Asano, Ranjan and Roos [1] formulate digital halftoning as a mathematical optimization probelm and obtain an approximation algorithm based on space-filling curves. So, the digital halftoning techniques based on space-filling curves seem to be promising. However, one of their serious disadvantages is that there is some difficulty when the size of an input image is not a power of 2 since most of recursively defined space-filling curves such as Hilbert and Peano curves are defined for square lattice planes of sizes of powers of 2. One advantage of the random space-filling curve proposed in this paper is that it can be defined even for illegular-shaped lattice planes under some reasonable conditions. A more precise description for an illegular-shaped lattice plane will be given later.

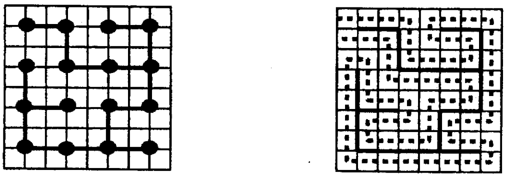
Another disadvantage of using a recursive space-filling curve comes from the shapes of the curves. Suppose that we draw space-filling curves by connecting verices which are consecutive in the order specified by the space-filling curves. Then, a straight gap is defined to be a horizontal or vertical line segment which does not intersect the space-filling curve. It should be noted that those gaps could be barriers against error propagation. Thus, long straight gaps are sometimes easily recognized in the resulting image. Figs. 1 show several representative space-filling curves which are defined recursively. It is easy to see that each space-filling curve contains long straight gaps. Especially, the serpentine rack have gaps which are as long as the side of an entire plane. On the other hand, since the random space-filling curve frequently changes its direction, it seldom contains long gaps. This is one of the advantages of the random space-filling curve.

This section presents an algorithm for generating a space-filling curve based on random choices. Let $A = (a_{ij})$, $i = 0, 1, \ldots, 2n - 1, j = 0, 1, \ldots, 2m - 1$ be a two-dimensional array with sides of even lengths. We first par-



(a) (b)

Figure 2: Partitioning an image matrix into 2×2 small submatrices (to the left) and its associated lattice graph (to the right).



(a) (b)

Figure 3: Maze which defines a random space-filling curve. (a) Spanning tree. (b) Traverse along the tree.

tition the entire array into 2×2 small arrays. We define a cell b_{ij} $(0 \le i \le n-1,\ 0 \le j \le m-1)$ to consist of elements $a_{2i,2j}, a_{2i+1,2j}, a_{2i,2j+1}, a_{2i+1,2j+1}$.

Now, we can imagine a lattice graph whose vertices are those b_{ij}'s and two vertices are joined by an edge if they are horizontally or vertically adjacent to each other. Let T be an arbitrary spanning tree of the lattice graph with a root vertex on the external boundary of the lattice plane. An example is illustrated in Fig. 2 in which lattice edges are represented by dotted lines and tree edges by solid lines. Then, starting with the root vertex on the external boundary, we repeat random choices among adjacent cells which have not been visited yet. When all of adjacent cells have been exhausted, we backtrack to the first cell which has unexplored adjacent cell and repeat random choices again. The process terminates when all the cells are visited. Then, we have a tree rooted at the vertex b_{ij} (see Fig. 3).

Now it is easy to see that the resulting tree serves as a connected wall which defines a maze on the rectangular grid plane. Thus, if we follow the wall while keeping one hand touching

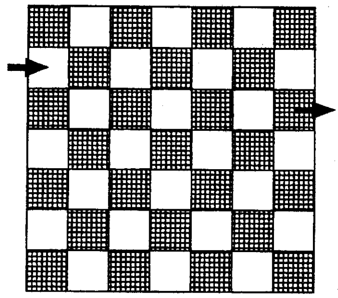


Figure 4: Is there any space-filling curve with starting and target positions specified on a lattice graph consisting of an odd number of lattice points?

the wall, we can traverse the entire image.

The above definition of a random space-filling curve on a rectangular lattice plane can be generalized to an illegular-shaped plane. It may have holes. The condition for a lattice region to satisfy is the following; It is a collection of 2×2 small lattice regions. We say that two such small regions are fully adjacent to each other if they share their horizontal or vertical sides of length two. Then, those small lattices must form a single connected component.

Figs. 5 6 show output images due to ordinary error diffusion method based on raster scan, and error diffusion along a random space-filling curve defined above. Regular patternings are recognized in Fig. 5 especially in its upper left and upper right sides while such patterns are not included in Fig. 6.

The above definition of a random space-filling curves is somewhat restrictive because it is generated by traversing along a spanning tree over an image. However, it is not easy to have a general random space-filling curve which makes a random choice at every location. Fig. 4 shows an example of a lattice plane which consists of an odd number of cells. When starting position and target position are specified in different parities, there is no space-filling curve between them.

3 Weighted Error Diffusion

It is sometimes difficult to use a random space-filling curve over an entire image due to the constraint on memory consumption, which requires us to partition an image into blocks and to perform halftoning for each such block separately. In such a case block boundaries tend to be recognized in the resulting image. Such problem due to block boundaries also occurs when recursively defined space-filling curves such as Peano or Hilbert curves are used. One method to have smooth boundaries is to carefully design weights for error distribution.

Our idea is as follows: As was stated above, error at a current pixel is distributed over neighboring pixels which have not been processed. Floyd and Steinberg [3] suggest some weights for each neighboring pixels which are determined probably according to empirical knowledge. Our idea is to distribute larger portion of the error to neighboring pixels which receive error from smaller number of pixels.

As a preporcessing step we compute for each pixel p the number of pixels from which errors are sent to p, which we denote by $d(p)$. Then, error e generated at a pixel p is distributed to its neighboring pixels $p_1, p_2, \ldots, p_k$ according to the formula:

$$r(p_i) = \frac{e}{d(p_i)} / \sum_{j=1}^{k} \frac{1}{d(p_j)}.$$

Fig. 7 shows an output image by error diffusion along Hilbert curve with usual distribution ratios and Fig. 8 shows one by error diffusion along the same curve with error distribution defined above.

This approach looks time and storage-consuming. Although it is hard to save storage consumption, its computation time is not a problem if we computed in advance the error distribution ratios when we designed a random space-filling curve.

Some other examples are shown in Figs. 9 (a) and (b). The output above is generated by traditional error diffusion and the one below by error diffusion along a random space-filling curve with adaptive error distribution method. The readers may recognize some regular patterns in the hood of a car in the output based on traditional error diffusion.

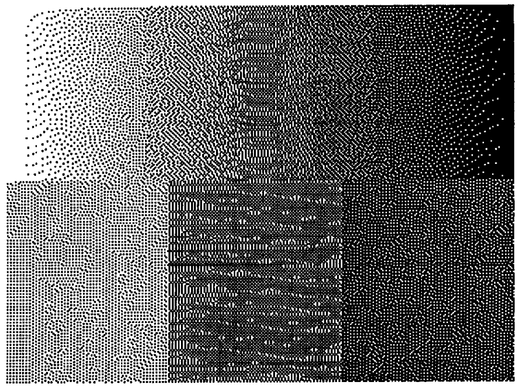


Figure 5: Output image by ordinary error diffusion method.

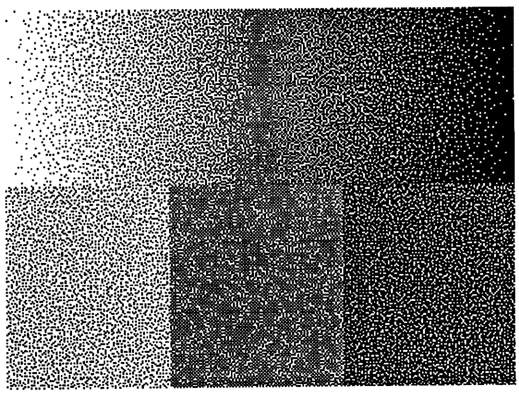


Figure 6: Output image by error diffusion along a random space-filling curve.

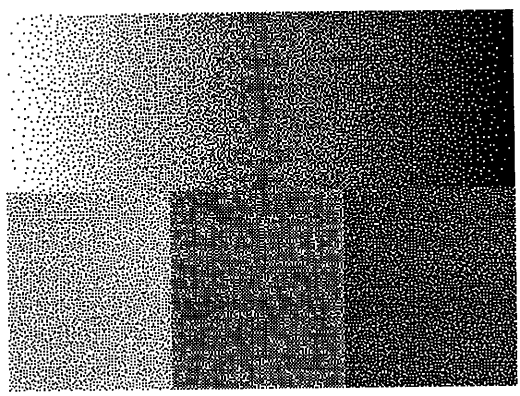


Figure 7: Output image by error diffusison along the Hilbert curve with ordinary distribution ratios.

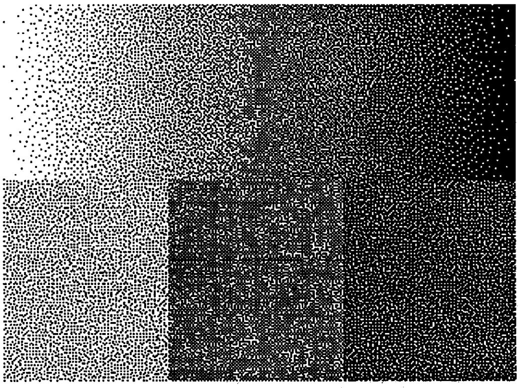


Figure 8: Output image by error diffusion along a random space-filling curve with weighted error distribution.

4 Concluding Remarks

In this paper we have proposed a new algorithm for digital halftoning based on a random space-filling curve induced by a random spanning tree. Using space-filling curves for halftoning is not new, but we experimentally showed that randomness is important to use space-filling curves as a guide for halftoning. Space filling curves have been extensively investigated for various applications in mind [1, 6, 8, 9]. To the author's best knowledge, this is the first efficient algorithm for generating random space-filling curve.

Experimental results show that error diffusion slong a random space-filling curve distributes black dots not to generate regular patterns in a region of uniform intensity while it may blur another region of non-uniform intensity. Therefore, the following starategy may be better: First we partition a given image into disjoint regions according to the uniformity of intensity levels, and then design a space-filling curve which is random in the uniform regions and raster scan in the non-uniform regions.

Acknowledgement

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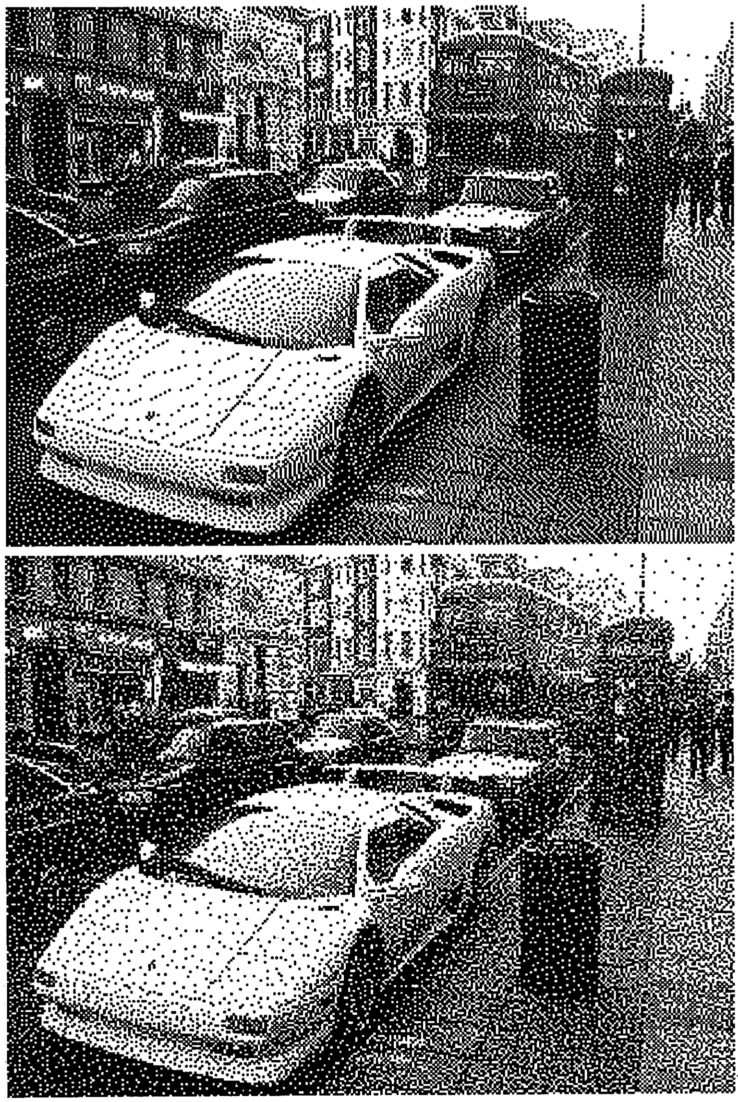

Figure 9: Output images due to (a) error diffusion method and (b) random space-filling curve.