プログラミング-言語・基礎・実践- 11－14
(1993. 3. 11)

並行計算のための簡約意味論について

本田 耕平　　吉田 展子

慶応義塾大学 大学院 計算機科学専攻

本論文では簡約に基礎を置いた並行プロセス計算のための等式理論を紹介する。この等式理論は観察や停止性の概念を用いずに相互模倣性を性格付けできる点が従来の研究と異なっている。我々はまず弱い等式理論と、それをもとに導出される強い等式理論を非同期プロセス計算、ν-計算に適用する。さらに、CCS、π-計算、λ-計算へこの等式理論を適用した結果についても簡単に触れる。

Reduction Theories for Concurrent Calculi

Nobuko Yoshida　　Kohei Honda

yoshida@mt.cs.keio.ac.jp　　kohei@mt.cs.keio.ac.jp

Department of Computer Science, Keio University
3-14-1, Hiyoshi, Kohoku-ku, Yokohama, 223, Japen

A formulation of semantic theories for processes which does *not* rely on the notion of observables or convergence, is studied. The new construction is solely based on reduction relation and equational reasoning, but can induce meaningful theories for processes, both in *strong* and *weak* settings. The resulting theories in many cases coincide with, and sometimes generalizes, observation-based formulation of behavioural equivalences. The construction is performed for a small system called ν-calculus. We also briefly summarize the results of its application to CCS, π-calculus and λ-calculus.

1 ν-calculus

Basic definitions for ν-calculus are presented. After defining terms and reduction relation, we formulate two transition relations based on distinct notions of observables, each of which induce quite different behavioural equivalences.

We assume the infinite set of *names*, ranged over by $a, b, c, \ldots$ or $x, y, z, \ldots$. Then the set of terms, $\mathbf{T}_\nu$, is given by the following grammar. $P, Q, R, \ldots$ range over the set.

$$P \ ::= \ \leftarrow av \ \mid \ ax.P \ \mid \ |x|P \ \mid \ P,\ Q \ \mid \ !ax.P \ \mid \ \Lambda$$

where "$!ax.P$ we assume $a \neq x$. Among terms, "$\leftarrow av$" denotes a *message* to a target a carrying a value v, while "$ax.P$" denotes an receptor which receives a message and instantiates the value in its body. In $ax.P$, x binds free occurrences of x in P (like x in $\lambda x.P$). "$|x|P$" is a *scope restriction* and x in $|x|$ binds free occurrences of x in P. "P, Q" is a concurrent composition of P and Q. "$!ax.P$" is a lazy replicator. "Λ" is a syntactic convention to denote nothing. *Free (resp. bound) names* in P is denoted by $\mathcal{FN}(P)$ (resp. $\mathcal{BN}(P)$). We also assume the usual notion of substitution, $[v/x]$ (or σ), and α-conversion. For technical convenience we hereafter identify α-convertible terms. Some conventions: we write $\leftarrow c$ and $c.P$ to mean we do not care the value to be communicated; $|xy|P$ denotes $|x|(|y|(P))$; Finally we will freely use parenthesis to be explicit about syntactic construction.

Reduction relation provides the basic notion of computing in the formalism. To formulate reduction we first stipulate a set of structural rules following Milner [14] (cf. [4]). We define $\equiv$ to be the smallest equivalence relation over terms generated by:

(1) $(P,Q),R \equiv P,(Q,R)$		(4) $P,Q \equiv Q,P$
(2) $P,\Lambda \equiv P$		(5) $!ax.P \equiv ax(P, !ax.P)$
(3) $\lvert x\rvert P,Q \equiv \lvert x\rvert(P,Q)$	$(x \notin \mathcal{FN}(Q))$	(6) $P \equiv Q$ then $P,R \equiv Q,R$ and $\lvert x\rvert P \equiv \lvert x\rvert Q$.

Let $\partial, \partial', \ldots$ range over a sequence of concurrent composition of terms of the forms $\leftarrow av$ and $ax.P$, and $\tilde{w}$ be a finite string of names. The main definition follows.

<u>DEFINITION</u> (Reduction relation)

(i) *One-step reduction, or simply reduction, denoted $\longrightarrow$, is the smallest relation over terms generated by the following rules.*

$$\text{COM}: \qquad |\tilde{w}|(\partial, \leftarrow av, ax.P, \partial') \ \longrightarrow \ |\tilde{w}|(\partial, P[v/x], \partial')$$

$$\text{STRUCT}: \qquad \frac{P_1' \equiv P_1,\ P_1 \rightarrow P_2,\ P_2 \equiv P_2'}{P_1' \rightarrow P_2'}$$

(ii) *Multi step reduction* $\twoheadrightarrow$ *is defined:* $\twoheadrightarrow \stackrel{\text{def}}{=} \rightarrow^*$.

We give some examples of reductions, along with several important expressions.

(i) Let $P \oplus Q \stackrel{\text{def}}{=} |c|(\leftarrow c,\ c.P,\ c.Q)$. Then: $P \oplus Q \rightarrow (P, |c|c.Q)$ and $P \oplus Q \rightarrow (Q, |c|c.P)$. Note $|c|c.Q$ is a term which never reduces nor interacts.

(ii) Let $\mathcal{FW}(ab) \stackrel{\text{def}}{=} !ay.\ \leftarrow by.$ Then: $\mathcal{FW}(ab), \leftarrow av \rightarrow \mathcal{FW}(ab), \leftarrow bv$. $\mathcal{FW}(ab)$ is called a *forwarder.*

(iii) Let $\mathcal{EQ}(ab) \stackrel{\text{def}}{=} (\mathcal{FW}(ab), \mathcal{FW}(ba))$. Then: $\mathcal{EQ}(ab), \leftarrow av \rightarrow \mathcal{EQ}(ab), \leftarrow bv \rightarrow \mathcal{EQ}(ab), \leftarrow av \rightarrow \ldots.$ $\mathcal{EQ}(ab)$ is called an *equator.* Note $\mathcal{EQ}(ab) \equiv \mathcal{EQ}(ba)$.

(iv) Let $\mathcal{I}(a) \stackrel{\text{def}}{=} \mathcal{FW}(aa)$. Then: $\mathcal{I}(a), \leftarrow av \rightarrow \mathcal{I}(a), \leftarrow av \rightarrow \ldots.$ $\mathcal{I}(a)$ is called an *identity receptor.*

(v) Let $\Omega \stackrel{\text{def}}{=} |o|(\leftarrow oo,\ \mathcal{I}(o))$. Then: $\Omega \rightarrow \Omega \rightarrow \Omega \rightarrow \ldots.$

We take $\approx_s$ and $\approx_a$ from [8], $\sim_s$ and $\sim_a$ are corresponding strong bisimilarities.

2 Reduction Theories for ν-calculus

2.1. ν-theories. *A ν-theory,* or simply a theory, is a formal theory, their formulae of the form $P = Q$, with at least the following axioms and rules.

(1)	$P = P$	(6)	$P = Q \Rightarrow P, R = Q, R.$
(2)	$P = Q \Rightarrow Q = P$	(7)	$P = Q \Rightarrow R, P = R, Q.$
(3)	$P = Q,\ Q = R \Rightarrow P = R$	(8)	$P = Q \Rightarrow \lvert x\rvert P = \lvert x\rvert Q.$
(4)	$P = Q \Rightarrow ax.P = ax.Q.$	(9)	$ax.P = by.Q \Rightarrow\ !ax.P = !by.Q.$
(5)	$P = Q$ when $P \equiv Q$		

Some notations:

(i) $\Im$, $\Im'$, ... range over ν-theories. The minimum (in the sense of (iv) below) ν-theory is denoted by $\Im_{\equiv}$.

(ii) If $P = Q$ is provable in $\Im$ then we write $\Im \vdash P = Q$, or sometimes $P =_{\Im} Q$. Specifically we often write $P \equiv_\nu Q$ for $\Im_{\equiv} \vdash P = Q$.

(iii) Given a set of equations $\mathcal{E}$, $\mathcal{E} + \Im$ is the result of adding equations as axioms to $\Im$. $\mathcal{E}+$ denotes $\mathcal{E} + \Im_{\equiv}$. $\Im + \Im'$ is a result of adding the set of equations from two theories to above rules[1]. We extend this to an arbitrary family of ν-theories, writing $\sum\{\Im_i\}_{i\in I}$, I being an index set.

[1]Note we do *not* take the union of the rules.

(iv) The relation induced by a theory $\Im$ is denoted by $|\Im|$. Given a family of ν-theories, the *maximum* (resp. *minimum*) theories are those whose corresponding relations are the maximum (resp. minimum) in that family. We say a ν-theory $\Im$ is a *subtheory* of another theory $\Im'$ if $|\Im| \subset |\Im'|$. If the inclusion is strict then the former is a *proper subtheory* of the latter.

(v) We say a theory is *consistent*, written $\Im \in \mathsf{Con}$, if it does not equate all possible pairs of terms (i.e. not $|\Im| = \mathbf{T}_\nu \times \mathbf{T}_\nu$). A theory is *inconsistent* if it is not consistent.

2.2. Reduction closure property. The notion of *states* is essential in concurrency. A term may change its meaning during its reductions so that "equality" in this setting means that two equated terms can go to an equated state again. But what is this equation here, if there is no notion of observables available? The purely equational reduction-based closure property to follow is one of answers to this question[2].

<u>DEFINITION</u> *A ν-theory $\Im$ is reduction-closed, if, whenever $\Im \vdash P = Q$, $P \twoheadrightarrow P'$ implies, for some Q', $Q \twoheadrightarrow Q'$ and $\Im \vdash P' = Q'$.*

We often call reduction-closed ν-theories simply *reduction theories.* The following property of reduction theories is important.

<u>PROPOSITION</u> *Let $\Im_1$ and $\Im_2$ be two reduction theories. Then $\Im_1 + \Im_2$ is also a (possibly inconsistent) reduction theory. This is extended to an arbitrary sum of reduction theories.*

Unfortunately the closure property alone may not induce any canonical equational constructions, except the minimum (which is e.g. $\Im_\equiv$) and the maximum (which is any inconsistent theory). To amend the situation, we refine our equational scheme by incorporating the notion of *insensitivity.*

2.3. Insensitivity and sound theories. Let C be a one-hole context and C_n a n-holes context, with each hole occurring exactly once in the latter. Then we define:

$$C \rightarrow C_n \quad \Leftrightarrow_{\text{def}} \quad \exists \sigma_1, ..., \sigma_n . \forall P \in \mathbf{T}_\nu .\ C[P] \rightarrow C_n[P\sigma_1]...[P\sigma_n]$$

and similarly $C \twoheadrightarrow C_n$. We read $C \rightarrow C_n$ (resp. $C \twoheadrightarrow C_n$) "the context C generically one-step (resp. multi-step) reduces to C_n". We also use the notation $C \equiv_\nu C'$ which is defined in a similar fashion.

[2]Essentially the same construction is referred to at the end of [15], though somewhat negatively.

DEFINITION *Let $C[\ ]$ be an arbitrary context. A term P_0 is insensitive if P_0 is in a set $\mathbf{T}'$ where for all $P \in \mathbf{T}'$, $C[P] \twoheadrightarrow P'$ implies $P' \equiv_\nu C'_n[Q_1]...[Q_n]$ where $C \twoheadrightarrow C'_n$ with $Q_i \in \mathbf{T}'$, $i = 1..n$.*

The set of insensitive terms is denoted Ins_ν. It is easy to see that this set is also closed in the way depicted in the definition. We call a reduction theory which equate insensitive terms *sound.*

Note we immediately get:

PROPOSITION $\Im_{\mathsf{ins}_\nu} \stackrel{\text{def}}{=} \{(P = Q) | P, Q \in \mathsf{Ins}_\nu\}+$ is sound and reduction-closed.

2.4. Intrinsic observables. An essential fact about reduction-closed ν-theories is that *sound and consistent theories are, a posteriori, automatically equipped with observables.* Since it is induced from reduction relation relative to the base equations which seem hardly questionable, the induced observables may be regarded intrinsic in the calculus, though confined to the weak setting.

Let a pair of terms be *incompatible*, written $P \# Q$, if for any sound $\Im$, $\Im \vdash P = Q \Rightarrow \Im \notin \mathsf{Con}$. One such pair becomes essential in deriving observability.

LEMMA $\leftarrow c \# \Lambda$, $c.\Lambda \# \Lambda$, and $(\leftarrow c, c.\Lambda) \# \Lambda$.

We now formulate a notion of "generic observable" intrinsic in sound theories, by a simple transition system.

$$|\tilde{w}|(\partial, \leftarrow av, \partial') \stackrel{\uparrow a}{\rightsquigarrow} |\tilde{w}|(\partial, \partial') \quad (a \notin \{\tilde{w}\}) \qquad \frac{P \equiv P' \quad P \stackrel{\uparrow a}{\rightsquigarrow} Q \quad Q \equiv Q'}{P' \stackrel{\uparrow a}{\rightsquigarrow} Q'}$$

THEOREM (Observability) *Let $\Im$ be a sound and consistent reduction theory and $\Im \vdash P = Q$. Then*

$$P \stackrel{\uparrow a}{\rightsquigarrow} P' \quad \Rightarrow \quad Q \twoheadrightarrow \stackrel{\uparrow a}{\rightsquigarrow} \twoheadrightarrow Q' \text{ for some } Q' \text{ with } \Im \vdash P' = Q'.$$

Note the transition relation by itself induces a version of bisimilarity which is obviously non-trivial[3]. Hence we have:

COROLLARY *Let $\Im_1$ and $\Im_2$ be consistent and sound. Then $\Im_1 + \Im_2$ is also consistent and sound. This extends to arbitrary sum of consistent and sound theories.*

[3]A relation is trivial if it is empty or universal.

It is clear now that the observability theorem assures us the existence of the maximum consistent sound theory.

PROPOSITION *Let* $P \cong Q \Leftrightarrow_{\text{def}} \Im \vdash P = Q$ *for some consistent and sound* $\Im$*. Then we define a theory* $\Im^*_\nu$ *as* $\Im^*_\nu \stackrel{\text{def}}{=} \{(P = Q) \mid P \cong Q\}+$*. Then* $\Im^*_\nu$ *is sound and consistent. Moreover it is maximum among such theories.*

2. 5. Strong theories and Bisimilarities. Reduction theories are essentially incepted in a "weak" semantic framework, since they do not care the number of reduction steps, less the termination. Notably we have $\Omega = \Lambda$ in any sound theories. The following construction now tells us that we can easily "derive" strong theories out of them. The construction may still be regarded natural and consistent, since a "weak" theory firstly tells us what is the abstract *meaning* of each term, then we refine the induced equations by considering exact steps a term needs to reach another semantic point.

3 Discussion

3. 1. Reduction theories for other Calculi. This section briefly summarized that results for applying our construction to not only other process calculi (CCS[11, 13], π-calculus[12, 14]), but also λ-calculus.

CCS. One essential issue in constructing reduction theories for CCS is that the summation is problematic in weak congruent theories in general. However we restore weak bisimilarity in our purely equational setting by considering the maximum sound theory among subset of CCS-contexts where a hole does not occur as a subexpression of a summand. In CCS the soundness property leads to synchronous bisimilarities, both in the weak and strong cases.

π-calculus. π-calculus[12, 14] is a superset of ν-calculus and is based on synchronous name passing. There are some versions of the calculus and reduction theories can be formulated for all of them; we, however, would like to refer to one interesting point on sound theories for the fragmentary π-calculus[14]. As night be expected, we have a result of generic synchronous observables for sound theories, i.e. input are also taken account. What was rather unexpected is the fact that the *equators* are also functional in the synchronous regime. This means that the maximum consistent sound theory in the fragmentary π-calculus contains an equation of the form:

$$\mathcal{EQ}_\pi(vw)|\bar{a}v.P \;=\; \mathcal{EQ}_\pi(vw)|\bar{a}w.P$$

where in π-calculus we write $\mathcal{EQ}_\pi(vw) \stackrel{\text{def}}{=} !v(x).\bar{w}x|!w(x).\bar{v}x$. The reason of this is that equators, by their very existence, *transforms* synchronous communication into asynchronous one, and then functions as in the asynchronous regime, i.e. *equates* names. Thus the maximum consistent sound theory for the fragment becomes strictly more general than "early" weak bisimilarity[4].

λ-calculus. Equational theories based on reduction and equation are extensively studied in terms of λ-calculus [3]. Specifically β-equality is subsumed in our reduction-closure property as noted, not to say many of our formal constructions are inspired by λ-theories. A natural question is; can we get any (interesting) observable if we equate insensitive terms? The answer is yes, at least in a certain family of sound theories where essential equations are beside α-equality, the equation of strongly unsolvable terms and the equation of PO_∞ terms[5]. We say a reduction theory is *ω-sound* if it is sound and, moreover, equate all PO_∞ terms. In the family of ω-sound theories, we can pick up *head-normal forms* as generic observables, though still without β-equality. Let $\Im$ be an ω-sound reduction theory over λ-terms. Then:

$$\Im \vdash M = N \;\Rightarrow\; (M \text{ has some head normal form} \;\Rightarrow\; N \text{ has some head normal form}).$$

Since termination (cf. Lazy λ-calculus[2, 16]) is not regarded as fundamental here, this notion of generic observable should be regarded as natural. An important corollary of this is again the existence of the maximum consistent theory in the family of theories, which coincides with $\mathcal{K}^*$. Further investigation in this line is left to future expositions.

3. 2. Further Issues. While reduction-based formulation of concurrency semantics has proved to be significant in varied formalisms, it remains to see how general it in fact is. Two further directions should be pointed out. Firstly we should ask whether the present framework can extend to other formalisms and other operators, and also to varied notions of equivalences (c.f. [1]). Secondly, and possibly more importantly, investigation of the formal mechanism underlying the present construction, which after all turns out to be effective in several significant systems, is essential. This should be related with study of formal models for concurrency, since e.g. our $\Im^*_\nu$ clearly reminds us of $\mathcal{K}^*$ which is a theory of several basic models of λ-calculus. Thus, for example, $\Im^*_\nu$ may be used as a hint to construct mathematical (i.e. syntax-free) models of ν-calculus. This line of investigation may also be helpful in understanding the nature of structural rules.

We hope further investigation of the reduction theories and related formulations, will contribute to deeper understanding of semantic structure of concurrent computation, and lead to their applications to significant semantic problems, both theoretical and pragmatic.

---

[4] In the strong setting, the theory corresponding to the maximum consistent sound theory coincides with "early" strong bisimilarity as formulated in [12].

[5] We say M is a PO_∞ term, if for any natural number n, there is M' such that $M \twoheadrightarrow \lambda x_1..x_{n'}.M'$ with $n' \geq n$[10, 16].

References

[1] Abramsky, S. : Observational Equivalence as a Testing Equivalence, *Theoretical Computer Science*, 53, 1987.

[2] Abramsky, S. : The Lazy Lambda Calculus, *Addison Wesley*, 1988.

[3] Barendregt, H. : *The Lambda Calculus: Its Syntax and Semantics.* North Holland, 1984.

[4] Berry, G. and Boudol, G. : The Chemical Abstract Machine. In *POPL*, 1990.

[5] Boudol,G., *Asynchrony and π-calculus*, a typescript, March 1992.

[6] Honda, K. and Tokoro, M., An Object Calculus for Asynchronous Communication, In: *Proc. of European Conference on Object-Oriented Programming*, LNCS, Springer-Verlag, July 1991. Keio CS report 92-001 (extended version), November 1992.

[7] Honda, K., *Notes on Soundness of Mapping from π-calculus to ν-calculus*, A typescript, April 1992.

[8] Honda, K., *Two bisimilarities in ν-calculus*, Submitted to *Theoretical Computer Science*, Keio CS technical report 92-002, September 1992.

[9] Honda, K. and Yoshida, Y., *Reduction Theories for Concurrent Calculi: Behavioural Semantics without Observables*, Keio CS technical report 92-003, October 1992.

[10] Longo, G., Set Theoretical Models of Lambda Calculus: Theory, Expansions and Isomorphisms, *Annals of Pure and Applied Logic*, 24:153-188, 1983.

[11] Milner, R., *Calculus of Communicating Systems.* LNCS 92, Springer-Verlag, 1980.

[12] Milner, R., Parrow, J.G. and Walker, D.J., *A Calculus of Mobile Processes. Part I/II.* ECS-LFCS-89-85/86, Edinburgh University, 1989

[13] Milner, R., *Communication and Concurrency.* Prentice Hall, 1989.

[14] Milner, R., Functions as Processes. In *Automata, Language and Programming*, LNCS 443, 1990.

[15] Milner,R. and Sangiriorgi,D., Barbed Bisimulation, ICALP, 1992.

[16] Ong,C-H.L., The Lazy Lambda Calculus: An Investigation into the Foundation of Functional Programming, Phd thesis in Imperial College, May 31, 1988.