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Gathering in Carrier Graphs
- Meeting via Public Transportation System -

Haozhi ZHENG† Ryota EGUCHI† Fukuhito OOSHITA‡ Michiko INOUE†
Nara Institute of Science and Technology† Fukui University of Technology‡

1. Introduction

The *gathering problem* is a fundamental problem in distributed computing. It requires that a set of agents with no direct communication and no common coordinate system reach the same node of a graph in a finite time.

Recently, distributed algorithms for highly dynamic graphs have been intensively studied [1]. These are dynamic graphs whose dynamics are continuously changing over time. The carrier graph (C-graph) is one class of these graphs that models a system where one or more carriers periodically visit sites in the system by following their routes. Agents can move with a carrier when some carrier comes to their current site. Practical examples of this model include public transportation systems like buses, planes, and satellites.

This paper considers the gathering problem in carrier graphs. This can be seen as a problem where people try to meet at some station (unknown in advance) in a public transportation system. We consider several assumptions on prior knowledge of agents such as the counts of agents or sites, and, on acquirable information at sites such as identifiers or the number of agents at the site or the site identifier, and for each assumption, clarify the solvability of the problem and propose algorithms for solvable cases.

Table 1. Solvability and time complexity under assumptions

Assumptions		Time Complexity		
Knowledge	Observation ability	Circular	Simple	Arbitrary
P	Any	$p+P$	$p+P$	$p+P$
k	Any	p	p	p
n	No	$2p$	$p+n(n-1)$	Impossible
Any	Agent ID	$3p$	$4p-1$	Impossible
No	Site ID	$2p$	$2p+1$	Impossible
n	Site ID	$2p$	$2p$	$2p$

We examine the solvability and propose algorithms (if solvable) for three classes of single-carrier graphs. We consider assumptions on agents including prior knowledge - the upper bound P on period, the number k of agents, and the number of sites - and observation abilities on agents' ID or sites' ID. The gathering problem's solvability and proposed algorithm's time complexity under each set of assumptions are shown in Table 1.

Furthermore, we also propose a gathering algorithm that terminates in finite rounds for carrier graphs with multiple carriers.

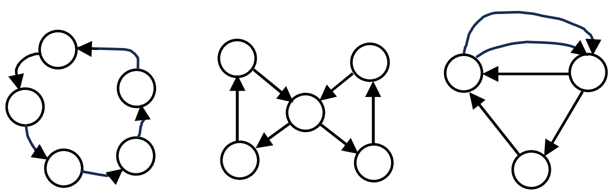
2. Problem Definition

2.1 Carrier Graph

We consider a system composed of a set S of n sites and a set C of m carriers. Each carrier $c \in C$ has a unique identifier $id(c)$ and an ordered sequence of sites $\pi(c) = \langle s_0, s_1, \ldots, s_{p(c)-1} \rangle, s_i \in S$, called a route, where the positive integer $p(c)$ is called a period of the route. The carrier c moves along the route in a cyclic manner.

Each route $\pi(c)$ defines an arc-labelled multi-graph $\vec{G}(c) = (S(c), \vec{E}(c))$, where $\vec{E}(c) = \{(s_i, s_{i+1}, i) : 0 \leq i < p(c)\}$. The set of all routes of carriers is denoted by $R = \{\pi(c) : c \in C\}$, and a period of R is defined as $p(R) = max\{p(c): c \in C\}$. When no ambiguity arises, we will simply denote $p(R)$ as p. The arc-labelled multi-graph $\vec{G}(C) = (S, \vec{E})$, where $\vec{E} = \bigcup_{c \in C} \vec{E}(c)$, is called *carrier graph*, or shortly, *C-graph*. Especially, a carrier graph with only one carrier is called a single carrier graph, or shortly, SC-graph.

SC-graphs can be categorized by the properties of their routes into three classes – circular, simple, and arbitrary. A route is simple if it has no multi arcs. A simple route is circular if it includes no repeated sites.



(a) Circular (b) Simple (c) Arbitrary

Fig. 1. Classes of routes and SC-graphs

2.2 The Gathering Problem

There are k mobile computational entities in the system $a_0, a_1, \ldots, a_{k-1} \in A$, called agents. Agents operate in a LOOK-MOVE-COMPUTE manner in each synchronous round. An agent can stay at the current site or move with one of the carriers.

The goal of the gathering problem is to gather all the agents within a finite time, that is to let every agent $a \in A$ move to the same site $s \in S$ and terminate, within finite time, regardless of the starting position.

3. Our Approaches

The key idea in addressing the gathering problem on general carrier graphs involves decomposing it into two parts: (1) a gathering problem on a static graph where each node exhibits one carrier, and (2) a gathering problem on a singular carrier graph. This decomposition is achieved by conceptualizing each carrier as a node within a static graph, called *meeting graph*. In this framework, solving the gathering problem on the meeting graph ensures that each agent is positioned along the route of a single carrier, thereby constituting the scenario for the gathering problem on a singular carrier graph.

3.1 Gathering on SC-graph

All the agents will arrive at the same site if every agent moves with the carrier when encountered, given that there is only one carrier. To solve the gathering problem, the key is to determine when to terminate the operation.

When P is known, agents can terminate if the number of agents has not changed in successive P rounds, as a sign that every agent is moving with the carrier.

When k is known, agents can easily determine whether every agent is moving with the carrier, then terminates when every agent shows up.

When n is known, n and $n(n-1)$ can be used as P on a circular or simple SC-graph respectively, given the fact that there are no multi-arc or repeated sites. Thus, agents can gather as P is known. But for arbitrary SC-graphs, we show the gathering problem is unsolvable.

When agents can observe other agents' IDs, special agents – *leader* and *landmark* (one landmark for circular SC-graphs while two different landmarks for simple SC-graphs) – can be elected. All agents will learn the leader's ID when the election is over since the leader will meet every other agent during the leader election process. The leader then moves to the location of the landmark and terminates with the landmark. Subsequently, other agents observing a carrier without a leader will recognize that it is time to head toward the leader. They will terminate upon encountering the leader again.

When sites' IDs are observable, agents can learn the site with the smallest ID and then gather there after visiting every site, which can be noticed when visiting the same site twice for circular SC-graphs (or the same arc twice for simple SC-graphs). The gathering is impossible for arbitrary SC-graphs since the end of visiting cannot be detected by agents. However, additional knowledge of n enables agents to be aware of every site is visited, so that the gathering is possible with knowledge of n.

Theorem 1. Let $\mathcal{A}$ be a set of assumptions. The gathering problem on a single carrier graph can be solved if $\mathcal{A}$ satisfies the conditions not marked as "Impossible" in Table 1 and is unsolvable if $\mathcal{A}$ satisfies only the conditions marked as "Impossible" in Table 1.

3.2 Gathering on General C-graph

When gathering on an anonymous C-graph with multiple carriers, we consider a hierarchical gathering strategy. That is, agents first gather in the meeting graph $H(C)$, which means arriving at the same carrier's route, then gathering on that carrier with algorithms for SC-graphs.

For an unknown C-graph, an exploration is required so that each agent maps the meeting graph $H(C)$. Ilcinkas et al. [2] propose an exploration algorithm EXPLORE-WITH-WAIT. Given the a priori knowledge of an upper bound $B = O(p)$ on the maximum period p, the worst-case time complexity is $\Theta(np)$. Given that the time complexity for gathering on a SC-graph is $O(p)$, the overall time complexity is $\Theta(np)$.

Theorem 2. The proposed algorithm solves the gathering problem on C-graphs in $\Theta(np)$ rounds.

4. Conclusion

We started an exploration of a variant of the gathering problem: gathering in carrier graphs. We analyzed several factors that affect the feasibility and the time complexity of gathering in single carrier graphs. Moreover, we extended our algorithms to solve the gathering problem in general carrier graphs.

Reference

[1] Casteigts et al. Time-varying graphs and dynamic networks. *IJPEDS*, 27(5):387–408, 2012.

[2] Ilcinkas et al. Exploration of carrier-based time-varying networks: The power of waiting. *TCS*, 841:50–61, 2020.