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Dimensionality reduction of spherical shell structure in diffractive imaging

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1 Introduction

Reconstructing a target object image by the Fourier phase retrieval is so called "*Diffractive imaging.*" The lost phase information in scattering needs to be estimated by the phase retrieval method. However, a retrieval result is not uniquely determined from the noise-contaminated diffraction pattern, because the distribution of phase-retrieved images spread widely on object-function space. The images distribution forms a spherical shell structure. Therefore, the average of the images has been used as a plausible reconstructed result at many experiments using X-ray, raiser and electron sources [1]. In this report, we introduced an universality of the spherical shell structure based on the finite iterations of phase-retrieval algorithm. Also, the capability using the phase-retrieved images with the reduction of the complexity of the images was presented by applying PCA (Principal Component Analysis).

2 Diffractive imaging

The Gerchberg-Saxton (GS) algorithm has been used for retrieving the Fourier phases using the Fourier-intensity measurement [2]. As shown in Fig. 1, the algorithm is constructed by repeating the forward and inverse Fourier transforms and applying the object and Fourier domain constraints. The observed diffraction pattern is used as the Fourier-domain constraint, and the object support and prior information of the target image correspond the object-domain constraint.

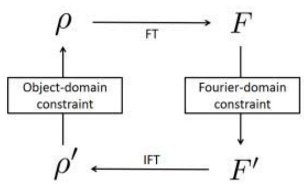

Figure 1: Schematic diagram of the GS algorithm

GS algorithm works to retrieve the Fourier phase with an optimal setting of object-domain constraint. Several methods applying the constraint have been proposed. The error reduction (ER) and hybrid input and output (HIO) methods have been fundamentally used as an object-update method [3]. The phase-retrieved images from an incomplete diffraction pattern form a spherical shell structure [1]. The average of the phase-retrieved images is obtained by the retrieved images, that is $\sum_{i=1}^{N} \hat{\rho}_i / N$, where $\hat{\rho}_i$ is each retrieved image $(i = 1, 2, ..., N)$ and N is the number of the images.

3 Structure of phase-retrieved images

We used a target object (256×256 pixels with a triangle cut of sky image) and perfect support fitting to the target. The figures are presented in left and middle of the top row in Fig. 3, respectively. The Fourier intensity of the target is presented at top right in Fig. 3. And also, the Poisson-noise contaminated intensities (total count $= 10^7, 10^6, 10^5$) are presented at bottom row in Fig. 3, respectively.

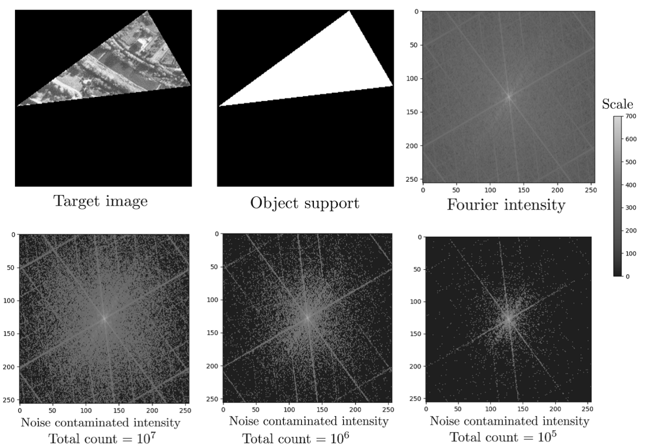

Figure 2: Top row: target image (left), object support (middle) and Fourier intensity of the target (right), Bottom row: Poisson-noise contaminated intensities for three types of total count.

As a fundamental setting of our numerical experiment, the GS algorithm with 2000 iterations was used for an initial start. 2000 random images as the initials were used for noise free and three kinds of noise-contaminated intensities, respectively. Concerning the object update procedure, HIO (10 iterations) and ER (10 iterations) were reciprocally used. The parameter of HIO algorithm was 0.8. By the phase-retrieval procedure with above setting, 2000 phase-retrieved images were obtained by using each Fourier intensity as the Fourier-domain constraint.

Let $\hat{\rho}_i^{\text{free}}$,$\hat{\rho}_i^{p7}$, $\hat{\rho}_i^{p6}$ and $\hat{\rho}_i^{p5}$ be the phase-retrieved images from noise-free, three kinds of noise intensities, where $i = 1, 2, \cdots, 2000$, and $p*$ denotes the Poisson noise case and total count 10^*. The averages for four cases were obtained as $\overline{\rho}_{\text{free}}$, $\overline{\rho}_{p7}$, $\overline{\rho}_{p6}$ and $\overline{\rho}_{\text{p5}}$. Let d_i^{free} be a distance between $\overline{\rho}_{\text{free}}$ and $\hat{\rho}_i^{\text{free}}$, where $i = 1, 2, \cdots, 2000$. And d_i^{p7}, d_i^{p6} and d_i^{p5} are also calculated.

Figure 3 presents four histograms using the distances between the averages and the phase-retrieved images in four cases, respectively. As concerning the histogram of noise-free case, a convergence of the phase-retrieving process is not found by the case of the noise free, because the phase-retrieval problem is not convex. However such the retrieval degree has been used as a good result in phase retrieval experiment. The distribution of the histograms becomes to spread from the averages. However, there is no retrieved image around the average in every case, and of cause, the target image is not be included in the retrieved images. The averages of the noise cases are plausible by the effectivity of the structure of the phase-received images.

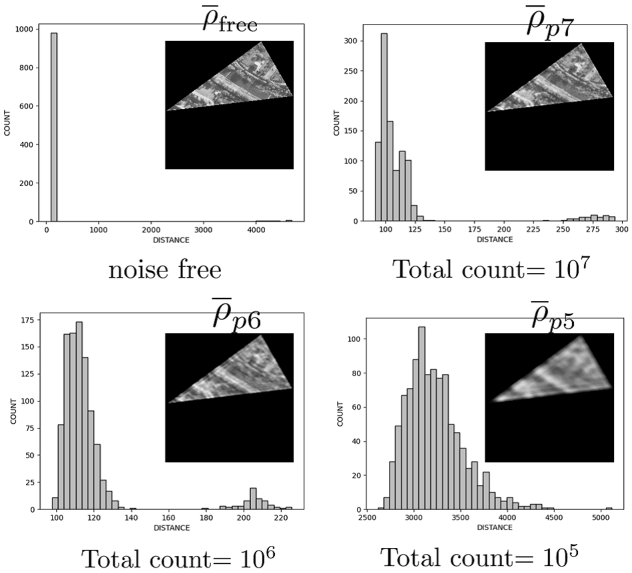

Figure 3: Four histograms of the distances between the average and each phase-retrieved image for noise-free case and three kinds of Poisson-noise cases. The averages are presented at top right of each histogram.

4 Dimensionality reduction

The dimension of object space is 65536 (256×256) in our case. The distribution of many phase-retrieved images spreads on 65536-dimension function space. Then, we applied PCA on all the images of all experiments above to prove that it hardly does damage to the spherical shell structure while most of the information is preserved. Table 1 presents the numbers of principle component concerning four type intensities. Three kinds of cumulative contribution rates (99%, 90% and 80%), are calculated. In the case of small noise (10^7) with 99% setting, the number of principle component is so large as to compare with the noise free case. The noise degree and the number of principle component are inversely proportional. However, enough phase-retrieved images are needed to form the structure of the spherical shell in the object function space.

Intensity	Noise free	10^7	10^6	10^5
Num.of p.c.(99%)	19	906	879	785
Num.of p.c.(90%)	8	475	421	261
Num.of p.c.(80%)	3	250	221	110

Table 1: The numbers of principle component for four types of intensity are presented. Three kinds of cumulative contribution rates are calculated.

5 Conclusions

In this report, we mentioned the invariant structure of the phase-retrieved images from random initials. Such the spherical shell structure is retained even in the Poisson-noise and noise-free cases. The principal component analysis is effective to maintain the structure, and the phase-retrieved images applied by PCA present the reduced dimension. Therefore, the application of PCA can significantly reduce the complexity of the images of phase-retrieved images without destroying its spherical shell structure. The mathematical relationships between the noise degree of intensity and dimension reduction by using PCA is our future work.

References

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