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# Effectiveness of Hilbert Huang Transform for analyzing gravitational wave data from supernova

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## 1. Introduction

The existence of Gravitational Wave (GW) was initially postulated by Albert Einstein in 1916 [1]. After around hundred years, in 14th of September 2015, the first GW was detected by the Laser Interferometric Gravitational Wave Observatory (LIGO) being the source as a Binary Black Hole (BBH) merger [2]. With the current sensitivity range of existing GW detectors, they are prominently detecting the GW from BBH merger or Binary Neutron Star merger.

With the sensitivity improvement of next generation GW detectors (3rd Generation), Einstein Telescope and Cosmic Explorer, they will be able to detect GW from Core Collapse Supernova (CCSN) explosions [3]. There are multiple GW excitation processes that occur inside a star before its explosion, and it is suggested that the GWs originating from the CCSN have a mode for each excitation process in terms of time-frequency representation. One of the main sources for GWs is the oscillations of the Proto Neutron Star (PNS). In the GW asteroseismology [4], those oscillations are called as eigenmodes of PNS. Basically, there are three prominent eigenmodes: *f*, *p* and *g* modes. Some simulation studies have shown that there is a correlation between *f* and $g_1$ (one particular mode of *g* modes) modes with the ramp up signal of GW generated by PNS in a such a way that the early phase and post phase of the ramp up signal in correspondence with the $g_1$ mode and *f* mode respectively [4]. Another study has shown that there is correlation between such eigenmodes and surface gravity of PNS [5].

One approach to identify such eigen modes of PNS is the time-frequency representation by a signal processing technique. For instance, one particular ramp up signal feature in a GW may consist of the different eigen modes as mentioned above. To distinguish the different modes, the time-frequency representation of conventional analysis such as Short Time Fourier Transform (STFT) doesn't have enough resolution because of the time-frequency uncertainty. On the other hand, the Hilbert Huang Transform (HHT) is not limited by time-frequency uncertainty, and thus, it provides a high resolution time-frequency representation [6].

Therefore, as a solution, this research work focuses on the application of HHT to the data analysis for the simulated GWs generated from CCSN.

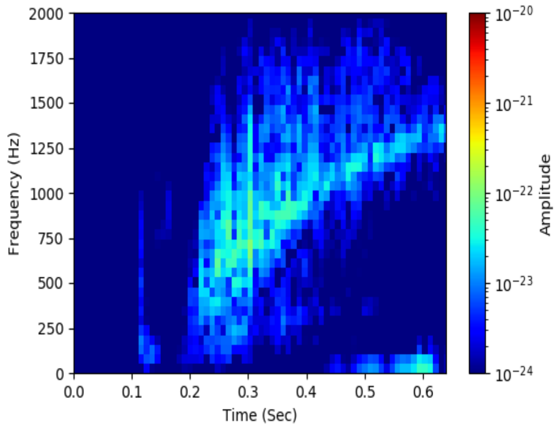
## 2. Hilbert Huang Transform

We briefly introduce HHT that has been used as a time-frequency analysis method. HHT consists of two sub parts: Empirical Mode Decomposition (EMD) and Hilbert Spectral Analysis (HSA). The key part of the HHT is EMD: the signal is decomposed into different local time components, which is known as Intrinsic Mode Function (IMF). The first step of the decomposition is the formation of upper and lower envelop by connecting local maxima and minima by cubic spline line separately. Then, the mean of upper and lower envelops is subtracted by the original signal to form the first proto-mode (0th iteration) of the IMF1. In this way, the IMF process may undergo for different number of iteration steps to obtain the IMF1 satisfying two conditions:

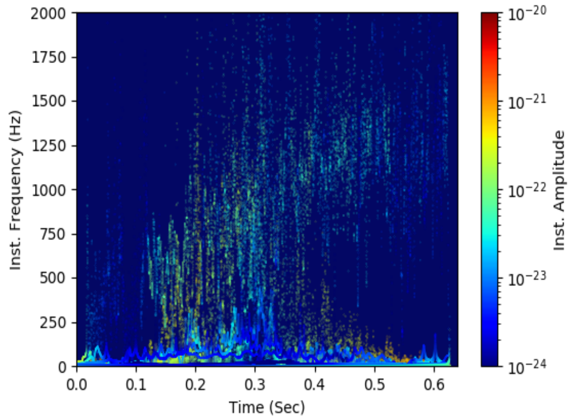
I. For all data set, the number of extrema and the number of zero crossing have to either be identical or differ at most by one,
II. At any data point, the mean values of the upper and the lower envelops defined by using local maxima and minima, respectively, have to be zero.

When a particular IMF is generated through the above procedure, that IMF is subtracted from the original signal to form the sub original signal for the next IMF. Then again, the above iteration procedure is applied to obtain the next IMF. In such a way, a set of IMFs can be extracted in the EMD process by the sifting the process of IMFs until the pre-defined stoppage criteria is satisfied. The physical meaning of obtaining IMFs is the separating different frequency components which are in the original signal in to different local time scales.

HSA is a type of time-frequency representation [6]. For the HSA process, instantaneous frequencies and instantaneous amplitudes are calculated by Hilbert Transform from each IMF component to form the time-frequency representation. The extracted instantaneous frequencies are physically meaningful due to the




(a)




(b)

***Figure 1*** *: The comparison between the STFT and HHT of LSK220-2D simulated GW. Panel (a) represents the spectrogram from STFT and panel (b) represents the time-frequency representation from HHT.*

elimination of riding waves in the original signal in the EMD process [6].

## 3. Waveform and Methodology

For this study, three simulated GW waveforms have been used [4]. All these waveforms are related to the PNS of CCSN and have been generated by 2D simulation of PNS models where they have considered different progenitor models.

The labels of the considered PNS models to generated GWs are LS220-2D, TGTF-2D and DD2-2D, respectively. The sampling frequency of all these three data sets is 16 kHz and the data length of LS220-2D, TGTF-2D and DD2-2D are 11722 samples, 5214 samples and 5212 samples, respectively.

In this study, to compare the results, for all these three GW waveforms, both STFT and HHT were applied. When the STFT is applied with the following parameters: FFT and overlap length are 300 samples, 150, respectively, and the Hann window was used.

For the HHT, as the stoppage criteria, the standard deviation method was considered [6].

## 4. Result and Discussion

The spectrogram from STFT and the time-frequency representation from HHT were generated for all three types GWs considered. In this paper, the only results related to the LSK2-2D has been shown in Figure 1(a) and (b). It can be clearly concluded that the resolution of the time-frequency representation from HHT is comparatively higher than that of the spectrogram obtained from STFT. With this high-resolution nature of HHT, it may be used to distinguish the different eigen modes composed in a ramp up signal since even very small change in the time-frequency representation could be precisely observed. In addition, HHT could be able extract the low frequency components with low amplitudes. It can be observed by Figure 1(b).

As our future work, we will introduce the ensemble EMD to suppress some problems of EMD, such as mode-splitting and mode-mixing [6]. Moreover, we will develop the method how to identify the modes on the time-frequency representation obtained from HHT.

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