Electronic Preprint for Journal of Information Processing Vol.26

Regular Paper

Further Analysis with Linear Programming on Blocking Time Bounds for Partitioned Fixed Priority Multiprocessor Scheduling

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Received: November 17, 2017, Accepted: May 10, 2018

Abstract: The recently developed FMLP$^+$ provides significant advantages for partitioned fixed priority scheduling, since it ensures asymptotically optimal $O(n)$ maximum priority-inversion blocking. The constraints under the FMLP$^+$ can be exploited to determine bounds on the blocking time. However, these bounds may be pessimistic since shared resources local to a processor do not incur priority-inversion blocking in some cases. Consequently, a schedulable task set may be erroneously judged as unschedulable because of these pessimistic values. Based on our analysis, additional constraints were added to compute the maximum blocking time bound of each task with linear programming and the corresponding worst-case response time. The results of experiments show our proposed strategy is less pessimistic than existing strategies. Meanwhile, we also demonstrate that local resource sharing should be used instead of global resource sharing where possible.

Keywords: partitioned fixed priority, multiprocessor scheduling, FMLP$^+$, blocking time bound, linear programming

1. Introduction

Since an increasing number of complex systems rely on computer control, real-time computing is becoming essential in a variety of fields.There are countless examples of applications requiring real-time embedded systems, including industrial automation, automotive networks, robotics, military systems, and even smart toys such as LEGO Mindstorms EV3 [1], [2], [3], [4].

Fixed priority preemptive partitioned scheduling algorithms for multiprocessor systems have been shown to be predictable [5]. The main advantage of using a partitioning approach in multiprocessor scheduling is that a wealth of real-time scheduling techniques and analyses for uniprocessor systems can be applied once an allocation of tasks to processors has been achieved [1]. The following optimality results for uniprocessor scheduling have a strong influence on research into partitioned multiprocessor scheduling. Rate monotonic (RM) priority assignment is the optimal policy for periodic task sets with implicit deadlines, if preemptive uniprocessor scheduling using fixed priority is considered [6]. On the other hand, since tasks are statically assigned to processors under partitioned scheduling algorithms, there are no job migrations and this serves to reduce overheads.

However, the task allocation problem is analogous to the bin packing problem. It is known to be NP-hard [7], which is the main disadvantage of applying a partitioning method to multiprocessor scheduling. Hence task allocation is a well-studied problem in the field of real-time systems. A synchronization-aware task allocation strategy has been proposed to reduce the scheduling penalties associated with remote task synchronization [8]. That is, tasks sharing a common mutex are bundled and then are co-located, transforming the shared mutex into a local mutex. This strategy is more efficient than the previous ones.

For the task synchronization, each task executes critical sections on its assigned processor in shared-memory systems. The FIFO Multiprocessor Locking Protocol (FMLP$^+$) [9], a refinement of the Flexible Multiprocessor Locking Protocol (FMLP) [10] for partitioned scheduling, is also a shared-memory locking protocol. Similarly, FIFO queues are employed to order requests for resources as well as blocked tasks. Although the FMLP cannot ensure asymptotically optimal priority-inversion blocking, the updated FMLP$^+$ assigns priorities to tasks: the effective priority of a lock holder is the time at which it requested the lock. It is this improved rule that ensures asymptotically optimal maximum priority-inversion blocking [9].

Concerning the calculation of blocking time bound, a linear-programming-based analysis technique, which is significantly less pessimistic than previous approaches, was developed in Ref. [11]. The problem of obtaining bounds on the maximum blocking can be transformed into a linear programming (LP) by imposing a few constraints, whereas previous methods require the analyst to explicitly consider each critical section. Thus, LP solvers provide a straightforward approach for obtaining bounds, such as the GNU Linear Programming Kit (GLPK) [12] or the

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CPLEX Optimizer developed by IBM [13].

1.1 Contributions

Under the FMLP$^+$, the bounds on maximum blocking time may be less pessimistic if local critical sections are considered. As is described in Ref. [8], local synchronization eliminates the scheduling penalties associated with global synchronization. We analyzed the solutions derived from the LP solver and determined how the above technique can be enhanced with additional constraints. As a result, those task sets which were erroneously judged as unschedulable are judged as schedulable with our proposed approach. Otherwise expensive processors with higher performance might be considered to improve the schedulability, which leads to higher costs. The effectiveness and merits of our strategies were demonstrated in the experiments. We also evaluated how much the average blocking time bounds were improved.

1.2 Organization

We introduce the task model and related assumptions in Section 2. We then review three kinds of delays, the blocking fraction, and the objective function in Section 3. We also state how to formalize blocking time bounds as a linear optimization problem, and how to compute the worst-case response time (WCRT) of each task. In Section 4, we analyze the pessimism of existing constraints through a numerical example. Additional constraints are developed in Section 5, and evaluations of our approach are provided in Section 6. Finally, we present our conclusions and areas for future work in Section 7.

2. Definitions

2.1 Assumptions

Fixed priority scheduling, with tasks having conventional RM scheduling priorities, is considered in this paper. Each task's worst-case execution time (WCET) and period are assumed to be known in advance. For simplicity, implicit-deadline task sets are also assumed. That is, each task's deadline is equal to its corresponding period.

All of the critical sections are assumed to be non-nested. Each job for a task requests and holds at most one resource at any time. Jobs release all resources before their completion. The FMLP$^+$, which ensures mutual exclusion, is utilized when two or more jobs access the same resource. If a job requires a locked resource, it must wait and incurs a delay until the requested resource is released. Semaphore protocols are used in this paper, under which jobs wait by suspending instead of spinning.

2.2 Task Model

Consider a real-time workload consisting of n sporadic tasks $\tau = \{T_1, T_2, \ldots, T_i, \ldots, T_n\}$ scheduled on m identical processors $\{P_1, P_2, \ldots, P_m\}$, whose cores have equal processing capabilities. Each task has a unique and fixed base priority under partitioned fixed priority scheduling. For brevity, tasks are ordered in strictly decreasing order of base priorities. That is, $i < j$ implies that T_i has a higher priority than T_j. Tasks are assigned to the m processors statically. The function $P(T_i)$ returns T_i's assigned processor.

Each task is considered to be an alternating sequence of normal execution segments and critical section execution segments [8]. T_i is described as follows:

$$T_i : (E_{i,1}, C_{i,1}, E_{i,2}, C_{i,2}, \ldots, E_{i,s(i)-1}, C_{i,s(i)-1}, E_{i,s(i)}),$$

where $E_{i,j}$ is the WCET of T_i's j^{th} normal execution, $C_{i,k}$ is the WCET of T_i's k^{th} critical section, $s(i)$ is the number of T_i's normal execution segment, and thus, T_i's critical section execution segment must be $s(i) - 1$. Therefore, the WCET of T_i is denoted by e_i such that

$$e_i = \sum_{j=1}^{s(i)} E_{i,j} + \sum_{k=1}^{s(i)-1} C_{i,k}.$$

In this paper, we mainly consider the following situation: $\forall E_{i,j} : E_{i,j} > 0$.

The deadline (= period) of T_i is denoted as d_i, and the utilization of T_i is defined as $u_i = e_i/d_i$. Let J_i denote a job of T_i, and J_i's response time is the difference between its finishing time and arrival time. T_i's WCRT r_i denotes the maximum value of any J_i's response time. T_i's bound on maximum blocking time is denoted by b_i.

Table 1 An example of a task set.

T_i	$P(T_i)$	d_i	$N_{i,1}$	$L_{i,1}$	$N_{i,2}$	$L_{i,2}$	$N_{i,3}$	$L_{i,3}$
$T_1 : (1, 1, 1, 2, 1)$	P_1	30	1	2	1	1	0	0
$T_2 : (1, 3, 1, 4, 1)$	P_2	40	0	0	1	3	1	4
$T_3 : (1, 5, 1)$	P_1	50	1	5	0	0	0	0
$T_4 : (1, 6, 1)$	P_2	60	0	0	0	0	1	6
$T_5 : (1, 7, 1)$	P_1	70	1	7	0	0	0	0
$T_6 : (1, 8, 1)$	P_2	80	0	0	0	0	1	8

2.3 Resources

The tasks share n_r serially reusable resources $l_1, l_2, \ldots, l_{n_r}$ besides the m processors. Here, $N_{i,q}$ is the maximum number of times that any J_i accesses l_q, and $L_{i,q}$ denotes T_i's maximum critical section length, which means the maximum duration that any J_i uses l_q in a single access. That is $L_{i,q} = 0$ if $N_{i,q} = 0$.

A resource is called a local resource in this paper if all of the tasks accessing the resource are assigned to the same processor. Conversely, a resource which is accessed by tasks allocated to different processors is said to be a global resource [14]. We let $P(l_q)$ denote the processor on which the local resource l_q is located. Consider **Table 1**. There are 6 tasks sharing 3 resources. Tasks T_1, T_3, and T_5 are assigned to processor P_1 and T_2, T_4, and T_6 are assigned to P_2. Resource l_1 is accessed by T_1, T_3, and T_5; l_2 by T_1 and T_2; and l_3 by T_2, T_4, and T_6. From the above definitions, l_1 and l_3 are local resources, while l_2 is a global resource.

Under the FMLP$^+$, priority of a task is raised to expedite request completion after it requests a resource.

3. Blocking Time Formulation

A linear-programming-based blocking analysis technique has been proposed which offers substantial improvements over prior blocking time bounds [11]. It can be adapted under various protocols besides the FMLP$^+$.

3.1 Delay

There are three kinds of delays common to all shared-memory

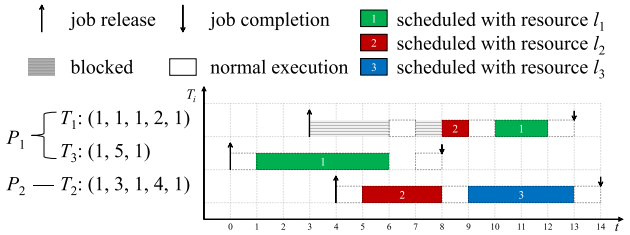


Fig. 1 An example of blocking fractions. J_1 arrives at t_3, J_3 at t_0, and J_2 at t_4. $R_1^{3,1,1}$ and $R_1^{2,2,1}$ cause J_1 to incur preemption delay and direct request delay, respectively.

locking protocols [9].

(1) Direct Request Delay: This arises under any protocol whenever a job J_i requests an unavailable resource. J_i can potentially incur blocking while waiting for the lock holder to finish its critical section. Direct request delay occurs only via resources which J_i requests.

(2) Indirect Request Delay: This arises if J_i waits for another job J_a to release a resource but J_a has been preempted by a third job J_b, thus increasing J_i's total acquisition delay. Indirect request delay can arise due to shared resources which J_i never accesses.

(3) Preemption Delay: This arises when J_i is preempted by a priority-boosted, lower-priority job. Thus, preemption delay affects even tasks which do not access shared resources.

3.2 Blocking Fraction

The concept of the blocking fraction was proposed in Ref. [11] to express partial blocking. For each T_x, $R_i^{x,q,v}$ denotes the v^{th} request for l_q by jobs of T_x from J_i's release until its completion. Here, $b_i^{x,q,v}$ denotes the blocking incurred by J_i due to the execution of $R_i^{x,q,v}$. The corresponding blocking fraction is as follows:

$$X_i^{x,q,v} = \frac{b_i^{x,q,v}}{L_{x,q}}, \tag{1}$$

where $X_i^{x,q,v} \in [0, 1]$. $X_i^{x,q,v}$ indicates the fraction of blocking time which was observed during J_i, out of the total blocking time that could arise from $R_i^{x,q,v}$. Here, $X_{i,D}^{x,q,v}$, $X_{i,I}^{x,q,v}$, and $X_{i,P}^{x,q,v}$ are the fractions of blocking due to direct request delay, indirect request delay, and preemption delay, respectively.

We refer to blocking fractions by means of an illustration. Consider **Fig. 1**. J_1 suffers blocking twice. Preemption delay is incurred by J_3 during $[3, 6)$ and direct request delay by J_2 during $[7, 8)$. We can see $b_1^{3,1,1} = 3$, $b_1^{2,2,1} = 1$ from Fig. 1 and $L_{3,1} = 5$, $L_{2,2} = 3$ from Table 1. Then,

$$X_{1,P}^{3,1,1} = \frac{b_1^{3,1,1}}{L_{3,1}} = \frac{3}{5} \quad \text{and} \quad X_{1,D}^{2,2,1} = \frac{b_1^{2,2,1}}{L_{2,2}} = \frac{1}{3}.$$

3.3 Objective Function

A task set is schedulable if each task's WCRT, which depends on its maximum blocking time bound, is less than or equal to its deadline (i.e., $r_i \leqslant d_i$ for each T_i). Therefore, we consider each task's bound on maximum blocking time to be the objective function.

Blocking depends on the access of both the local and global resources. Let b_i^l and b_i^r denote bounds on the maximum local and remote blocking time, respectively. Similarly, τ^l and τ^r denote the sets of the local and remote tasks. $N_{x,q}^i$ denotes the number of requests by T_x for l_q from J_i's release until its completion. $N_{x,q}^i$ can be bounded for a sporadic task T_x [9]. The objective function is described as follows [11]:

$$\text{maximize} \quad b_i, \quad i = 1, 2, \ldots, n \tag{2}$$

where

$$b_i = b_i^l + b_i^r, \tag{3}$$

$$b_i^l = \sum_{T_x \in \tau^l} \sum_{q=1}^{n_r} \sum_{v=1}^{N_{x,q}^i} (X_{i,D}^{x,q,v} + X_{i,I}^{x,q,v} + X_{i,P}^{x,q,v}) \cdot L_{x,q},$$

$$b_i^r = \sum_{T_x \in \tau^r} \sum_{q=1}^{n_r} \sum_{v=1}^{N_{x,q}^i} (X_{i,D}^{x,q,v} + X_{i,I}^{x,q,v} + X_{i,P}^{x,q,v}) \cdot L_{x,q},$$

$$\tau^l = \{T_x | P(T_x) = P(T_i) \wedge x \neq i\},$$

$$\tau^r = \{T_x | P(T_x) \neq P(T_i)\},$$

$$N_{x,q}^i = \left\lceil \frac{r_i + r_x}{d_x} \right\rceil \cdot N_{x,q}. \tag{4}$$

The objective function is used to obtain each T_i's theoretical maximum blocking time caused by other tasks $(T_1, T_2, \ldots, T_{i-1}, T_{i+1}, \ldots, T_n)$ in the same task set, i.e., T_i's bound on maximum blocking time. In other words, each task has its own maximum blocking time bound. When we compute T_i's maximum bound, we consider how T_i is delayed by other tasks and neglect other tasks' blocking time. We have to compute all task bounds one by one so as to do the analysis of schedulability.

3.4 Linear Optimization

Figure 1 outlines the computation of the blocking fraction based on its theoretical definition. The worst-case scenarios in hard real-time issues are the main concern. However, Fig. 1 does not represent the worst-case scenario, and $b_1^{3,1,1} = 3$ or $b_1^{2,2,1} = 1$ is not the maximum blocking time bound, either. Only when a task's maximum blocking time bound is obtained shall we be able to make the corresponding scenario.

For the FMLP$^+$, the bound on maximum blocking time can be formalized as a linear optimization problem in terms of the blocking fractions (variables), Eq. (2) (objective function) mentioned above, and Constraint 1, 9–14 in Ref. [11].

3.5 Response Time Analysis

We use response time analysis to determine each T_i's r_i [8], [15]. Under blocking conditions, the response time of a task with a fixed priority can be calculated by the following recurrent relation.

(1) Initial Condition: Iteration starts with Eqs. (5a) and (5b), which are the first points in time that T_i and T_x could possibly complete.

$$\begin{cases} r_i^{(0)} = e_i & \text{(5a)} \\ r_x^{(0)} = e_x & \text{(5b)} \\ N_{x,q}^{i,(0)} = \left\lceil \dfrac{r_i^{(0)} + r_x^{(0)}}{d_x} \right\rceil \cdot N_{x,q}. & \text{(5c)} \end{cases}$$

After each $N_{x,q}^{i,(0)}$ is substituted into the constraints, we can solve the LP and obtain $b_i^{l,(0)}$ and $b_i^{r,(0)}$, the initial bounds on the maximum local and remote blocking time respectively.

(2) Recurrence: If $r_i^{(u)} = r_i^{(u-1)}$, then $r_i^{(u)}$ is the actual WCRT for T_i; that is, $r_i = r_i^{(u)}$. Otherwise, we have to compute each r_i iteratively with Eq. (6) until r_i converges for each task.

$$r_i^{(u)} = e_i + b_i^{(u-1)} + \sum_{\substack{T_h \in P(T_i) \\ h<i}} \left\lceil \frac{r_i^{(u-1)} + b_h^{r,(u-1)}}{d_h} \right\rceil \cdot e_h, \qquad (6)$$

Here, $b_i^{(u-1)} = b_i^{l,(u-1)} + b_i^{r,(u-1)}$. Once r_i is calculated, the feasibility of T_i is guaranteed if and only if $r_i \leqslant d_i$. Conversely, a task set is judged as unschedulable if $r_i > d_i$ for at least one T_i.

4. Analysis on Pessimism

A numerical example is presented to demonstrate the pessimistic results derived under the existing constraints. Recall Table 1. We made use of GLPK to solve the generated LPs of each task. Concerning T_1, the solution can be seen in the first row of **Table 2**. Other variables equal to 0, such as $X_{1,I}^{2,2,1}$, $X_{1,D}^{4,3,1}$, and $X_{1,P}^{6,3,1}$, are not listed in the table, because they do not alter the value of the objective function.

First, we consider the blocking incurred by J_1 due to direct request delays. Since $X_{1,D}^{3,1,1} = 1$ and $X_{1,D}^{5,1,1} = 1$, J_1 must incur blocking twice when requesting l_1. However, there is no way for J_3 or J_5 to execute if J_1 arrives or resumes earlier than (or at the same time as) J_3 or J_5 (see **Fig. 2** (a)). Conversely, J_1 cannot request l_1 if J_3 or J_5 is the lock holder of l_1, because of priority-boosted J_3 or J_5 and J_1's unfinished normal execution segment (see Fig. 2 (b)). Thus, it is impossible that $R_1^{3,1,1}$ or $R_1^{5,1,1}$ causes J_1 to incur a direct request delay.

Similarly, $R_1^{4,3,1}$ and $R_1^{6,3,1}$ also cause J_1 to incur an indirect request delay twice in spite of l_2 which J_1 never accesses. However, only when J_1 waits for J_2 to release l_2 but J_2 has been preempted by J_4 or J_6 does $b_1^{4,3,1}$ or $b_1^{6,3,1}$ occur. Since J_2 in a critical section has the highest priority on P_2, $b_1^{4,3,1}$ or $b_1^{6,3,1}$ cannot occur.

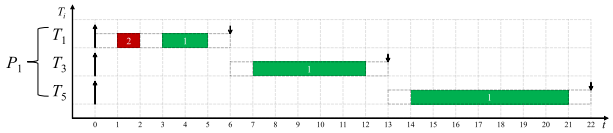
It is possible for $b_1^{2,2,1}$ to occur because T_1 and T_2 are assigned to two different processors. J_2 can affect J_1 directly as long as J_2 has locked l_2 when J_1 issues a request for l_2.

On the other hand, there are other feasible solutions due to the symmetry of Constraint 1 in Ref. [11]. Now that J_1 is never directly delayed by J_3 or J_5, the priority-boosted J_3 and J_5 are likely to preempt J_1. $X_{1,D}^{2,2,1} = X_{1,P}^{3,1,1} = X_{1,I}^{4,3,1} = X_{1,P}^{5,1,1} = X_{1,I}^{6,3,1} = 1$ is another optimal solution. Here, $X_{1,P}^{3,1,1} = 1$ and $X_{1,P}^{5,1,1} = 1$ cannot change b_1 (or r_i) but they provide a better result than $X_{1,D}^{3,1,1} = 1$ and $X_{1,D}^{5,1,1} = 1$. Of course, the values of $X_{1,I}^{4,3,1}$ and $X_{1,I}^{6,3,1}$ are still pessimistic.

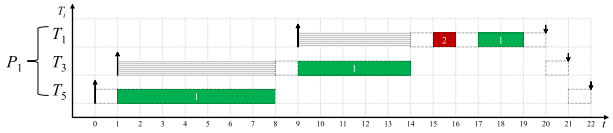
Overall, the actual execution time for J_1 is much lower than the theoretical upper bounds. It is impossible to make a corresponding figure for the worst-case scenario for J_1. We computed b_1 ($b_1 = 29$ in Table 2) by Eq. (3) and obtained $r_1 = 40$ and $r_2 = 51$, as listed in the second row of **Table 3** after iteratively calculating r_i. Both T_1 and T_2 miss their deadlines, and so the task set is regarded as unschedulable. It is therefore necessary for us to derive more realistic and less pessimistic values.

Table 2 Comparison of the blocking fractions and the maximum blocking time bounds.

method	$X_{1,D}^{2,2,1}$	$X_{1,D}^{3,1,1}$	$X_{1,P}^{3,1,1}$	$X_{1,I}^{4,3,1}$	$X_{1,D}^{5,1,1}$	$X_{1,P}^{5,1,1}$	$X_{1,I}^{6,3,1}$	b_1
existing method	1	1	0	1	1	0	1	29
proposed method	1	0	1	0	0	1	0	15



(a) J_1, J_3, and J_5 arrive at t_0. It is impossible for J_3 or J_5 to block J_1. Note that: if $C_{1,1}$ is delayed by another task on a remote processor, J_1 cannot be directly delayed by J_3 or J_5, because of $E_{1,2}$. In this case, T_1 is equivalent to another task: $(1, 2, 1)$.



(b) J_1, J_3, and J_5 arrive at t_9, t_1, and t_0, respectively. J_3 is blocked by J_5 because the effective priority of J_5 in a critical section is higher than the base priority of J_3, and similarly, J_1 is blocked by J_3. Note that: $R_3^{5,1,1}$ causes J_3, $R_1^{3,1,1}$ causes J_1 to incur preemption delay instead of direct request delay.

Fig. 2 As is depicted in Fig. 2 (a), higher-priority jobs arrive earlier than lower-priority ones, whereas Fig. 2 (b) depicts the situation in which lower-priority jobs arrive earlier than higher-priority ones. No matter which job arrives earlier, local resources will never incur direct request delay under the FMLP$^+$.

Table 3 Comparison of the WCRT.

T_i	T_1	T_2	T_3	T_4	T_5	T_6
d_i	30	40	50	60	70	80
r_i (existing method)	40	51	26	26	28	38
r_i (proposed method)	21	25	20	26	22	28
improvement (%)	47.50	50.98	23.08	0	21.43	26.32

5. Improvements

Since the above example showed us that the bounds on the maximum blocking time obtained by the original constraints were pessimistic, we tried to reduce pessimism through additional constraints.

5.1 Preliminaries

To simplify the necessary lemma and theorems, we define $\tilde{J}_i$ as a job starting with a normal execution segment (i.e., $E_{1,1} \neq 0$). The lemma is then expressed as follows.

Lemma 1 Under the FMLP$^+$, if $\tilde{J}_i$ in a critical section is preempted by another job J_a, then J_a must be in an earlier-initiated global critical section.

Proof: Suppose not. Then J_a is in a local critical section, which was initiated earlier than $\tilde{J}_i$ but has not yet finished. Since J_a's critical section is local, J_a cannot be affected by jobs on other processors. Thus, there must exist a job J_b which is local to J_a and is executing its critical section. J_a will continue executing its local critical section as soon as J_b finishes its critical section. $\tilde{J}_i$ cannot execute its critical section because J_a's effective priority is higher than $\tilde{J}_i$'s at that time. It is impossible that J_a preempts $\tilde{J}_i$ in a critical section. Contradiction. □

5.2 Theorems

We were able to deduce the following theorem for the direct request delay after obtaining $X_{1,D}^{3,1,1} = 0$ and $X_{1,D}^{5,1,1} = 0$ in the example above.

Theorem 1 $\tilde{J}_i$ never incurs direct request delays due to other jobs in local critical sections under the FMLP$^+$.

Proof: Suppose not. Then there exists a lower-priority job J_x local to $\tilde{J}_i$, which delays $\tilde{J}_i$ directly. According to the FMLP$^+$, lock holders are scheduled in order of increasing lock-request time [11]. J_x must request a local lock earlier than $\tilde{J}_i$. $\tilde{J}_i$ has also requested the same lock before J_x releases the lock. But the effective priority of a lock holder is higher than any other local jobs in normal execution segments under the FMLP$^+$. Thus, $\tilde{J}_i$ cannot execute its normal execution segment before its critical section when J_x is in the critical section. $\tilde{J}_i$ cannot issue the request if $\tilde{J}_i$ does not finish its normal execution segment first. Contradiction. □

Similarly, the second theorem about indirect request delays can be derived from the fact that $X_{1,I}^{4,3,1} = 0$ and $X_{1,I}^{6,3,1} = 0$.

Theorem 2 Under the FMLP$^+$, $\tilde{J}_i$ never incurs indirect request delays caused by J_b's local critical sections if $\tilde{J}_i$ waits for $\tilde{J}_a$ to release a resource.

Proof: Suppose not. From the definition of an indirect request delay, $\tilde{J}_a$ must be preempted by a third job J_b. From the Lemma, it is impossible for $\tilde{J}_a$ to be preempted by J_b in a local critical section. Contradiction. □

By analysis, we also surmised that the preemption delay is related to the global critical sections. Let $\tilde{s}(i)$ denote the number of global resources accessed by T_i.

Theorem 3 The number of times that priority-boosted, lower-priority jobs $\tilde{J}_x$ in critical sections can preempt $\tilde{J}_i$ is at most $1 + \tilde{s}(i)$ under the FMLP$^+$.

Proof: From the Lemma, $\tilde{J}_i$ must be in a normal execution segment. Other lower-priority jobs local to $\tilde{J}_x$ can delay $\tilde{J}_i$ with at most one critical section whenever $\tilde{J}_i$ resumes. According to Theorem 1, $\tilde{J}_i$ cannot be suspended due to other jobs in local critical sections if $\tilde{J}_i$ does not have global critical sections. That is, the number of times that $\tilde{J}_i$ may be preempted by $\tilde{J}_x$ depends on the number of global resources accessed by $\tilde{J}_i$. In addition to the first normal execution segment of $\tilde{J}_i$, $\tilde{J}_x$ has at most $1 + \tilde{s}(i)$ opportunities to affect J_i. □

5.3 Additional Constraints

We now consider an additional constraint arising from Theorem 1.

Additional Constraint 1 In any schedule of τ under the FMLP$^+$:

$$\sum_{T_x \in \tau^i} \sum_{l_q \in l_{\tilde{q}}^i} \sum_{v=1}^{N_{x,q}^i} X_{i,D}^{x,q,v} = 0,$$

where $\tau^i = \tau \setminus \{T_i\}$ is the set of all tasks except T_i. $l_{\tilde{q}}^i = \{l_q | P(l_q) = P(T_i)\}$ denotes the local resources on the processor which T_i is assigned to. Likewise, the following additional constraint is based on Theorem 2.

Additional Constraint 2 In any schedule of τ under the FMLP$^+$:

$$\sum_{T_x \in \tau^i} \sum_{l_q \in l_{\tilde{q}}^i} \sum_{v=1}^{N_{x,q}^i} X_{i,I}^{x,q,v} = 0.$$

In practice, local resource sharing is adopted as much as possible to avoid the penalties associated with remote task synchronization. The benefits of Additional Constraint 1 and 2 are that blocking incurred by local synchronization can be eliminated. The third constraint can then be derived from Theorem 3.

Additional Constraint 3 In any schedule of τ under the FMLP$^+$:

$$\sum_{T_x \in \tau^{ll}} \sum_{q=1}^{n_r} \sum_{v=1}^{N_{x,q}^i} X_{i,P}^{x,q,v} \leqslant 1 + \tilde{s}(i).$$

where $\tau^{ll} = \{T_x | P(T_x) = P(T_i) \wedge x > i\}$ is the set of local, lower-priority tasks. The number of global resources will shrink if the task allocation algorithm of transforming global resource sharing into local sharing is used. $\tilde{s}(i)$ will also grow smaller so that the pessimism of $X_{i,P}^{x,q,v}$ can be limited.

The three additional constraints above cannot be used independently but only in combination with Constraint 1 and 9–14 in Ref. [11].

Table 4 List of Notations.

T_i	Task i
τ^i	the set of all tasks except T_i
τ^l	the set of local tasks
τ^r	the set of remote tasks
τ^{ll}	the set of local, lower-priority tasks
J_i	a job of T_i
$\tilde{J}_i$	a job starting with a normal execution segment
l_q	resource q
$l_{\tilde{q}}^i$	local resources on the T_i's assigned processor
$P(l_q)$	the processor on which local resources l_q are
m	the number of processors
n	the number of tasks
n_r	the number of resources
$\tilde{s}(i)$	the number of global resources accessed by T_i
$N_{x,q}^i$	the number of requests by T_x for l_q from J_i's release until its completion
$L_{i,q}$	T_i's maximum critical section length
$N_{i,q}$	the maximum number of times that any J_i accesses l_q
$R_i^{x,q,v}$	the v^{th} request for l_q by jobs of T_x
$b_i^{x,q,v}$	actual blocking due to the execution of $R_i^{x,q,v}$
$X_i^{x,q,v}$	the proportion of blocking time actually incurred by J_i

5.4 Review of the Example

We now return to the numerical example in Table 1. We add the three additional constraints and compute each task's bound on maximum blocking time. As the last column of Table 2 lists, b_i has been greatly reduced from 29 to 15. The new value obtained by our proposed method is closer to the actual maximum blocking time than by the existing one.

The second row of Table 2 shows that $R_1^{2,2,1}$ might cause J_1 to incur direct request delay as before. Yet $X_{1,D}^{3,1,1} = X_{1,D}^{5,1,1} = 1$ is now $X_{1,P}^{3,1,1} = X_{1,P}^{5,1,1} = 1$. Although J_1 may still be affected by J_3

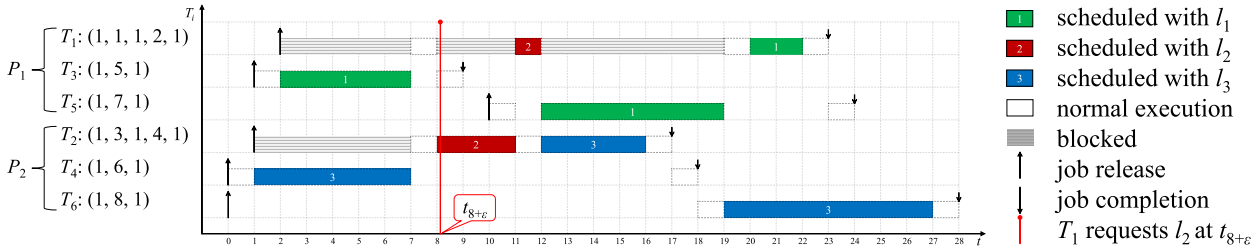


Fig. 3 A worst-case scenario for J_1 under the FMLP$^+$. From the second row of Table 2, $b_1 = 15$. On P_1, J_1, J_3, and J_5 arrive at t_2, t_1, and t_{10}, respectively; J_2 arrives at t_1; and both J_4 and J_6 arrive at t_0 on P_2. J_1 is successively affected by J_3 with the direct request delay, by J_2 with preemption delay and by J_5 with the direct request delay.

and J_5's critical sections, this is because Additional Constraint 1 works that the rate of occurrence of both kinds of delays is now more reasonable. On the other hand, $X_{1,I}^{4,3,1} = X_{1,I}^{6,3,1} = 1$ becomes $X_{1,I}^{4,3,1} = X_{1,I}^{6,3,1} = 0$, leading to less pessimism that J_1 cannot be indirectly delayed by J_4 or J_6 due to Additional Constraint 2.

We can now estimate $b_1^{3,1,1} = 5$, $b_1^{2,2,1} = 3$, and $b_1^{5,1,1} = 7$ from Eq. (1). As **Fig. 3** depicts, the schedule results in a worst-case scenario for J_1. Next, we explain in detail how J_1 is delayed by J_3, J_2, and then by J_5.

(1) $b_1^{3,1,1} = 5$: J_3's critical section begins and its priority is boosted at t_2. J_3's effective priority is higher than J_1's until t_7, though J_1 has the highest base priority among all tasks. Therefore, J_1 is blocked by J_3 as soon as it arrives.

(2) $b_1^{2,2,1} = 3$: From the figure, both J_1 and J_2 request l_2 at t_8. Then J_1 should have acquired the lock due to its higher base priority. In theory, we have to consider the maximum blocking. That is, J_1 requests l_2 after J_2 does. We suppose that J_1 issues a request for l_2 at $t_{8+\varepsilon}$ instead of t_8, where ε is a sufficiently small positive number (i.e., $\varepsilon > 0, \varepsilon \to 0$). As a result, J_2 acquires the lock and J_1 remains suspended until t_{11}. Thus,

$$b_1^{2,2,1} = \lim_{\varepsilon \to 0}[11 - (8 + \varepsilon)] = 3.$$

(3) $b_1^{5,1,1} = 7$: In the same way as $b_1^{3,1,1}$, J_1 is preempted by the priority-boosted J_5 at t_{12}. Also, J_5 is preempted by J_1 at t_{11} because the lock holders are scheduled in order of increasing lock-request time.

On the other hand, J_5 could not cause J_1 to incur a preemption delay at t_{12} if J_1 did not access the global resource l_2, or if l_2 was a local resource instead of a global one. Without accessing a global resource, J_1 could not be delayed by J_2 on the remote processor. That is, J_5 could not be executed immediately as soon as it released at t_{10}. As is described in Additional Constraint 3, preemption delay is related to the number of task's global critical sections. This example also illustrates that a global critical section may incur additional penalties.

Based on the new blocking time bounds, we repeated the experiment in Section 3.5. The new WCRT of each task is listed in the third row of Table 3. All of the tasks are completed before their deadlines. Unlike the result presented in Section 4, the whole task set is now schedulable. By comparison, a task set might be judged as unschedulable without additional constraints.

6. Experiments

The main purpose of our experiments was to verify whether results could become less pessimistic if there are local resources available after task allocation.

6.1 Task Generation

The following parameter settings were used in the experiments:

(1) Critical sections
- $n_r = 8$
- $N_{max} \in \{1, 3\}$
- $N_{i,q} \in [0, N_{max}]$
- $L_{i,q}$: 2 kinds of uniform distributions

(2) Task sets
- $m = 8$
- $n \in [m, 10 \cdot m]$
- $d_i \in [10\,\text{ms}, 100\,\text{ms}]$
- $u_i \in [0.1, 0.2]$

In order to generate critical sections, we considered three parameters: $N_{i,q}$, $L_{i,q}$, and the number of resources n_r. $N_{i,q}$ was related to the maximum request times N_{max} and randomly chosen from $\{0, 1, \ldots, N_{max}\}$. T_i did not access l_q if $N_{i,q} = 0$. Otherwise the corresponding $L_{i,q}$ was uniformly distributed over the range $[50\,\mu\text{s}, 100\,\mu\text{s})$ (short) and $[100\,\mu\text{s}, 500\,\mu\text{s}]$ (long).

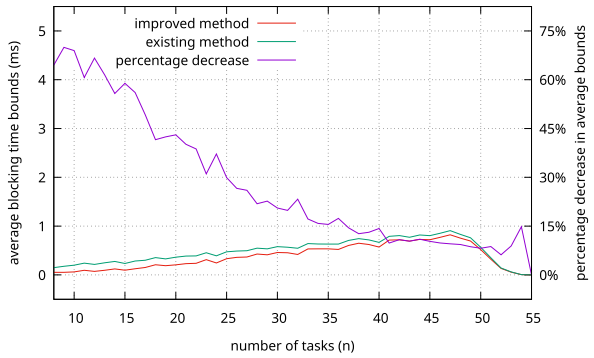
For simplicity, we considered only one multiprocessor platform with 8 processors. The number of tasks n ranged from $n = m$ to $n = 10 \cdot m$. d_i was chosen from a uniform distribution ranging over $[10\,\text{ms}, 100\,\text{ms}]$; u_i was also uniformly distributed over $[0.1, 0.2]$.

6.2 Allocation

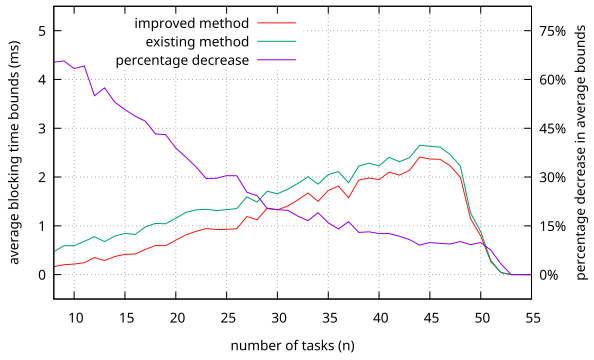
Our priority was to determine whether our additional constraints could eliminate pessimism. The more local resources available after allocation, the more effective additional constraints for local resources might be. Therefore, we considered the synchronization-aware task allocation algorithm in Ref. [8]. First, tasks which share the same resource were bundled together, and then the bundles which could not be assigned as a single task to a processor were split into bundles or tasks that would fit any existing processor.

6.3 Comparison

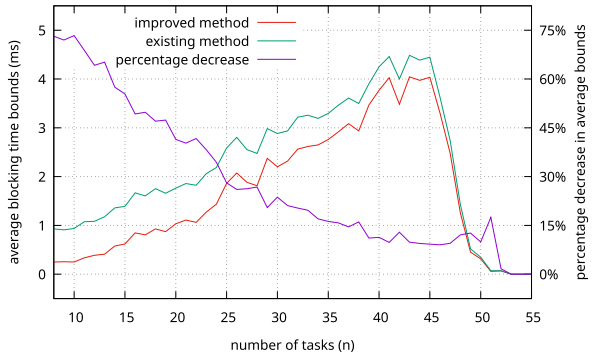
We used the same settings of parameters as Ref. [11]. To show the merits of our proposed method more clearly, we added a new



(a) short critical section, $N_{max} = 1$

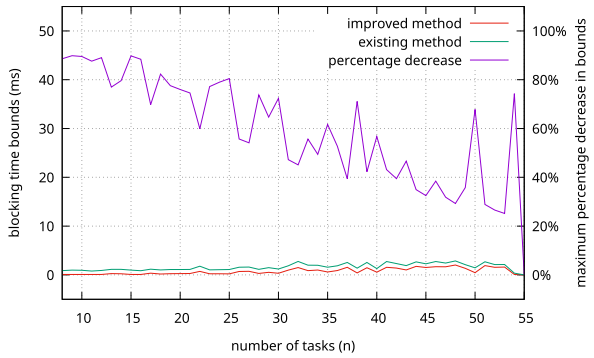


(b) long critical section, $N_{max} = 1$

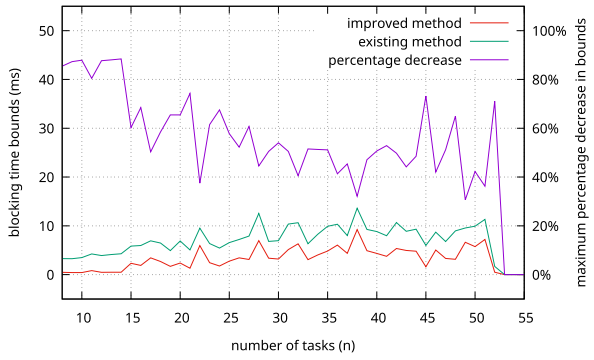


(c) long critical section, $N_{max} = 3$

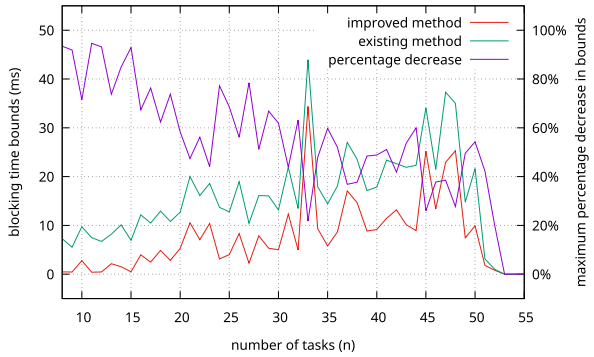
Fig. 4 Average blocking time bounds under three combinations of parameters.



(a) short critical section, $N_{max} = 1$



(b) long critical section, $N_{max} = 1$



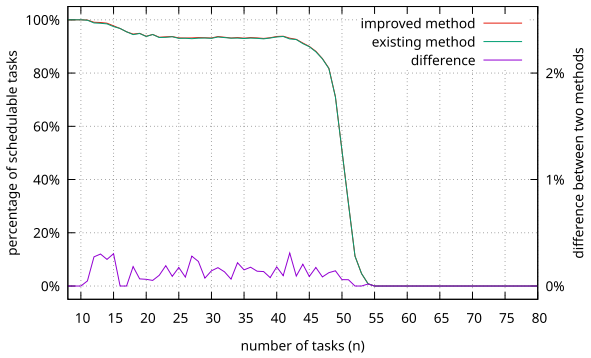
(c) long critical section, $N_{max} = 3$

Fig. 5 Maximum percentage decrease in blocking time bounds under three combinations of parameters. We draw purple curves to show the maximum decrease between two methods. Yet red or green ones only represent the blocking time bounds in the case of maximum difference under the improved or existing method, respectively. They are not discussed independently.

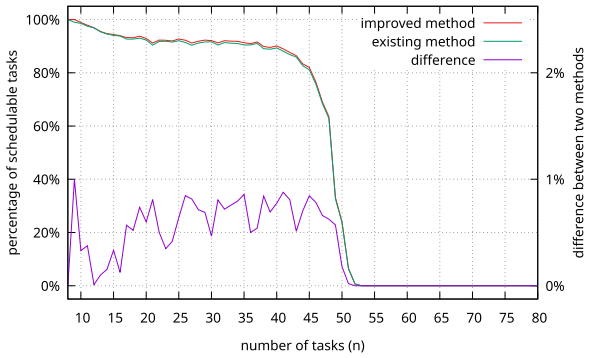
type of critical section length (long). Under the same settings, we tested 100 task sets for each n, calculated all task blocking time bounds for each task set, and counted the number of schedulable tasks (i.e., $r_i \leqslant d_i$) separately. Then we averaged the blocking time bounds of all tasks in 100 task sets as well as the proportion of schedulable tasks. The number of schedulable tasks was in particular set to zero unless the corresponding task set could be partitioned.

On the other hand, those tasks whose blocking time bounds were significantly improved had a large effect on schedulability directly or indirectly. Some tasks became schedulable directly under our proposed method, whereas they were regarded as unschedulable under the existing one. Although the schedulability of other tasks did not change, they affected several remote lower-priority tasks. In addition to the average blocking time bound, we collected the maximum percentage decrease in blocking time bound among 100 task sets for each n.

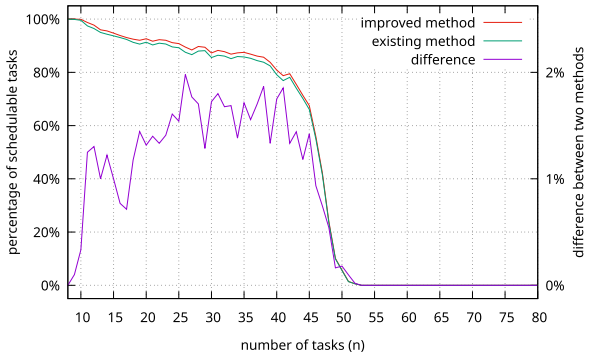
Firstly, we consider the parameter for the short critical section. As is depicted in **Fig. 4** (a), we used the following parameter settings: $N_{max} = 1$. Since task sets were found to be unschedulable if $n > 55$, the blocking time bounds were not computed. The average blocking time bound of each task set slightly decreases under the conditions of short critical sections. From **Fig. 6** (a) no clear difference can be seen between the improved method and the existing method. This is due to the slight reduction in blocking time bounds that our proposed approach brings, giving only a small improvement in the schedulability. Therefore, a curve was added to show the difference in percentage of schedulable tasks (the same as Fig. 6 (b) and Fig. 6 (c)). However, **Fig. 5** (a)



(a) short critical section, $N_{max} = 1$



(b) long critical section, $N_{max} = 1$



(c) long critical section, $N_{max} = 3$

Fig. 6 Percentage of schedulable tasks under three combinations of parameters.

evidently shows the maximum difference in blocking time bound for each n between two methods. For $n < 13$: the maximum percentage decrease is up to about 90%. The percentage decrease in Fig. 4 (a) also illustrates that the bounds on maximum blocking time are effectively reduced with our strategy.

Next, we consider Fig. 4 (b) for the experiment with a long critical section and $N_{max} = 1$. Compared with the existing method, our strategy lowers the average blocking time bound by more than 27%. Most significantly, for $n = 9$, the blocking time bound decreases by as much as 65.7%. In Fig. 5 (b), the maximum decrease in blocking time bound can be seen obviously under the circumstance of long critical sections. Figure 6 (b) also shows a clearer advantage for our proposed method than Fig. 6 (a). The schedulability also rises because the bounds on maximum blocking time are less pessimistic. Especially at $n = 9$, the percentage of schedulable tasks improves 1%. Although the request times are unchanged, the additional constraints perform better if the critical sections are longer.

Finally, we consider the case when $N_{max} = 3$. More request times also increase the total length of the critical section. The result depicted in Fig. 4 (c) shows a larger advantage for the improved method than in Fig. 4 (b): about a 30% decrease in the average blocking time bound and 73% decrease in the maximum blocking time bound at $n = 10$. On the other hand, Fig. 5 (c) illustrates the greatest improvement in the maximum decrease among Fig. 5. Significantly at $n = 8, 9, 11, 12$ or 15, the percentage drops more than 90%. Figure 6 (c) shows a higher schedulability for the improved method when the number of supported tasks is between 11 and 45. For $n = 26$, schedulability increases by nearly 2 percentage points (maximal value). The results of experiments reveal that the additional constraints work better for more request times.

Overall, since the schedulability is relevant to the task's WCRT and deadline, our proposed method sometimes brings a small improvement in the schedulability if task sets are randomly generated. On the contrary, shorter deadlines may lead to a bigger improvement. Nevertheless, the additional constraints make the blocking time bounds less pessimistic under the different combinations of parameters.

7. Conclusion

Using a numerical example, the existing approach is shown to result in pessimistic values. From a theoretical perspective, we succeed in solving the problem through the introduction of three additional constraints on the local critical section. As long as there exist local resources after task allocation, the additional constraints can function. The results of experiments indicate that it is more efficient to utilize both the original and additional constraints together, especially under the condition that the critical sections are relatively long. They also illustrate that the pessimism of maximum blocking time bound is reduced significantly. Although the schedulability rises slightly, our results still represent an improvement over the existing method.

In our future work, we intend to include further constraints under different protocols besides the distributed locking protocols. We shall also try to apply our strategy to the spin execution control policy as well as the global scheduling algorithm.

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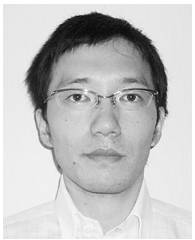

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