

Approximation Algorithms for the Traveling Salesman with a Drone

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Abstract: We examine a routing problem arising when an unmanned aerial vehicle (UAV), or *drone*, is used in a combination with a truck for the last-stretch of parcel deliveries to end customers. Agatz et al. (ERIM Report Series, Reference No. ERS-2015-011-LIS (2015)) proposed a $(2 + \alpha)$ -approximation algorithm for the Traveling Salesman Problem with Drone (TSP-D), with the minimum spanning tree heuristic for the TSP and with no drone deliveries. With this work, we propose an improved, $\min\{3/2 + \alpha, 1 + \sqrt{n}\}$ -approximation algorithm for the TSP-D where n is the number of customers that need to be served.

Keywords: vehicle routing, NP-hardness, approximation algorithm

1. Introduction

The last-stretch (or last-mile) delivery services have changed dramatically over the years. Traditionally, delivery trucks have been used to manage these tasks. However, recent technological innovations have arisen in which Unmanned Aerial Vehicles (UAV-s), or *drones* could be utilized to support parcel deliveries [3], [4], [11]. With the prediction that drones could enhance the competency of delivery operations, major delivery and logistic companies are investigating the benefits of unmanned delivery drone services.

There is an emerging body of research that considers the combination of a truck and a drone for the last-stretch delivery problem, see, e.g., Ha et al. [6], Murray and Chu [8], and Ponza [10]. All of these works ask to concurrently resolve a route for both the truck and the drone to complete all deliveries with the least amount of time.

Apart from the combination of a truck and a drone as heterogeneous delivery vehicles, there are several works in the literature that consider a setting where the last-stretch delivery can only be performed by the drone, see, e.g., Mathew et al. [7] and Othman et al. [9].

In this study, we undertake a scenario in which a delivery truck and a drone collaboratively perform the *last-stretch* delivery of parcels to customers' doorsteps. This work builds upon the study of Agatz et al. [1], where they established the Traveling Salesman Problem with Drone (TSP-D). They work on the collaboration of a single truck and a single drone to serve all customer locations by either the truck or the drone, and ask to find the shortest tour in terms of time to complete all parcel deliveries. In their study,

they assume that both the truck and the drone travel on the same road network. The truck and the drone are required to start and end their route at a designated depot. The drone has unit payload capacity, and it never delivers two parcels consecutively without rendezvousing with the truck. The pickup of parcels from the truck can only take place at the customer locations. In other words, the drone can only land on and depart from the truck while it is parked at a customer location or the depot. They also considered that the drone's speed is a constant α times higher than the truck's speed, and allowed the drone's launch and rendezvous to be at the same point. They proposed a $(2 + \alpha)$ -approximation algorithm for the TSP-D, with the minimum spanning tree heuristic for the TSP and without drone deliveries. In addition, they formulate this problem as a mixed-integer programming (MIP) and expand various "Truck First, Drone Second" heuristic procedures, based on local search and dynamic programming.

With this work, we propose an improved approximation algorithm for the TSP-D. We show that in metric graphs, there is a $\min\{3/2 + \alpha, 1 + \sqrt{n}\}$ -approximation algorithm for the TSP-D, where n is the number of customers that need to be served.

The remainder of this paper is organized as follows. Section 2 outlines the basic notation, the problem models for this paper, and establishes the NP-hardness of the problem. Section 3 describes the approximation algorithm for the problem, and finally, Section 4 concludes the paper.

2. Preliminaries

2.1 Notation

The set of reals (resp., nonnegative reals) is denoted by \mathbb{R} (resp., \mathbb{R}_+).

The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$, respectively. For vertices $u, v \in V(G)$, we use uv to refer to an edge $e \in E(G)$ such that e is incident to u and v . We call u and v the *end vertices* of the edge uv . For a set $E' \subseteq E(G)$ of edges, we write $V(E')$ for the set of all end vertices of edges

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in E' . A graph G is *complete* if every two vertices $u, v \in V(G)$ are adjacent. The *degree* of a vertex $u \in V(G)$ in a graph G is the number of edges in $E(G)$ incident to u . For convenience, given a collection \mathcal{G} of graphs, we write $V(\mathcal{G})$ for the set $\bigcup_{G \in \mathcal{G}} V(G)$.

A *subgraph* G' of a graph G is a graph such that $V(G') \subseteq V(G)$, and $E(G') \subseteq E(G)$, and we write $G' \subseteq G$. A graph $G' \subseteq G$ is an *induced* subgraph of G if it holds that $E(G') = \binom{V(G')}{2} \cap E(G)$, and we also say that G' is induced by $V(G')$. Given a graph G and a set $V' \subseteq V(G)$, we write $G[V']$ for the subgraph of G induced by V' . In addition, for a subset $V'' \subseteq V(G)$, we write $G - V''$ for the graph $G[V(G) - V'']$. Given a graph G and a set $E' \subseteq E(G)$, we write $E(G) - E'$ for the graph $(V(G), E(G) \setminus E')$.

Given a graph G , a *matching* $M \subseteq E(G)$ is a subset of edges such that each vertex in $V(G)$ is incident to at most one edge in M . A matching $M \subseteq E(G)$ is *perfect* if it holds that $V(M) = V(G)$. Given a graph G and a matching $M \subseteq E(G)$, a vertex $v \in V(G)$ such that $v \notin V(M)$ is called *exposed*.

A *path* $P = (v_1, v_2, \dots, v_p)$ is a graph such that $V(P) = \{v_1, v_2, \dots, v_p\}$ and $E(P) = \{v_i v_{i+1} \mid i = 1, 2, \dots, p-1\}$. Such a graph P is also called a v_1, v_p -path, and we define the *length* of P to be $p-1$. A path with length 2 is called a 2-path. A path $P = (v_1, v_2, \dots, v_p)$ such that $v_i \neq v_j$ holds for $1 \leq i \neq j \leq p$ is called a *simple path*. Given a path $P = (v_1, v_2, \dots, v_p)$, for two integers, i and k such that $1 \leq i < k \leq p$, let $P[v_i, v_k]$ denote the path $(v_i, v_{i+1}, \dots, v_k)$.

A *cycle* $C = (v_1, v_2, \dots, v_p)$ is defined to be a path (v_1, v_2, \dots, v_p) such that $v_1 = v_p$. If $v_i \neq v_j$ for all $1 \leq i \neq j < p$, then the cycle C is called a *simple cycle*.

Given a graph G and an *edge weight* function $w : E(G) \rightarrow \mathbb{R}_+$, we say that the graph G is *weighted* by w , and write (G, w) . For convenience, for any vertex $v \in V(G)$, we define that $w(vv) = 0$. For a subset $E' \subseteq E(G)$, let $w(E')$ denote $\sum_{e \in E'} w(e)$. For brevity, let $w(G)$ denote $w(E(G))$. For a collection \mathcal{G} of graphs, define $w(\mathcal{G}) \triangleq \sum_{G \in \mathcal{G}} w(G)$.

A weighted graph (G, w) is said to be *metric* if the edge weight function w satisfies the *triangle inequality*, that is, for all $u, v, q \in V(G)$ it holds that

$$w(uv) \leq w(uq) + w(qv). \quad (1)$$

2.2 Problem Model

Let C be the set of customers that need to be served. In practice, it might not be possible for all customers to be served by the drone, for instance, their parcels might be too heavy, and since the drone is battery powered, the range of delivery might be limited. On the other hand, we might wish to specify certain customers to be served exclusively by the drone. Let A be the set of customers that can only be served by the truck, and let B be the set of customers that can only be served by the drone. We have that $C = A \cup B$. In this work, we deal with the special case when $A = B$. In addition, we are given the location of a special point r , the depot. For any two points $u, v \in C \cup \{r\}$, let $w(uv)$ be the time that it takes for the truck to travel between them. Moreover, the truck and the drone are required to start and end their route at a designated depot, r . The drone has unit payload capacity, and it never delivers two parcels consecutively without rendezvousing

with the truck. The pickup of parcels from the truck can only take place at the customer locations. In other words, the drone can only land on and depart from the truck while it is parked at a customer location or the depot. According to the model of Agatz et al. [1], the drone uses the same road network as the truck, but it is $\alpha \geq 1$ times faster than the truck. An illustration of the problem scenario is shown in **Fig. 1(a)**.

Agatz et al. [1] have modeled the TSP-D as a problem of finding a pair of cycles with a special structure in a weighted graph (G, w) , such that both cycles contain a distinguished vertex $r \in V(G)$, and the union of both cycles contains all vertices in $V(G)$. The objective is to minimize an involved cost function dependent on the weight w , and an input parameter $\alpha \geq 1$.

For our purposes, we present a slightly different but equivalent model. Let (G, w) be a weighted graph representing a road network such that $V(G) = C \cup \{r\}$, a special vertex $r \in V(G)$ representing the depot, and a real parameter $\alpha \geq 1$ that is the ratio between the drone's and the truck's speed. As a feasible route for last-stretch deliveries, we seek to find a pair (R, \mathcal{P}) of a cycle $R = (r = r_1, r_2, \dots, r_p = r)$ in the graph G and an ordered tuple of 2-paths $\mathcal{P} = (P_1, P_2, \dots, P_k)$ such that

- (i) each path $P_i \in \mathcal{P}$ is of the form $P_i = (r_h, d_i, r_\ell)$, such that $r_h, r_\ell \in V(R)$, $d_i \in V \setminus V(R)$ and $h \leq \ell \leq p$;
- (ii) for any two paths $P_i, P_j \in \mathcal{P}$ with $i < j$ such that $P_i = (r_h, d_i, r_\ell)$ and $P_j = (r_m, d_j, r_q)$, it holds that $\ell \leq m$; and
- (iii) $V(R) \cup \{d_1, d_2, \dots, d_k\} = V(G)$.

In terms of the routing scenario, R is the truck route, and each $P \in \mathcal{P}$ is the route that the drone takes to deliver a parcel from the truck to a customer and return to the truck. Condition (i) states that if the drone picks up a parcel from the truck at location $r_h \in V(R)$, then it cannot return to the truck at some earlier location, and must return to the truck before the truck itself returns to the depot r ; condition (ii) states that the drone must rendezvous with the truck before setting off to another delivery; and condition (iii) simply states that all customers must be served. Henceforth, given a graph G , a vertex $r \in V(G)$, and an ordered pair (R, \mathcal{P}) of cycle $R \subseteq G$ and ordered tuple \mathcal{P} of 2-paths in G , we say that the ordered pair (R, \mathcal{P}) is a *feasible* route if R and \mathcal{P} satisfy the conditions (i)-(iii) above. An illustration of a feasible route of the given problem scenario is depicted in **Fig. 1(b)**.

Before defining an objective function, we introduce some additional notation. Given a graph G and a designated vertex $r \in V(G)$, let $(R = (r = r_1, r_2, \dots, r_p = r), \mathcal{P} = (P_1, P_2, \dots, P_k))$ be a feasible route. For each 2-path $P_i = (r_h, d_i, r_\ell) \in \mathcal{P}$, let $R_i = (r_h, r_{h+1}, \dots, r_\ell)$ be the path $R[r_h, r_\ell]$. In the practical setting, the truck and the drone rendezvous at location r_h , and the drone departs from the truck at location r_h , serves the customer d_i , and returns back to the truck at location r_ℓ , while the truck proceeds on its route from location r_h to location r_ℓ . Then, the time it takes for the drone to reach the location r_ℓ is given by $w(P_i)/\alpha$, while for the truck to reach r_ℓ is $w(R_i)$. Both vehicles must be at location r_ℓ at the same time to rendezvous. If the drone flies faster than the truck, then it will wait for the truck at r_ℓ . On the other hand, if the truck is faster than the drone, then it will wait

for the drone at r_ℓ . Then, the time before the next rendezvous location r_ℓ is given by $\max \{w(R_\ell), w(P_\ell)/\alpha\}$. In addition, define $\bar{E}_R \triangleq E(R) \setminus \bigcup_{i=1}^k E(R_i)$ to be the set of edges where the drone is docked to the truck while the truck delivers parcels. Then, we define $\text{cost}(R, \mathcal{P})$ to be

$$\text{cost}(R, \mathcal{P}) \triangleq w(\bar{E}_R) + \sum_{i=1}^k \max \left\{ w(R_i), \frac{1}{\alpha} w(P_i) \right\}. \quad (2)$$

Formally, we get the following problem.

THE TRAVELING SALESMAN PROBLEM WITH DRONE - TSP-D

Instance: A weighted graph (G, w) with a designated vertex $r \in V(G)$, and a real number $\alpha \geq 1$.

Objective: Find a feasible route (R, \mathcal{P}) such that $\text{cost}(R, \mathcal{P})$ is minimized, or report that none such route exists.

In particular, let *metric* TSP-D be the TSP-D such that the graph (G, w) is metric. Note that an instance of the metric TSP-D always has a feasible route.

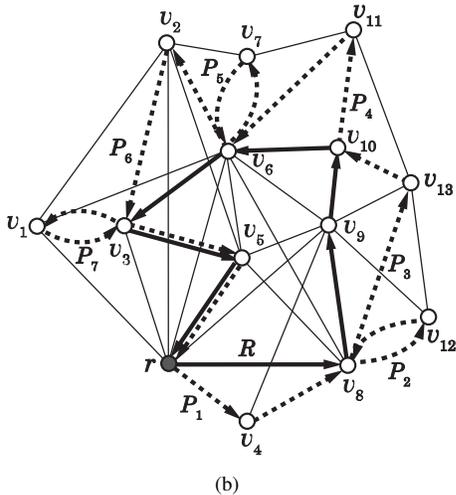
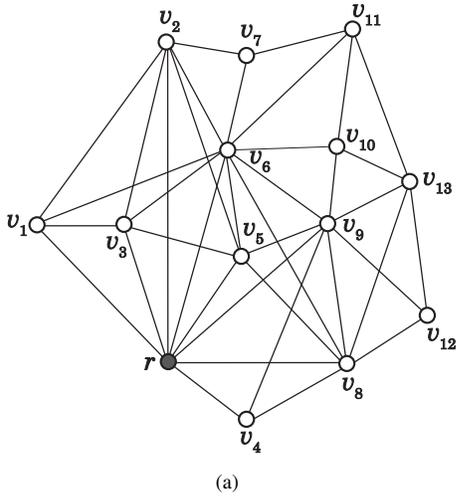


Fig. 1 (a) An illustration of the problem scenario. The customers to which parcels need to be delivered are represented by white circles. The depot is depicted by a vertex r . (b) A feasible route for the given problem scenario in (a). The truck's route, R , is shown by bold arrows, and the 2-paths that give the drone's delivery routes are shown by dashed arrows (P_1, P_2, \dots, P_7) . Parallel bold and dashed arrows means that the drone is docked to the truck.

Observation 1 For an instance $I = ((G, w); r \in V(G); \alpha \geq 1)$ of the metric TSP-D, a Hamiltonian cycle R in G gives a feasible route (R, \emptyset) to I such that the truck delivers all parcels to each of the customers.

Next, we provide a lower bound on the cost of an optimal route to the TSP-D via the following observation, also given by Agatz et al. [1].

Observation 2 For an instance $I = ((G, w); r \in V(G); \alpha \geq 1)$ of the metric TSP-D, let (R^*, \mathcal{P}^*) be an optimal route for I . Then, it holds that

$$\text{cost}(R^*, \mathcal{P}^*) \geq \max \left\{ w(R^*), \frac{1}{\alpha} w(\mathcal{P}^*) \right\}. \quad (3)$$

2.3 NP-hardness

We will show the NP-hardness of the problem TSP-D by a reduction from the Hamiltonian s, t -Path Problem (see e.g., [5]).

Theorem 1 The metric TSP-D is NP-hard.

Proof. Let $(H = (V, A); s, t \in V)$ be an instance of the Hamiltonian s, t -Path Problem, i.e., a simple undirected graph H with a vertex set V , terminals $s, t \in V$, and edge set A , as illustrated in **Fig. 2(a)**. Let $|V(H)| = n$. We construct an instance $I = ((G, w); r \in V(G); \alpha \geq 1)$ of the TSP-D with metric edge weight w as shown in **Fig. 2(b)**. First, construct three new vertices, $r, p, x \notin V$, and let G be the complete graph over the vertex set $V(G) = V \cup \{r, p, x\}$. Define $E' \triangleq A \cup \{rp, ps, rt, xr, xp\}$. Then we define $w' : E' \rightarrow \mathbb{R}_+$ to be

$$w'(uv) = \begin{cases} 0 & \text{if } uv = rp, \\ 1 & \text{if } uv \in A, uv = ps, \text{ or } uv = rt, \\ \alpha \cdot \frac{n+1}{2} & \text{if } u = x \text{ and } v \in \{p, r\}. \end{cases} \quad (4)$$

Then, let $G' = (V(G), E')$, and we define $w : E(G) \rightarrow \mathbb{R}_+$ to be $w(uv) \triangleq \min \{w'(P) \mid P \in G' \text{ is a } u, v\text{-path}\}$. (5)

It is easy to verify that for any three vertices in $V(G)$, the triangle inequality, Eq. (1), is satisfied, i.e., the graph (G, w) is metric. Finally, we take any value $\alpha \geq 1$. We show that the constructed instance $I = ((G, w); r \in V(G); \alpha \geq 1)$ of the metric TSP-D has a feasible route (R, \mathcal{P}) such that $\text{cost}(R, \mathcal{P}) = n + 1$, if and only if the graph H admits a Hamiltonian s, t -path, see **Fig. 2(c)**.

First, we demonstrate the “if” direction by showing how a Hamiltonian s, t -path P_H in H can be used to construct a feasible route (R, \mathcal{P}) in $I = ((G, w); r \in V(G); \alpha \geq 1)$ such that $\text{cost}(R, \mathcal{P}) = n + 1$. If the instance $(H = (V, A); s, t \in V)$ has a feasible path $P_H = (s, r_2, \dots, t)$, then we construct a feasible route (R, \mathcal{P}) as follows. Let $R = (r, p, s, r_2, \dots, t, r)$ be a cycle in G , such that $R[s, t] = P_H$. Immediately we have that $w(R) = n + 1$, since the cycle R comprises edges of weight 0 and 1 by Eq. (4). Further, we construct a single 2-path $P_1 = (p, x, r)$ in I , for which by Eq. (4) it holds that $w(P_1) = \alpha \cdot (n + 1)$, and let $\mathcal{P} = (P_1)$. Notice that in this case we get $R_1 = R[p, r]$ and $\bar{E}_R = \{rp\}$. By Eq. (4) we have $w(\bar{E}_R) = 0$, and therefore, by Eq. (2), it follows that

$$\text{cost}(R, \mathcal{P}) = w(\bar{E}_R) + \max \left\{ w(R_1), \frac{1}{\alpha} w(P_1) \right\} = n + 1,$$

and this concludes the proof of the “if” direction. An illustration of a feasible route (R, \mathcal{P}) is shown in Fig. 2(d).

Next, to show the opposite direction of the claim, we show that if the instance $I = ((G, w); r \in V(G); \alpha \geq 1)$ has a feasible route $(R, \mathcal{P} = (P_1, P_2, \dots, P_k))$ with $\text{cost}(R, \mathcal{P}) = n + 1$, then the graph H has a Hamiltonian s, t -path. We will do this by showing that such a route (R, \mathcal{P}) is exactly of the form as described earlier, i.e., $R = (R_1 = (r, p, s, r_2, \dots, t, r))$ such that $R[s, t]$ is a Hamiltonian path in $G[V]$ using only edges from the set A , and $\mathcal{P} = (P_1 = (p, x, r))$. First, notice that for the special vertex x it holds that $x \notin V(R)$, otherwise by $\alpha \geq 1$ we immediately get $w(R) > n + 1$, which by Observation 2 contradicts the assumption that $\text{cost}(R, \mathcal{P}) = n + 1$. Hence, we have $x \in V(\mathcal{P})$. Next, let $P_j = (r_h, x, r_t) \in \mathcal{P}$ be the 2-path such that $x \in V(P_j)$. Notice that by Eq. (4), unless $\{r_h, r_t\} = \{p, r\}$, we immediately get $w(P_j) > \alpha(n + 1)$, and hence it must hold that $P_j = (p, x, r)$, and $w(P_j) = \alpha(n + 1)$. Then, we will get

$$\begin{aligned} \text{cost}(R, \mathcal{P}) &= w(\bar{E}_R) + \max\{w(R_j), \frac{1}{\alpha}w(P_j)\} \\ &+ \sum_{1 \leq i \neq j \leq k} \max\{w(R_i), \frac{1}{\alpha}w(P_i)\}, \end{aligned}$$

which by Eq. (4) can be $n + 1$ if and only if $w(\bar{E}_R) = \sum_{1 \leq i \neq j \leq k} \max\{w(R_i), \frac{1}{\alpha}w(P_i)\} = 0$ and $w(R_j) \leq n + 1$. Therefore, it must hold that $\mathcal{P} = (P_1 = (p, x, r))$, and for (R, \mathcal{P}) to be feasible, it must hold that $V \cup \{r, p\} \subseteq V(R)$, which means that $R[s, t]$ is a Hamiltonian s, t -path in $G[V]$ using only edges from the set A , as required. \square

3. Approximation Algorithms

First, we provide three technical lemmas which are necessary for the analysis.

Lemma 1 For an instance $I = ((G, w); r \in V(G); \alpha \geq 1)$ of the metric TSP-D, let (R^*, \mathcal{P}^*) be an optimal route for I . For each $d \in V(\mathcal{P}^*) \setminus V(R^*)$, let $c_d \in V(R^*) \setminus V(\mathcal{P}^*)$ satisfy

$$w(c_d d) = \min\{w(cd) \mid c \in V(R^*)\}. \quad (6)$$

(i) Then, for each $d \in V(\mathcal{P}^*) \setminus V(R^*)$, it holds that

$$w(rd) \leq \frac{1}{2}w(R^*) + w(c_d d). \quad (7)$$

(ii) Furthermore, it holds that

$$\sum_{d \in V(\mathcal{P}^*) \setminus V(R^*)} w(c_d d) \leq \frac{1}{2}w(\mathcal{P}^*).$$

Proof. (i) Let us observe that $d \in V(\mathcal{P}^*) \setminus V(R^*)$ and $c_d \in V(R^*) \setminus V(\mathcal{P}^*)$ hold. Notice that by the triangle inequality, it holds that

$$w(rd) \leq w(c_d d) + w(rc_d).$$

Moreover, for each $c_d \in V(R^*) \setminus V(\mathcal{P}^*)$ it holds that

$$w(rc_d) \leq \frac{1}{2}w(R^*).$$

Then, we get

$$w(rd) \leq w(c_d d) + \frac{1}{2}w(R^*),$$

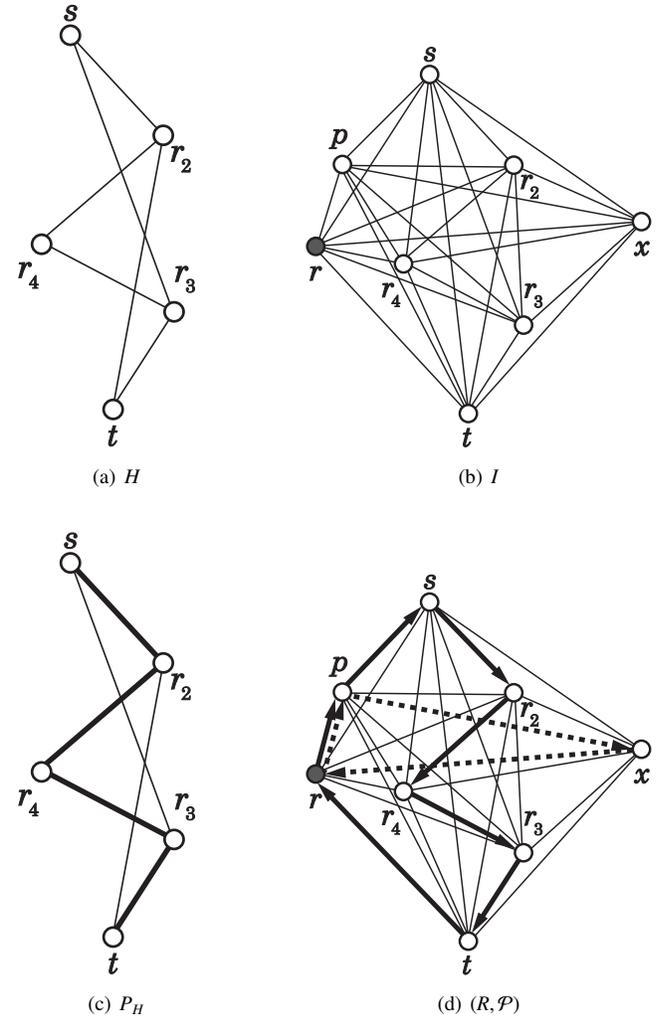


Fig. 2 A reduction from the Hamiltonian s, t -Path Problem to a metric TSP-D instance $I = ((G, w); r \in V(G); \alpha \geq 1)$. (a) An instance of the Hamiltonian s, t -Path Problem, $(H = (V, A), s, t \in V)$. (b) A metric TSP-D instance $I = ((G, w); r \in V(G); \alpha \geq 1)$. (c) A Hamiltonian s, t -path P_H in H . (d) A feasible route (R, \mathcal{P}) in the instance I .

as required.

(ii) If $d \in V(\mathcal{P}^*) \setminus V(R^*)$, then for some 2-path $P = (c_h, d, c_t) \in \mathcal{P}^*$ it holds that $d \in V(P)$. Then, it holds that

$$2w(c_d d) \leq w(c_h d) + w(d c_t) = w(P).$$

By summing over all $d \in V(\mathcal{P}^*) \setminus V(R^*)$, we get

$$\sum_{d \in V(\mathcal{P}^*) \setminus V(R^*)} w(c_d d) \leq \frac{1}{2} \sum_{P \in \mathcal{P}^*} w(P) \leq \frac{1}{2}w(\mathcal{P}^*),$$

as required. \square

Lemma 2 For an instance $I = ((G, w); r \in V(G); \alpha \geq 1)$ of the metric TSP-D, let (R^*, \mathcal{P}^*) be an optimal route for I . Then, for any minimum spanning tree T^* of (G, w) it holds that

$$w(T^*) \leq w(R^*) + \frac{1}{2}w(\mathcal{P}^*). \quad (8)$$

Proof. Let T_R be a spanning tree of the graph R^* . Then, it holds that

$$w(T_R) \leq w(R^*). \quad (9)$$

Next, we define $F \subseteq E(G)$ to be

$$F \triangleq \{uw \mid u \in V(\mathcal{P}^*) \setminus V(R^*) \text{ and } v \in V(R^*) \text{ such that it holds that } w(uv) = \min \{w(uq) \mid q \in V(R^*)\}\}.$$

Then, by Lemma 1(ii) it holds that

$$w(F) \leq \frac{1}{2}w(\mathcal{P}^*). \quad (10)$$

Further, using the spanning tree T_R and the set of edges F , we can construct a spanning tree T of the graph G , such that it holds that

$$w(T) \leq w(T_R) + w(F). \quad (11)$$

Finally, as T^* was taken to be a minimum spanning tree, it holds that $w(T^*) \leq w(T)$, from which the claim follows. \square

Lemma 3 For an instance $I = ((G, w); r \in V(G); \alpha \geq 1)$ of the metric TSP-D, let (R^*, \mathcal{P}^*) be an optimal route for I . For a subset $J \subseteq V(G)$ such that $|J|$ is even, let M_J be a minimum cost matching with $V(M_J) = J$. Then, it holds that

$$w(M_J) \leq \frac{1}{2}(w(R^*) + w(\mathcal{P}^*)). \quad (12)$$

Proof. The cycle R^* and the 2-paths in \mathcal{P}^* can be shortcut to a Hamiltonian cycle H of G , such that

$$w(H) \leq w(R^*) + w(\mathcal{P}^*).$$

This cycle H can be shortcut into a cycle H_J such that $V(H_J) = J$, and by the triangle inequality it holds that

$$w(H_J) \leq w(H).$$

The cycle H_J contains two disjoint matchings M_1 and M_2 such that $V(M_1) = V(M_2) = J$ and $w(M_1) + w(M_2) = w(H_J)$. Then, it holds that

$$w(M_J) \leq \min\{w(M_1), w(M_2)\} \leq \frac{1}{2}w(H_J), \quad (13)$$

as required. \square

Theorem 2 For an instance $I = ((G, w); r \in V(G); \alpha \geq 1)$ of the metric TSP-D, a feasible route (R, \emptyset) such that

$$\text{cost}(R, \emptyset) \leq (3/2 + \alpha) \cdot \text{cost}(R^*, \mathcal{P}^*) \quad (14)$$

can be constructed in polynomial time.

Proof. Refer to Observation 1 that a Hamiltonian cycle R in G gives a feasible route (R, \emptyset) . Then, it holds that

$$\text{cost}(R, \emptyset) = w(R). \quad (15)$$

We use the Christofides algorithm [2] to construct a Hamiltonian cycle in (G, w) , shown in **Fig. 3**. By the Christofides algorithm [2], for a minimum spanning tree T^* of G , where J is the set of vertices with odd degree in T^* , and a minimum cost matching M_J such that $V(M_J) = J$, R is constructed such that

$$w(R) \leq w(T^*) + w(M_J). \quad (16)$$

By Lemmas 2 and 3, it holds that

$$\text{cost}(R, \emptyset) \leq w(T^*) + w(M_J)$$

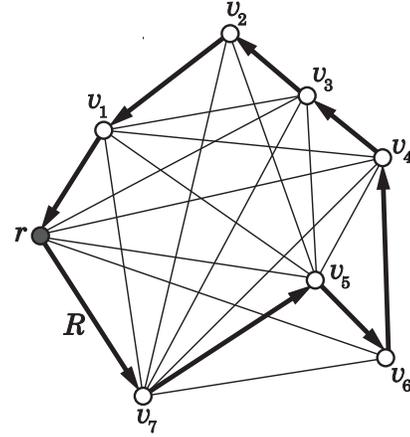


Fig. 3 A feasible truck route when only the truck delivers parcels to each of the customers, which are presented by white circles, and returns back to the depot, r , after serving all customers.

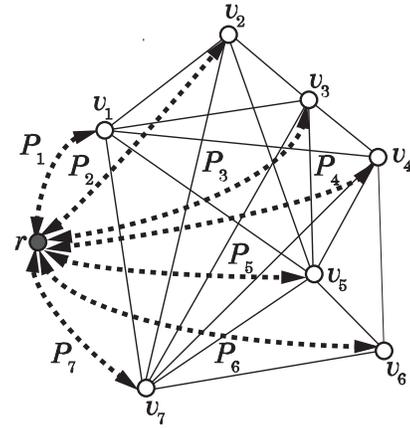


Fig. 4 A feasible drone route when only the drone delivers parcels from the depot, r , to each of the customers, presented by white circles, and returns back to the depot, r , after serving each customer.

$$\begin{aligned} &\leq w(R^*) + \frac{1}{2}w(\mathcal{P}^*) + \frac{1}{2}w(R^*) + \frac{1}{2}w(\mathcal{P}^*) \\ &= \frac{3}{2} \cdot w(R^*) + w(\mathcal{P}^*) \\ &\leq \frac{3}{2} \cdot \text{cost}(R^*, \mathcal{P}^*) + \alpha \cdot \text{cost}(R^*, \mathcal{P}^*), \quad (17) \end{aligned}$$

as required. \square

Let $\mathcal{P} = (P_1, P_2, \dots, P_n)$ be such that each $P_i \in \mathcal{P}$ is of the form $P_i = (r, u_i, r)$, $u_i \in V(G) \setminus \{r\}$, and for $i \neq j$, $u_i \neq u_j$. Notice that $R = (r)$ is a trivial cycle with a single vertex, and $((r), \mathcal{P})$ gives a route.

Theorem 3 For an instance $I = ((G, w); r \in V(G); \alpha \geq 1)$ of the metric TSP-D such that $V(G) = \{r, u_1, u_2, \dots, u_n\}$, let (R^*, \mathcal{P}^*) be an optimal route for I . Let $\mathcal{P} = (P_1, P_2, \dots, P_n)$ be such that $P_i = (r, u_i, r)$, $u_i \in V(G) \setminus \{r\}$. Then, for the route $((r), \mathcal{P})$ it holds

- (i) $((r), \mathcal{P})$ is a feasible route for I , and
- (ii) $\text{cost}((r), \mathcal{P}) \leq (1 + n/\alpha) \cdot \text{cost}(R^*, \mathcal{P}^*)$.

Proof. (i) Notice that $((r), \mathcal{P})$ as illustrated in **Fig. 4**, satisfies the conditions (i)-(iii) in Sect. 2.2, from where it holds that $((r), \mathcal{P})$ is a feasible route for I .

(ii) For each $d \in V(\mathcal{P}^*) \setminus V(R^*)$, let $c_d \in V(R^*)$ be such that $w(c_d d) = \min \{w(cd) \mid c \in V(R^*)\}$. By Lemma 1, it holds that

$$\begin{aligned}
 \text{cost}((r), \mathcal{P}) &= \frac{2}{\alpha} \sum_{d \in V(G) \setminus \{r\}} w(rd) \\
 &= \frac{2}{\alpha} \sum_{d \in V(\mathcal{P}^*) \setminus V(R^*)} w(rd) + \frac{2}{\alpha} \sum_{d \in V(R^*)} w(rd) \\
 &\leq \frac{2}{\alpha} \sum_{d \in V(\mathcal{P}^*) \setminus V(R^*)} w(c_d d) + \frac{2}{\alpha} \sum_{d \in V(\mathcal{P}^*) \setminus V(R^*)} \frac{1}{2} w(R^*) \\
 &\quad + \frac{2}{\alpha} \sum_{d \in V(R^*)} \frac{1}{2} w(R^*) \\
 &\leq \frac{1}{\alpha} w(\mathcal{P}^*) + \frac{n}{\alpha} w(R^*) \\
 &\leq \text{cost}(R^*, \mathcal{P}^*) + \frac{n}{\alpha} \cdot \text{cost}(R^*, \mathcal{P}^*) \quad (\text{by Eq. (3)}) \\
 &= (1 + n/\alpha) \cdot \text{cost}(R^*, \mathcal{P}^*),
 \end{aligned}$$

as required. \square

Theorem 4 For an instance $I = ((G, w); r \in V(G); \alpha \geq 1)$ of the metric TSP-D such that $|V(G)| = n + 1$, let (R^*, \mathcal{P}^*) be an optimal route for I . Then, a feasible route (R, \mathcal{P}) such that

$$\text{cost}(R, \mathcal{P}) \leq (\min\{3/2 + \alpha, 1 + \sqrt{n}\}) \cdot \text{cost}(R^*, \mathcal{P}^*) \quad (18)$$

can be constructed in polynomial time.

Proof. If $\alpha \leq \sqrt{n}$, then by Theorem 2, we can construct a feasible route (R, \emptyset) such that

$$\text{cost}(R, \emptyset) \leq (3/2 + \alpha) \cdot \text{cost}(R^*, \mathcal{P}^*). \quad (19)$$

Further, if $\alpha \geq \sqrt{n}$, then by Theorem 3, we can construct a feasible route $((r), \mathcal{P})$ such that

$$\begin{aligned}
 \text{cost}((r), \mathcal{P}) &\leq \left(1 + \frac{n}{\alpha}\right) \cdot \text{cost}(R^*, \mathcal{P}^*) \\
 &\leq (1 + \sqrt{n}) \cdot \text{cost}(R^*, \mathcal{P}^*). \quad (20)
 \end{aligned}$$

Finally, by Eqs. (19) and (20), it holds that

$$\text{cost}(R, \mathcal{P}) \leq (\min\{3/2 + \alpha, 1 + \sqrt{n}\}) \cdot \text{cost}(R^*, \mathcal{P}^*),$$

as required. \square

4. Conclusion

In this work, we have investigated the deployment of an unmanned aerial vehicle, or drone, to support a truck for the last-stretch deliveries of parcels to customers' doorsteps. We built upon the work of Agatz et al. [1], who introduced the TSP-D, and showed that it can be approximated within a $(2 + \alpha)$ -approximation ratio. We could show that metric problem instances with n customers that can be served by either the drone or the truck can be approximated within a $\min\{3/2 + \alpha, 1 + \sqrt{n}\}$ -approximation ratio. It might be possible to find a special type of instances that can be solved in polynomial time [9].

In the problem that we investigated, both the truck and the drone are allowed to perform deliveries. As future work, it would be interesting to impose limitations on customers that can be served by the drone, e.g., customers with bulky or heavy parcels. On the other hand, due to the novelty effect, some customers may demand to be served by the drone, and this can provide a different extension of the TSP-D.

The approximation results rely on the triangle inequality, and
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this property cannot be easily applied when for the parameter α it holds that $\alpha < 1$, and it is an open question as to the approximability of this problem setting.

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