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自己検証非決定性ならびにラスベガスマルチヘッド２次元有限オートマタ*

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1 Introduction and Definitions

The comparative study of the computational power of nondeterministic, deterministic, and randomized computations is one of the central tasks of complexity theory. In this paper we focus on the relationships between Las Vegas and determinism and between Las Vegas and nondeterminism.

Recently, Duris, Hromkovic and Inoue [1] proved, for the first time, a strong separation between nondeterminism, Las Vegas, and determinism for finite automata with two-dimensional squared inputs.

Very recently, Inoue, Tanaka, Ito and Wang [4] proved, for the first time, a strong separation among nondeterminism, Las Vegas, and determinism, for computing models with strings as its inputs. This paper proves a strong sparation among nondeterminism, Las Vegas and determinism for three-way (simple) multihead finite automata [3] with two-dimansional squared inputs.

A *four-way two-dimansional k-head finite automaton* (2-kHA) M is a finite automaton with k read-only input heads operating on two-dimansional input tapes surrounded by boundary symbols #. These heads can move up, down, left, or right. We denote by $L(M)$ the set (language) of all inputs accepted by M.

A four-way two-dimansional *simple* multihead finite automaton (SP2-MHA) is a 2-MHA whose only one head (called the *"reading"* head) is capable of distinguishing the symbols in the input alphabet, and whose other heads (called *"counting"* heads) can only detect whether they are on the boundary symbols or a symbol in the input alphabet.

A *three-way multihead finite automaton* (TR2-MHA) (resp., *three-way simple multihead finite automaton* (TRSP2-MHA)) is 2-MHA (resp., SP2-MHA) all the heads of which cannot move up. As usual, we define nondeterministic and deterministic versions of those automata. The states of these automata are considerd to be divided into three disjoint sets of working, accepting, and rejecting states. No action is possible from any rejecting or accepting state.

A *self-verifying nondeterministic* 2-MHA (resp., TR2-MHA, SP2-MHA, TRSP2-MHA) is a 2-MHA (resp., TR2-MHA, SP2-MHA, TRSP2-MHA) with four types of states: working, accepting, rejecting, and neutral ("I do not know") ones. There is no possible move from accepting, rejecting, and neutral states. The self-verifying nondeterministic device M is not allowed to make mistakes. If there is a computation of M on an input x finishing in an accepting (resp., rejecting) state, then x must be in $L(M)$ (resp., x must not be in $L(M)$). For every input y, there is at least one computation of M that finishes either in an accepting state (if $y \in L(M)$) or in a rejecting state (if $y \notin L(M)$).

A *Las Vegas* 2-MHA (resp., TR2-MHA, SP2-MHA, TRSP2-MHA) A may be viewed as a self-verifying nondeterministic 2-MHA (resp., TR2-MHA, SP2-MHA, TRSP2-MHA) with probabilities assigned to every nondeterministic branching. The probability of a computation of A is defined through the transition probabilities of A. We require for $y \in L(A)$ (resp., $y \notin L(A)$) that A reaches an accepting (resp., rejecting) state with a probability of at least $\frac{1}{2}$.

For each $k \geq 1$, let $2-kHA$ denote a two-dimensional k-head finite automaton. In order to represent different kinds of $2-kHA$'s, we use the notation $XYZ2-kHA$, where

(1) $\begin{cases} X = TR : \text{three-way} \\ \text{there is no } X : \text{four-way.} \end{cases}$

(2) $Y \in \{D, N, SVN, LV\}$,
D: deterministic, N: nondeterministic,
SVN: self-verifying nondeterministic,
LV: Las Vegas;

(3) $\begin{cases} Z = SP : \text{simple} \\ \text{there is no } Z : \text{non-simple;} \end{cases}$

We denote by $\pounds[XYZ2-kHA]$ the class of languages accepted by XYZ2-kHA's.

Let Σ be a finite set of symbols. A two-dimensional tape over Σ is a two-dimensional rectangular array of elements of Σ. The set of all two-dimensional tapes over Σ is denoted by $\Sigma^{(2)+}$. Given a tape $x \in \Sigma^{(2)+}$, we let $l_1(x)$ be the number of rows of x and $l_2(x)$ be the number of columns of x. If $1 \leq i \leq l_1(x)$ and $1 \leq j \leq l_2(x)$, we let $x(i,j)$ denote the symbol in x with coodinates (i,j). We define $x[(i,j),(i',j')]$, only when $1 \leq i \leq i' \leq l_1(x)$ and $1 \leq j \leq j' \leq l_2(x)$, as the two-dimensional tape z satisfying the following:

(i) $l_1(z) = i' - i + 1$ and $l_2(z) = j' - j + 1$,
(ii) for each $k, r (1 \leq k \leq l_1(x), 1 \leq r \leq l_2(x))$, $z(k,r) = x(k+i-1, r+j-1)$.

Particularly, for each $i \leq i \leq l_1(x)$, $x[i,*]$ denotes $x[(i,1),(i,l_2(x))]$, that is the i-th row of x.

2 Determinism versus Las Vegas for Three-way Machines

We first prove a strong separation between deterministic and Las Vegas TRSP2-MHAs.

Theorem 2.1. For each $k \geq 2$,
$$\pounds[TRDSP2-kHA] \subsetneq \pounds[TRLVSP2-kHA].$$

Proof. For each $k \geq 2$, let
$$\begin{aligned} T_1(k) = \ & \{ x \in \{0,1\}^{(2)+} \mid \exists n \geq k+1 [l_1(x) = l_2(x) = n \\ & \wedge \exists i_1, i_2, \cdots, i_k (1 \leq i_1 < i_2 < \cdots < i_k \leq n-1)[\\ & x(1,i_1) = x(1,i_2) = \cdots = x(1,i_k) = 1 \\ & \wedge \forall i (1 \leq i \leq n-1, i \neq i_1, i_2, \cdots, i_k)[x(1,i) = 0] \\ & \wedge [(x(2,i_1) = 1 \wedge \ x(2,n) = 0) \\ & \vee (x(2,i_2) = x(2,i_3) = \cdots = x(2,i_k) = 1 \\ & \wedge \ x(2,n) = 1)]]]\}. \end{aligned}$$

$T_1(2k)$ is a witness language for the theorem. Language $T_1(2k)$ is accepted by a TRLVSP2-kHA M which acts as follows. Let R and $H_1, H_2, \cdots, H_{k-1}$ be the reading and counting heads of M, respectively.
First, M checks whether the first row of x has exactly

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*Self-Verifying Nondeterministic and Las Vegas Multihead Two Dimentional Finite Automata

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2k '1's. Let $x(1,i_1), x(1,i_2), \cdots, x(1,i_k)$, where $1 \le i_1 < i_2 < \cdots < i_k \le n-1$, be these 2k '1's on the first row. Then, M chooses one of the following two actions ① and ② with probability $\frac{1}{2}$.

① M checks whether $x(2,i_1) = 1$.
If $x(2,i_1) = 1$ and $x(2,n) = 0$, then M enters an accepting state.
If $x(2,i_1) \neq 1$ and $x(2,n) = 0$, then M enters a rejecting state.
If $x(2,n) = 1$, then M enters a neutral state, whther or not $x(2,i_1) = 1$.

② For each $j(1 \le j \le k-1)$, M moves H_j to the cell of x with coordinates (i_{2j}, i_{2j+1}). Then, M checks by using R whether $x(2,i_{2k}) = 1$, and for each $j(1 \le j \le k-1)$. M checks by using R and H_j whether $x(2,i_{2j}) = 1$ and $x(2,i_{2j+1}) = 1$. (It is an easy exercise to see that these actions can be done.)
If $x(2,i_{2l}) = 1$ for each $l(2 \le l \le k)$ and $x(2,n) = 1$, then M enters an accepting state.
If $x(2,i_{2l}) \neq 1$ for some $l(2 \le l \le k)$ and $x(2,n) = 1$, then M enters a rejecting state.
If $x(2,n) = 0$, then M enters a neutral state whether or not $x(2,i_l) = 1$ for each $l(2 \le l \le k)$.

The proof of "$T_1(2k) \notin \pounds[TRDSP2-kHA]$" is omitted here.

We next prove a strong separation between deterministic and Las Vegas TR2-MHAs.

Theorem 2.2. For each $k \ge 2$,
$\pounds[TRD2-kHA] \subsetneq \pounds[TRLV2-kHA]$.

Proof. For each $k \ge 2$, let

$T_2(k) = \{x \in \{0,1\}^{(2)+} \mid \exists n \ge 2b(k)+1[l_1(x) = l_2(x) = n \wedge [(x[1,*] = x[2b(k),*] \wedge x(2b(k)+1,n) = 0) \vee (\forall i(2 \le i \le b(k))[x[i,*] = x[2b(k)-i+1,*]] \wedge x[2b(k)+1,n] = 1)]]\}$,
where $b(k) = \binom{k}{2}$.

$T_2(k)$ is a witness language for the theorem. The details of the proof are omitted here.

3 Self-verifying Nondeterminism versus Nondeterminism for Three-way Machines

This section proves a strong separation between self-verifying nondeterminism and nondeterminism, for three-way machines.

Theorem 3.1.

$$\pounds[TRN2-1HA] - \bigcup_{1 \le k < \infty} \pounds[TRSVNSP2-kHA] \neq \phi.$$

Proof. For each $n \ge 2$, let

$T_1 = \{x \in \{0,1\}^{(2)+} \mid l_1(x) = l_2(x) = n \wedge x[1,*] \neq x[2,*]\}$.

T_1 is a witness language for the theorem. The details of the proof are omitted here.

Theorem 3.2.

$$\pounds[TRN2-2HA] - \bigcup_{1 \le k < \infty} \pounds[TRSVN2-kHA] \neq \phi.$$

proof. Let,

$T_2 = \{x \in \{0,1,2\}^{(2)+} \mid \exists n \ge 3[l_1(x) = l_2(x) = n$
$\wedge$(there exists an integer i, $3 \le i \le n$, such that
(i)$x[(i,1),(n,n)] \in \{2\}^{(2)+}$,
(ii)$\forall j(1 \le j \le i-1)$[the jth row of x is of the form $w_j 2 w'_j$ for $w_j, w'_j \in \{0,1\}^+$], and
(iii)$\exists k, \exists l(1 \le k < l \le i-1)$[(the kth row of x is $w_k 2 w'_k$) $\wedge$ (the lth row of x is $w_l 2 w'_l$)
$\wedge\, w_k = w_l \wedge w'_k \neq w'_l$])]\}.

T_2 is a witness language for the theorem. The details of the proof are omitted here.

4 Self-verifying Nondeterminism versus Las Vegas for Three-way Simple Multi-head Machines

Theorem 4.1. For each $k \ge 2$,

$$\pounds[TRSVN2-1HA] - \bigcup_{1 \le k < \infty} \pounds[TRLVSP2-kHA] \neq \phi.$$

Proof. For any $n \ge 2$, let

$L = \{x \in \{0,1\}^{(2)+} \mid \exists n \ge 1[l_1(x) = l_2(x) = n \wedge \exists i(0 \le i \le n-1), \exists z \in \{0,1\}^*[if\ x[2,*] = 0^i 1 z,\ then\ x(1,i+1) = 1]]\}$.

L is a witness language for the theorem. The details of the proof are omitted here.

5 Four-way Self-verifying Nondeterminism versus Las Vegas

For four-way machines, we have:

Theorem 5.1. For each $k \ge 1$,
(1) $\pounds[SVN2-kHA] = \pounds[LV2-kHA]$, and
(2) $\pounds[SVNSP2-kHA] = \pounds[LVSP2-kHA]$.

Proof. We prove only (1), because the proof of (2) is the same. "$\pounds[LV2-kHA] \subseteq \pounds[SVN2-kHA]$" is obvious, because every $LV2-kHA$ can be viewed as a $SVN2-kHA$. The simulation of a $SVN2-kHA$ by a $LV2-kHA$ can be done by using a mixture of the proofs of Theorem 1 in [5] and Theorem 1 in [2].

6 Conclusion

This study proved a strong separation among nondeterminism, Las Vegas, and determinism for $2-MHA$s. Unsolved problems in this study are :

(1) £[TRLVSP2-1HA] $- \bigcup_{1 \le k < \infty}$ £[TRDSP2-kHA] $\neq \phi$?

(2) £[TRLV2-1HA] $- \bigcup_{1 \le k < \infty}$ £[TRD2-kHA] $\neq \phi$?

(3) £[TRSVN2-1HA] $- \bigcup_{1 \le k < \infty}$ £[TRLV2-kHA] $\neq \phi$?

(4) £[DSP2-2HA] $\subsetneq$ £[LVSP2-2HA]?

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