Fillet Operations with Loop Subdivision Surfaces

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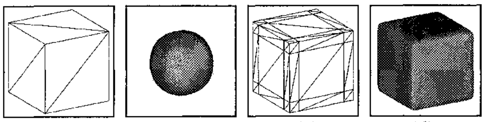
Wei-zhong Liu*
Department of Applied Mathematics
Dalian University of Technology, China

Kunio KONDO†
Department of Information and Computer Sciences
Saitama University, Japan

1. Introduction

Surfaces can be generated easily by subdividing polyhedral networks recursively, but over simple or rough polyhedral network, it is difficult to get the surfaces which designers want to model. To solve this problem, complex or precise polyhedral networks have to be defined. For example, for getting a cube-like shape (Figure 1(d)) with Loop subdivision method, a polyhedron shown in Figure 1(c) has to be defined, we can't get cube-like shape by subdividing a simple cube (Figure 1(a)). In this paper, a set of rules for making fillet operations with Loop subdivision are presented. In our method, according to the sharpness of each edge of polyhedral networks, the initial polyhedral networks are subdivided one step by using the rules proposed here, and then the Loop subdivision method is used to generate surfaces.



(a) (b) (c) (d)

Figure 1: Fillet operations with Loop subdivision surfaces. (a)The original polyhedron. (b)The 2nd Loop subdivision shape. (c)The modified polyhedron. (d) The 2nd Loop subdivision over modified polyhedron.

2. Definitions and Regular process

Fillet value describes the sharpness of an edge and is defined as the rate of two distances. Referring to Figure 2, the fillet value on edge E_0 is defined as $\frac{d1}{d2}$, edge (P_{02},P_{05}) termed fillet line on the face F_0. The two fillet values of an edge can be different. The fillet value of an edge also can be 0. Now we assume the fillet value is not equal to 0. The regular fillet process can be described as following steps (Referring to Figure 3):

Step 1: For each edge E_i, according to its sharpness, a parameter called fillet value is assigned to it.

Step 2: For each edge E_i linking vertex V_k, V_j and shared by face F_s, F_t, four intersection points of the E_i are generated. They are called fillet edge points, termed P_{si} and P_{ti}. Corresponding to V_j, there are two points P_{sj} and P_{tj} are generated on edge E_i, but only one point will be used to generate new faces, the useful point is termed P_j.

*劉偉中 大連理工大学応用数学科, 中国
†近藤邦雄 埼玉大学情報システム工学科、日本

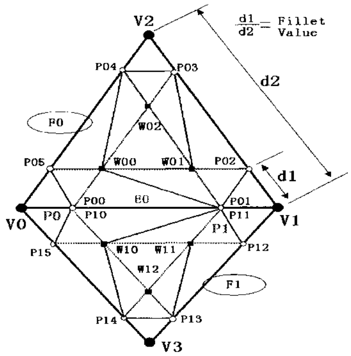


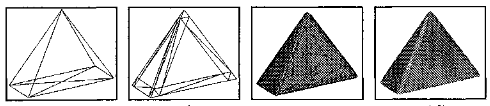
Figure 2: The definition of fillet value on edge E_0. Solid circle nodes are the vertices of the polyhedron, hollow circle nodes are the fillet edge points. Solid square nodes are fillet face points.

Step 3: For each vertex V_i on a face F_s, an intersecttion point of two fillet lines, corresponding to the two contiguous edges that have common end point V_i, is generated. This point is called fillet face point termed W_{si}(Solid square nodes in Figure 2).

Step 4: For each face F_i, a new face is generated by linking all the fillet face points W_{ij} on the face F_i.

Step 5: For each edge E_i on face F_s, linking vertices V_j,V_k, two new faces are generated by linking (P_{sj}, P_{sk},W_{sj}) and (P_{sk},W_{sk},W_{sj}) respectively.

Step 6: For each vertex V_i, two faces are generated on each face that V_i is common to. They are linked by V_i, the fillet face point and two fillet edge points. For example, in Figure 2, for V_0 on face F_0, the two faces are (V_0,P_0,P_{05}) and (P_0,W_{00},P_{05}).



(a) (b) (c) (d)

Figure 3: The regular fillet operation on a pyramid. (a) A pyramid. (b) The modified pyramid after fillet operations. (c) The 1st Loop subdivision over modified pyramid. (d) The 2nd Loop subdivision over modified pyramid.

After subdividing the pyramid according to above steps, subdivision process will be carried out recursively. Figure 3 illustrates the procedure of making fillet operations on a pyramid. The process described above is an ideal case. Generally, according to different fillet values of edges, some rules are necessary. In the following sections, five rules are introduced for dealing with general cases.

3. The rules of fillet operation

3.1 Rule 1

In general, the fillet values of the edges of a polyhedron are not equal each other. Here we give Rule 1. Referring to Figure 4, for each edge E_i linking vertices (V_j,V_k), common to faces F_s and F_t, the fillet values are λs and λt. If λs is larger than λt, P_{si} will be treated as useful fillet edge point P_i.

3.2 Rule 2

Referring to Figure 5, for a face F_0 and a vertex V_0 of F_0, two edges of F_0 having common end point V_0 are E_0 and E_1. If the useful fillet edge points P_0 and P_1 are not P_{00} and P_{05}, through point W_{00} make three lines (W_{00},P_{25}), (W_{00},P_{10}) and (W_{00}, V_0). Then we get two faces (V_0,P_0,W_{00}) and (V_0, W_{00},P_1).

3.3 Rule 3

Fillet operations will not be carried out on the edges that are on the same plane. These edges are called interior edges. For example, in Figure 6, the two faces F_0 and F_1 are on the same plane, edge E_0 are called interior edge, the fillet operations will not be done on E_0.

3.4 Rule 4

Rule 4 deals with the case that the fillet value of an edge is 0. Referring to Figure 7, for an edge E_0 linking vertices V_0 and V_1, shared by F_0 and F_1, if the fillet value of E_0 on the face F_0 equals to 0, the fillet edge points P_{02} and P_{05} on the F_0 will not be generated. If the fillet point P_{25} on edge E_1 of F_2 is generated, through point P_{01} make a line (P_{01},P_{25}), then we get one face (V_1,P_{25},P_{01}).

3.5 Rule 5

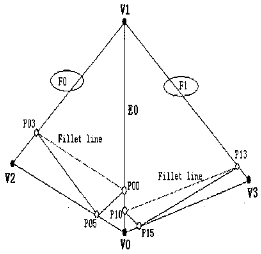
Rule 5 is called corner fitting process. It is carried out if there is a fillet operation on one neighbor edge of V_i. Referring to Figure 8, F_0~F_3 are on the same plane, so there are no fillet operations on edge (V_0,V_2), (V_0,V_3) and (V_0,V_4). For edge (V_0,V_3), there is no fillet edge point generated by fillet operations, we create a new point **P** on this edge.

4. Examples

Figure 9 is the example we made by using the above rules.

5. Conclusions

In this paper, some rules for implementing the fillet operations with Loop subdivision surfaces are proposed. The fillet operations are implemented by subdividing a polyhedral network one step with given sharpness of edges of the polyhedral network. The operations are simple and effective. With these rules, users can implement fillet operations over polyhedra easily and effectively and control the final shape easily by modifying the fillet values. The fillet operations proposed here will strengthen the functions of systems that use subdivision methods to model surfaces.



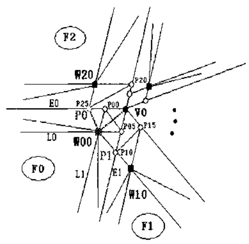
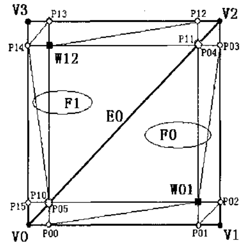


Figure 4: Rule 1 *Figure 5: Rule 2*
The fillet values of edges are not equal each other.



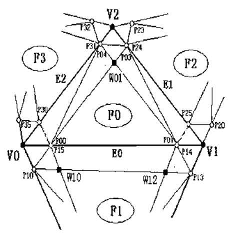


Figure 6: Rule 3
Fillet subdivision process will not be made on interior edges.

Figure 7: Rule 4
On surface F_0, the fillet value on edge E_0 is 0

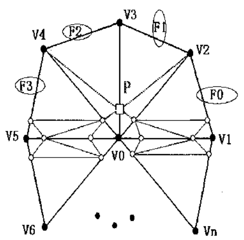
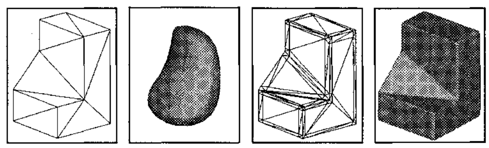


Figure 8: Corner fitting around vertex V_0. F_0~F_3 are on the same plane. The vertex P is the new created point.



(a) (b) (c) (d)

Figure 9: The fillet operations on a polyhedron. (a) A polyhedron. (b) Loop Surfaces after 2 iterations on 9(a). (c)The polyhedron after fillet operations on 9(a). (d) Loop surfaces after 2 iterations on 9(c)

References

C. Loop, Smooth Subdivision Surfaces Based on Triangles, Master Thesis, University of Utah, Department of Mathematics, 1987.