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Recognition of triangles for perspective projection

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Abstract

Method to recognize solid objects by triangulation are now used in general. In order to recognize a solid object using picture taken by a camera, it is important to investigate the recognition of the planar contours of several triangles of tetrahedrons which construct the solid object approximately . It has been shown that it is useful to get information for the recognition that all image of triangles in the phothos have the images of their inscribed circles [2], where affine trasformations are used. However it has the problems that camera angles are restricted. In this paper, our purpose is to expand the camera angles, that is, to show that those assigned triangles is possible to be recognized if image tangent points are known on image of its inscribed circles of the triangles. A system of algebraic equations are obtained whose unknown is the angle between the direction of camera viewpoint and the normal of the face of a triangle.

1 Introduction

In [1], planar contours are recognized by the invariant value $I(\tau) = \int |\mathbf{x} - \mathbf{x}_1\ \mathbf{x}^{(1)}|dt$ whose parameter τ is so called affine arc length $\tau = \int abs(|\mathbf{x}^{(1)}\mathbf{x}^{(2)}|)^{\frac{1}{3}}dt$. Here $\mathbf{x}$ is the position variable and $\mathbf{x}_1$ is one reference point which is fixed. $\mathbf{x}^{(i)}$ denotes ith derivative. In [1], it was shown that $I(\tau)$ is useful for smooth contours. However if a contour includes lines, then we cannot use this method, since $\tau(s)$ is identically equal to 0 on lines. In the author's paper [2], another new method to make a smooth contour of an inscribed circle is proposed. It was shown that original angles of triangles are reproduced only from the photographed image without original information, if it is assumed that the inscribed circles are attached to their triangles, that is, original angles are reproduced by the invariance of eccentric angle under the affine transformation for their inscribed circles. By these angles, $I(\tau)$ is calculated and even triangles are recognized.

In [1] and [2], pseudo-perspective projections (affine transformations) are used. However in practical consideration, perspective projections are better to be used.

In this paper, it is shown that the idea in [2] is also useful for perspective projection, using the image of isosceles triangles consisting of tangent points and vertices.

Proposition 1 *Let a* **tangent point** *is defined as the intersection point of the inscribed circle and an edge of the triangle. Let* $\mathbf{e}_i$ *be a tangent point on the edge* $\mathbf{b}_i\mathbf{b}_{i+1}$ *for* $i = 0, 1, 2$ *for the inscribed circle of a triangle* $\triangle\mathbf{b}_0\mathbf{b}_1\mathbf{b}_2$. *The three isosceles triangles* $\triangle\mathbf{b}_0\mathbf{e}_0\mathbf{e}_2$, $\triangle\mathbf{b}_1\mathbf{e}_1\mathbf{e}_0$ *and* $\triangle\mathbf{b}_2\mathbf{e}_2\mathbf{e}_1$, *are constructed and each one is constructed from one vertex and two tangent points. It holds that two angles are equal:* $\angle\mathbf{e}_1\mathbf{e}_0\mathbf{b}_1 = \angle\mathbf{e}_0\mathbf{e}_1\mathbf{b}_1$; $\angle\mathbf{e}_0\mathbf{e}_2\mathbf{b}_0 = \angle\mathbf{e}_2\mathbf{e}_0\mathbf{b}_0$; $\angle\mathbf{e}_2\mathbf{e}_1\mathbf{b}_2 = \angle\mathbf{e}_1\mathbf{e}_2\mathbf{b}_2$.

The method exploits the property that the back-projected ones from the image triangles $\triangle\bar{\mathbf{b}}_0\bar{\mathbf{e}}_0\bar{\mathbf{e}}_2$, $\triangle\bar{\mathbf{b}}_1\bar{\mathbf{e}}_1\bar{\mathbf{e}}_0$ and $\triangle\bar{\mathbf{b}}_2\bar{\mathbf{e}}_2\bar{\mathbf{e}}_1$ are 3 isosceles triangles and so 2 back-projected angles for each triangle must have equal angles. This property is used to obtain the equations for the angle between the normal of face of triangle and the direction of camera view point.

2 Perspective projection

A perspective projection is defined as a central projection with center where the camera is located.

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Definition 1 perspective projection *Let* $^t(X, Y, Z)$ *be a point on the planar contour. Let* $\mathbf{x} = \binom{x}{y}$ *be a point in the image 2-dimensional space for camera canvas. Then the image point in 2-dimensional is* $\mathbf{x} = \frac{f}{Z}\binom{X}{Y}$. *Here* f *is the focal length and* Z *is the length from camera to a point in the planar contour.*

The perspective projection and its inverse projection are obtained in the authors paper [2] as follows.

Proposition 2 *Let* $(-a_1, -a_2, 1)$ *be normal vector to face of triangle. Let* $\bar{\mathbf{x}}$ *be a point in two dimensional plane of the first and the second elements of coordinate system on the camera canvas. Let* $\bar{\mathbf{x}} = H(\mathbf{v})$ *be a perspective projection from a point* $\mathbf{v}$ *in the original plane to the image point* $\bar{\mathbf{x}}$. *Then* $H(\mathbf{v}) = \frac{fA\mathbf{v}}{^t\mathbf{a}A\mathbf{v}+\zeta}$. *Let* $\mathbf{v} = H^{-1}(\bar{\mathbf{x}})$ *be an inverse projection of* H *from an image point* $\bar{\mathbf{x}}$ *to the original point and then* $H^{-1}(\bar{\mathbf{x}}) = \frac{\zeta A^{-1}\bar{\mathbf{x}}}{fs(\bar{\mathbf{x}})}$. *Here* $s(\mathbf{x}) = 1 - \frac{^t\mathbf{a}\mathbf{x}}{f}$ *and* t *denotes a transpose and put*

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}; A = \begin{pmatrix} \{\sqrt{1+a_1^2}\}^{-1} & -a_1a_2\{\sqrt{1+a_1^2+a_2^2}\sqrt{1+a_1^2}\}^{-1} \\ 0 & \sqrt{1+a_1^2}\{\sqrt{1+a_1^2+a_2^2}\}^{-1} \end{pmatrix}.$$

3 Determination of three angles of original triangle

The three angles of an original triangle are determined by vertices of the image triangle, if its gradient $\mathbf{a}$ and its focal length f are given. Note that the angle has no relation with the distance ζ from the camera to the origin of the space of triangle. Put $\Lambda = (A^tA)^{-1}$; $\tilde{\mathbf{a}} = \binom{a_2}{-a_1}$.

Theorem 1 *Assume that the focal length* f *and the gradient* $\mathbf{a} = \binom{a_1}{a_2}$ *are given. Then the original angle* $\angle \mathbf{b}_2\mathbf{b}_0\mathbf{b}_1$ *is given with respect to the image points vertices as*

$$\angle \mathbf{b}_2\mathbf{b}_0\mathbf{b}_1 = cos^{-1}\frac{< B_1, B_2 >_\Lambda}{\|B_1\|_\Lambda \|B_2\|_\Lambda} \quad where \quad B_1 = \bar{\mathbf{b}}_2 - \bar{\mathbf{b}}_0 - \frac{\tilde{\mathbf{a}}}{f} \mid \bar{\mathbf{b}}_2, \bar{\mathbf{b}}_0 \mid \quad B_2 = \bar{\mathbf{b}}_1 - \bar{\mathbf{b}}_0 - \frac{\tilde{\mathbf{a}}}{f} \mid \bar{\mathbf{b}}_1, \bar{\mathbf{b}}_0 \mid .$$

4 Calculation of gradient $\binom{a_1}{a_2}$ of original space

Since some one of three triangles say $\triangle \mathbf{b}_0\mathbf{e}_0\mathbf{e}_2$ is an isosceles triangle, it holds that two angles are equal; $\angle \mathbf{e}_1\mathbf{e}_0\mathbf{b}_1 = \angle \mathbf{e}_0\mathbf{e}_1\mathbf{b}_1$ and hence $< \mathbf{b}_1 - \mathbf{e}_0, \mathbf{e}_1 - \mathbf{e}_0 >=< \mathbf{b}_1 - \mathbf{e}_1, \mathbf{e}_0 - \mathbf{e}_1 >$. This property is expressed by the image tangent points $\bar{\mathbf{e}}_i$ and image vertices $\bar{\mathbf{b}}_i$ as $\gamma < \mathbf{f}, \mathbf{g} >= 0$. Here $\gamma = \frac{\zeta^2}{f^2s^2(\bar{\mathbf{e}}_0)s^2(\bar{\mathbf{e}}_1)s(\bar{\mathbf{b}}_1)}$; $\mathbf{f} = s(\bar{\mathbf{e}}_0)(s(\bar{\mathbf{e}}_1)\bar{\mathbf{b}}_1 - s(\bar{\mathbf{b}}_1)\bar{\mathbf{e}}_1) + s(\bar{\mathbf{e}}_1)(s(\bar{\mathbf{e}}_0)\bar{\mathbf{b}}_1 - s(\bar{\mathbf{b}}_1)\bar{\mathbf{e}}_0)$; $\mathbf{g} = \Lambda(s(\bar{\mathbf{e}}_0)\bar{\mathbf{e}}_1 - s(\bar{\mathbf{e}}_1)\bar{\mathbf{e}}_0)$. Scalar γ is eliminated. The equation $< \mathbf{f}, \mathbf{g} >= 0$ is obtained with unknowns a_1 and a_2, which is independent in ζ.

Theorem 2 *Let the focal length* f *be given. Let the image tangent points and vertices are given. Then* $\mathbf{a}$ *is obtained by the system of algebraic equations with respect to* a_1 *and* a_2 *with degree 3;* $< \mathbf{f}, \mathbf{g} >= 0$.

References

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