A Method of Visible Surface Computation with Finite Element Technique

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1 Introduction

The research of visible surface computation has been being done for a long time. The purpose of this research is to produce a complete description of a surface that is only partially constrained by available data. The problem to find a visible-surface depending on the data given has been formulated in a regularization framework by D.Terzopoulos and others[1][2]. The result of the desired surface is a solution of a partial differential equation. When discretized with finite element technique, this equation gives rise to a large sparse linear system. To solve this large sparse linear system takes very long time. In their method, the rectangle is used as the basic element to discretize the working area with finite elment technique. In fact, in many cases there are many intersection points in the mesh where are no available data. This is one of the reasons that make the sparse linear system very large. And in many cases, the available data are distributed randomly to a degree in the working area. In those cases, it is difficult to find a uniform rectangular mesh that can make all of the available data on its intersection points or make the mesh in a very small scale when the available data are in a quite number. In order to solve this problem, here we propose to use triangle as the basic element to replace the rectangle to discretize the working area. We will only use the points that the available data exist as the vertices of the triangles. So we can reduce the size of the sparse linear system to save processing time.

The remainder of this paper is organized as the following: Section 2 discusses the shape function on a triangular element. Section 3 expresses the implementation of the computation we propose, and Section 5 concludes with discussions about future work.

2 Shape Function

To make a appropriate shape function on the element is very important. Considering both of the surface that we will to describe and the complication of the calculation, we choose : $[\xi_1, \xi_2, \xi_3, \xi_1\xi_2, \xi_2\xi_3, \xi_1\xi_3, \xi_1{}^2\xi_2+\frac{1}{2}\xi_1\xi_2\xi_3(3(1-\mu_3)\xi_1-(1+3\mu_3)\xi_2+(1+3\mu 3)\xi_3), \xi_2{}^2\xi_3+\frac{1}{2}\xi_1\xi_2\xi_3(3(1-\mu_1)\xi_2-(1+3\mu_1)\xi_3+(1+3\mu 1)\xi_1), \xi_3{}^2\xi_1+\frac{1}{2}\xi_1\xi_2\xi_3(3(1-\mu_2)\xi_3-(1+3\mu_2)\xi_1+(1+3\mu 2)\xi_2)]$ as the basis to create the shape function polynomialsy[3]. Here, (ξ_1, ξ_2, ξ_3) is triangular coordinate in a triangular element and μ_1, μ_2, μ_3 are constans related to the side length of the triangular element repectively. Then the shape function on a triangular element can be written as in equation(1):

$$
\begin{aligned}
p(\xi_1, \xi_2, \xi_3) = \\
& c_0\xi_1 + c_1\xi_2 + c_2\xi_3 + c_3\xi_1\xi_2 + c_4\xi_2\xi_3 + c_5\xi_1\xi_3 + \\
& \tfrac{1}{2}c_6\xi_1\xi_2\xi_3(3(1-\mu_3)\xi_1 - (1+3\mu_3)\xi_2 + (1+3\mu 3)\xi_3) + \\
& \tfrac{1}{2}c_7\xi_1\xi_2\xi_3(3(1-\mu_1)\xi_2 - (1+3\mu_1)\xi_3 + (1+3\mu 1)\xi_1) + \\
& \tfrac{1}{2}c_8\xi_1\xi_2\xi_3(3(1-\mu_2)\xi_3 - (1+3\mu_2)\xi_1 + (1+3\mu 2)\xi_2) + \\
& c_6\xi_1{}^2\xi_2 + c_7\xi_2{}^2\xi_3 + c_8\xi_3{}^2\xi_1 \qquad (1)
\end{aligned}
$$

In order to calculate the coefficients in equation(1), we make the shape function go through some spcific points p_1,p_2,p_3,m_1,m_2,m_3, o_1,o_2,o_3 on the element as showed in figure(1). Here, $m_i, i = 1..3$ are the middle points of the sides and $o_(ii), i = 1..2$ are the moddle points between O and the vertices of the element respectively. The coordinates of the specific points are calculated with the points around them. We get:

$p = f_p(p_1, p_2, p_3, p_{11}, p_{22}, p_{33})$
For example:
$p_{m_2} = \frac{1}{4}(p_1 + p_3 + p_O + p_{O_2})$
$= \frac{1}{4}(p_1 + p_3) + \frac{1}{12}(p_2 + p_{22} + 2p_1 + 2p_3)$.
Now, with those specific points and equation(1) we can get coefficient functions:
$c_i = f_c(p_1, p_2, p_3, p_{11}, p_{22}, p_{33}), i = 0..8$
Then we change triangular coordinate to natural coordinate in equation(1) to get the shape function on the element. We get:

$$v = \sum_{n=0}^{k} c_n x^i y^j \qquad (2)$$

Here, $0 < i, j < 5$, c_n is a linear function of $(p_1, p_2, p_3, p_{11}, p_{22}, p_{33})$ and k is the number of the terms in the polynomial.

3 Implementation of the computation

Ar first, we calculate the energy in the working area with the equation(3)[1]. Here, v is shape function, S is working area, s is the element area, C is the set of all of the available data and β is coefficient.

$$E(v^s) = \frac{1}{2}\sum_S \int\int_s (v^s_{xx})^2 + 2(v^s_{xy})^2 + (v^s_{yy})^2 dxdy + \frac{\beta}{2}\sum_{(x_i,y_i)\in C}[v^s(x_i, y_i) - c(x_i, y_i))]^2 \qquad (3)$$

We use a Gaussian Quadrature numerical formula for triangle to integrate conveniently on the triangular elements. We use 7 sampling points to intergrate. Then for every element we can get a equation as following:

$$E_s(v^s) = f_e(P^s_1, P^s_2, P^s_3, P^s_{11}, P^s_{22}, P^s_{33}) \qquad (4)$$

and we can get a matrix form of the equations from all of the elements[1]:

$$E(\mathbf{v}^s) = \frac{1}{2}(\mathbf{v}^s, \mathbf{A}\mathbf{v}^s) - (\mathbf{f}^s, \mathbf{v}^s) \qquad (5)$$

As the same as in [1], when obtain the minimum of $E(\mathbf{v}^s)$, we can get:

$$\mathbf{A}^s\mathbf{u}^s = \mathbf{f}^s \qquad (6)$$

The solution of this linear system is the data that we will use to construct the surface.

4 Conclusion

In this paper, we proposed a method for reconstruction of visible-surface with finite element technique. This method can be used for the data that are distributed randomly. As futur work we are going to try to triangulate the working area and create the linear system quickly.

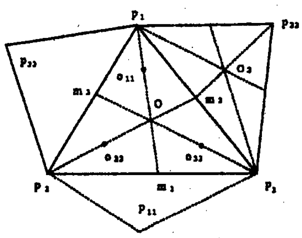

図 1: a triangular element

References

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[3] S.Bernhard," Modified shape functions for the three-node plate bending element passing the patch test ",Int.j.Number.mtchods eng.,vol.26, PP. 705-715(1988).