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Introduction of Economic-Oriented Fairness to a Process Algebra

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1 Introduction

Faireness[2), 5)] is one of the important notion for programming language including concurrency (or parallelism) and nondeterminism such as process algebras like CCS[4)]. This ensures that having to repeatedly choose among a set of alternatives, no alternative will be postponed forever. For example, assume, in a network system, a node P sends a message to either a note Q or R repeatedly. When the fairness is not supposed on this network system, Q (or R) may be able to receive no message.

In general, fairness is implicitly assumed on practical systems. So the both Q and R will receive a message eventually in a practical system. However, the fairness does not say how much frequency Q and R recieve messages. In spite that Q recieves several thousand messages, R may recieve only one message. In practial meaning, is it "fair"? This paper introduce more quantitative fairness for a process algebra. 2) calls such fairness is "economic-oriented". So we call this fairness economic-oriented fairness.

2 Process Algebra

We assume the reader have the knowleage of process algebra such as CCS. In this paper, we adopt processs whose semantics are decided by a transition relation $\rightarrow$[4)]. Let $\mathcal{A}$ be a set of all *atomic actions* and $\mathcal{P}$ a set of all processes. We assume that this $\mathcal{A}$ has no silent action. The transition relation $\rightarrow \subseteq \mathcal{P} \times \mathcal{A} \times \mathcal{P}$.

3 Economic-Oriented Fairness to a Process Algebra

The economic-oriented fairness in this paper ensure that having to repeatedly choose among a set of alternatives, each expectative of selection of each alternative is equal at each transition step. More precisely, the expectation of each selection equals the others. This section introduces the new process operator $\prod$, which is a composition oparator except subprocesses of $\prod$ assumed the economic-oriented fairness. We argue about the probabilistic transition function satisfying the economic-oriented fairness.

For $N(\geq 1)$ and any i where $1 \leq i \leq N$, let $P_i \in \mathcal{P}$ and $n_i \geq 0$. Let $\prod_{i=1}^{N} n_i : P_i$ be a process which satisfies the following:

$$\frac{P_j \xrightarrow{a} Q}{\prod_{i=1}^{N} n_i : P_i \xrightarrow{a} \prod_{i=1}^{N} n_i' : P_i'} \quad 1 \leq j \leq N, a \in \mathcal{L}$$

where $n_i' = n_i (i \neq j)$ or $n_j + 1 (i = j)$ and $P_i' = P_i (i \neq j)$ or $Q (i = j)$. We call the above new processes *compositional processes*. In the case of $N = 2$, $\prod_{i=1}^{2} n_i : P_i$ is described by $P_1 \,{}_{n_1}\|_{n_2} P_2$. Each n_i represents how many actions are executed from P_i's initial process to P_i.

For example, $P\,{}_2\|_3 Q$ shows that its initial process $P_0\,{}_0\|_0 Q_0$ executed five actions, where two were executed from P_0 and three were from Q_0 in some order, and became $P\,{}_2\|_3 Q$. If P performs an action and becames P', then its result process is $P'\,{}_3\|_3 Q$.

Next, we introduce the probabilistic labeled transition relation $\rightarrow_p$ for compositional processes. The probabilistic transition relation $\rightarrow_p \subseteq \mathcal{C} \times \mathcal{A} \times \mathcal{C} \times Pr$ where $\mathcal{C}$ is a set of all compositional processes and $Pr = \{pr : 0 \leq pr \leq 1\}$. We omit (P, a, Q, p) by $P \xrightarrow{a}_p Q$, which means P can perform the action a at probability p, and after a action, it becomes Q. The probability p of $\rightarrow_p$ is defined by the function $\mu : \mathcal{C} \times \mathcal{A} \times \mathcal{C} \longrightarrow [0,1]$ is a total function called the *probabilistic transition function*, satisfying the following restriction: $\forall P \in \mathcal{P}$,

$$\sum_{a \in \mathcal{A}, Q \in \mathcal{C}} \mu(P, a, Q) = 1$$

Note that the probabilistic transition relation does not consider a transition for a silent action. We can easily extend the above transition relation to the one for an action sequence. That is, $\mu(P, \varepsilon, P) = 1$ and $\forall t \in \mathcal{A}^*$, if $\mu(P, t, Q) = p$ and $\mu(Q, a, R) = q$, then $\mu(P, t \cdot a, R) = p \cdot q$. It is obvious that $\forall m \geq 0$. $\sum\{p : \exists Q$ such that $P \xrightarrow{t}_p Q, t \in \mathcal{A}^m\} = 1$,

In below, we define the economoc-oriented fairness for compositional processes. Institutively, the economical-fairness on a compositonal process $P = Q\,{}_0\|_0 R$ is considered as follows. For an action sequence $t = a_1 a_2 \cdots a_{2m}$, suppose $P \xrightarrow{t} P'$. Then this transition is fair if t contains m actions performed by the subprocesses of Q and the other m actions performed by the subprocesses of R. Even if a_1 to a_m are by Q's, and a_{m+1} to a_{2m} are by R's, this transition is fair. Its first N actions, however, appers no

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b's. It seem to be not fair until at least the N-th action. So we need the probabilistic transition and define this fairness by the expectative number of actions performed by each subprocesses.

Definition 3.1 The probabilistic transition function μ on compositional processes is economical-fair, if for any initial composition process $P = \prod_{i=1}^{N} 0 : P_i$, let $m \leq 1$ and $S_m = \{Q : P \xrightarrow{t}_p Q, t \in \mathcal{A}^m\}$. Then the sum of the expectative number of actions by any P_j in sequences in m actions satisfies;

$$\sum_{Q \in S_m} \{p \cdot n_j : P \xrightarrow{t}_p Q, Q \equiv \prod_{i=1} N n_i : P'_i\} = m/N$$

□

The probabilistic transition relation by the economical-fair probabilistic transition function is also called economical-fair. Figure 1 shows an example of an economical-fair probabilistic transition tree. In this figure, the probabilistic transition relation $R \xrightarrow{p} S$ shows $\sum\{q : \exists S.\exists a \in Act.R \xrightarrow{a}_q S\} = p$. $mP+nQ$ described under the each compositional process in this figure means that the sum of the expectative number of actions by P's subprocesses is m and that by Q's is m. For example, at each $P'\,_1\|_1 Q'$, the both expectative number of actions by P's and Q's subprocesses are $2/6 \cdot 1$. The expectative number of actions in two actions sequence from $P\,_0\|_0 Q$ is $1P + 1Q$.

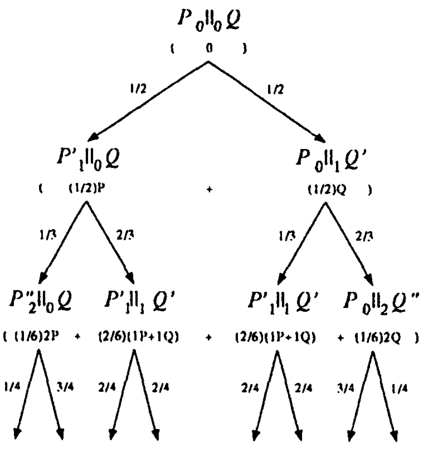

Figure 1: An example of a part of an economical-fair probabilistic transition tree.

There are many economical-fair probabilistic transition function. For example, the probabilistic transition function destributing equal probability to each subprocess of a compositonal process is economical-fair. In practical, however, some probabilistic priorties may be preferable. If $P\,_0\|_0 Q \xrightarrow{t} P'\,_{2m}\|_m Q'$ then the transition probability by Q' may be two times of probability by P'. Finally, we define such a probabilistic transition function, and show this function is economical-fair.

Definition 3.2 $\mu_2 : \mathcal{C} \times \mathcal{A} \times \mathcal{C} \longrightarrow [0,1]$ is a function satisfying the following condition:

$$\sum\{\mu_2(\prod_{i=1}^{N} n_i : P_i, a, \prod_{i=1}^{N} n'_i : P'_i)\} = (\prod_{i \neq k} n_i) / \sum \prod_{i \neq k} n_i$$

where $n'_i = n_i (i \neq k)$ or $n_i + 1 (i = k)$ and $P'_i = P_i (i \neq k)$ or $Q (i = k)$ such that $P_i \xrightarrow{a} Q$. □

Proposition 3.3
1. *μ_2 is an economical-fair probabilistic transition function.*
2. *For $\prod_{i=1}^{N} n_i : P_i$, $\sum\{\mu(P_i, a, Q_i)\} / \sum\{\mu(P_j, a, Q_j)\} = n_j / n_i$.*

Proof By the definition of μ_2. □

4 Conclusion

In this paper, we propose the notion of economical-fairness for the compositional processes. The compositional operator $\prod$, however, is just interleave, since each subprocess of $\prod$ cannot communicate to the other subprocess. So, we need to deal with such communication, i.e. a silent action to define the probabilistic transition function. In next, we would like to consider the economical-fairness for users on processes. Under the economical-fairness, subprocesses of a compositional process seem to be fair in practical. However, suppose that P has only one communication port and Q has three ports. Also suppose each port is assigned to each user respectivley. Under the economical-fairness in this paper, does $P\,_0\|_0 Q$ supply fair communication opportunity to each user? The notion of the new economic-oriented fairness based on each communication port is necessary.

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