Incorporating Trading Costs into Optimization of Asset Allocation

4N−2

Atsushi YOSHIMOTO
Investment Tech. Dev. Comm., Sumitomo Life Insurance Co.

1. Introduction

Trading costs are the major concern for portfolio managers. Due to non-linearity of the cost function, the ordinary quadratic programming solution technique cannot be applied to solve portfolio optimization problems subject to trading costs. In this paper, I address the portfolio optimization problem in which trading costs are directly incorporated. The cost is assumed to be a V-shaped function of a difference between an existing and a new portfolios. The portfolio optimization system called POSTRAC (Portfolio Optimization System with TRAding Cost) is proposed. POSTRAC is designed to predict a future return of security by means of both the simple regression model (SRM) and the moving average model (MAM) as well as to perform the optimization process. The nonlinear programming software, GAMS, [1], is incorporated into the system.

2. Portfolio Optimization Problem

The portfolio optimization problem considered here is formulated by:

Problem NLP

$$J(\underline{x}^*) = \text{maximize } [\underline{x}' \cdot \underline{f}(\cdot) - \underline{c}' \cdot \underline{1} - \lambda \cdot \underline{x}' \cdot \underline{V} \cdot \underline{x}]$$

subject to

$$c_i = k|x_i - x^o{}_i|, \ \forall i$$
$$\underline{x}' \cdot \underline{1} = 1$$
$$\underline{x} \geq \underline{0}$$

where $\underline{x}$ is a portfolio, $\underline{f}(\cdot)$ is an expected return vector function of securities, $\underline{c}$ is a trading cost vector, and $\underline{V}$ is a variance-covariance matrix of securities. λ is a coefficient to calibrate a risk on the return unit (risk tolerance) and k is a constant trading cost per change in proportion of the i-th security.

As seen at the first constraint, a trading cost vector is a function of a difference between an existing portfolio, $\underline{x}^o$, and a new portfolio, $\underline{x}$. It is assumed that a trading cost for a given security is V-shaped. The second constraint implies that a fund is to be invested fully on risky and riskless securities. The last constraint is to prohibit short sales and borrowings. No additional constraints, such as turnover constraints, minimum trading size constraints are considered.

3. System Development

The moving average model (MAM) and the simple regression model (SRM) were used to predict a future return of security. In MAM, a future return of the i-th security at period T, $r_{i,T}$, is predicted by:

$$r_{i,T} = ER_{i,T} + \epsilon_{i,T}$$

where $ER_{i,T}$ is calculated by:

$$ER_{i,T} = \frac{1}{T - t_0} \sum_{t=t_0}^{T-1} r_{i,t} \ , \quad (T > t_0)$$

$\epsilon_{i,T}$ is a disturbance term with $E[\epsilon_{i,T}] = 0$. $r_{i,t}$ is the observed data for the return of the i-th security at period t. t_0 is the starting period and T is the current period for the problem, at which a new portfolio will be constructed.

Introducing an exogenous variable to predict a future return of security, SRM was built. The underlying assumption for SRM is that a future return of the security responds to an exogenous variable with a lapse of time. Letting $R_{i,T-1}$ be an exogenous variable for the i-th security, SRM used here was built as follows:

$$r_{i,T} = \beta_0 + \beta_1 \cdot R_{i,T-1} + \epsilon_{i,T}$$

where β_0 and β_1 are coefficients to be estimated.

For Problem NLP, an expectation of $r_{i,T}$ becomes the i-th element of $\underline{f}(\cdot)$. A covariance of the i-th and j-th security returns becomes an element of the i-th row at the j-th column of $\underline{V}$.

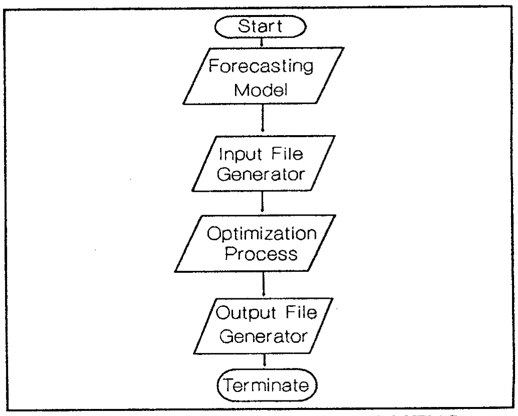

Figure 1. The structure of POSTRAC

Incorporating both MAM and SRM as well as the optimization model, GAMS, the portfolio optimization system called

POSTRAC (Portfolio Optimization System with TRAding Costs) was proposed. Figure 1 describes the structure of the system.

4. Model Experimentation

Analysis of trading costs with two forecasting models was achieved for the global asset allocation. Japan, the U.K., the U.S., Germany, Canada, and France were considered for the investment. In each country, two security indices, i.e., stock market index and Salomon Brothers bond performance index, were used. Data for both indices were from the database serviced by Datastream International [2]. Japanese one-month CD (certificate of deposit) was used as the riskless security, obtained from the database called CAPITAL [3]. Foreign securities were hedged by the one-month forward exchange rate, whose data were Barclays Bank US dollar exchange rate quotes from the database by Datastream International. No foreign exchange exposure was considered.

As for the exogenous variables for security return, the yield spread was used for the stock indices, and the spread between long-term (10 years) and short-term (3 months) yield-to-maturities was used for the bond indices. A return of security at period t was regressed on the corresponding exogenous variable at period (t-1), so that a future return was predicted by using data at the current period. Data were available monthly from January 1985 to July 1991. Twenty-four month long data were used for prediction. Thus, the analysis period was from January 1987 to June 1991. Trading costs were assumed to be 1% per change in proportion of security. It was applied for all securities. λ was set equal to 40. All data for exogenous variables were used from the database by Datastream International, too.

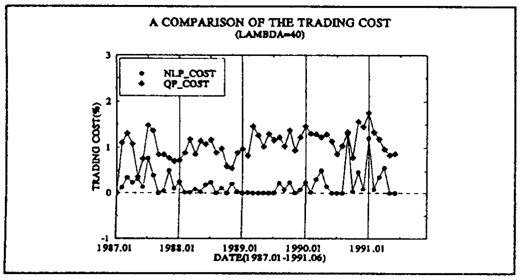

Figure 2. Trading cost by SRM

A comparison of efficient frontiers showed that the portfolio can be improved by taking trading costs into account in the optimization framework. In using the simple regression model, it was observed that when the trading costs were not considered into the optimization framework, the calculated trading cost (QP_COST) varied within the high range from 0.5% to 2% at most periods. In contrast, the solution of Problem NLP did not have such a high cost. The trading cost (NLP_COST) was constantly low, ranging from 0% to 0.5% with a few outliers (Figure 2). Note that 2% of the trading costs means a new portfolio is constructed completely different of the existing one.

When applying the moving average model, one may realize that the expectation on a future return of securities does not change in sudden as opposed to the simple regression model. Figure 3 depicts the total trading cost in the case of the moving average model. QP_COST ranged from 0.5% to 1.5% in most cases. NLP_COST was almost zero over the time horizon.

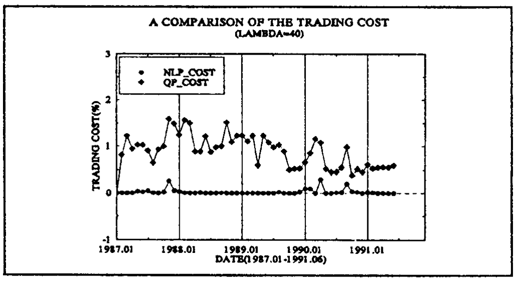

Figure 3. Trading cost by MAM

5. Conclusions

Ignoring trading costs often results in an inefficient portfolio performance. Inefficiency of such a portfolio could be reduced by using additional constraints called turnover constraints and/or minimum trading size constraints. However, searching for an optimal solution or optimal tradeoff between costs of portfolio revision and returns from securities still remains unresolved. By formulating the cost function directly into the return of the portfolio, an optimal tradeoff can be searched. When a V-shaped cost function is considered, the nonlinear programming solution technique is the one needed to solve the problem. Using the proposed system, it was shown that the cost imposed on the portfolio revision could be largely reduced in the case of both MAM and SRM.

6. References

[1]:Brooke, A., D. Kendrick, and A. Meeraus. 1988. GAMS: A user's guide. The Scientific Press. California. 289p.
[2]:"Indices, interest and exchange rates", Datastream International.
[3]:"Economic and financial information (TK Series)", Nomura Research Institute. (in Japanese)