2L-1

NP∩coNP-集合の多項式時間到達可能性について

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Let us recall that every set A in NP∩coNP has the structure in the form

$$A=\{x\mid(\exists y,|y|=p(|x|))Q(x,y)\}=\{x\mid(\forall y,|y|=p(|x|))R(x,y)\}$$

with two defining P-predicates Q,R and a defining polynomial p. We just call it a P-predicate structure of A. To investigate the complexity of A, we now turn our attention to its P-predicate structures. Let us say y a symmetric solution for A on an input x if $|y|=p(|x|)\wedge(Q(x,y)\vee\neg R(x,y))$ is satisfied. The symmetric solutions for A directly represent the complexity of P-redicate structures of A. For example, if we can access such a solution easily then A is easily recognizable. Hence it is less complex. So we may observe how intricate we access symmetric solutions for A.

Let us call f an access function for a set A∈NP∩coNP if f computes a symmetric solution for A on each input. This paper aims at investigating the computational complexity of each access function for NP∩coNP-sets. To see the complexity of computing access functions, let us use the following classes of functions on Σ^* (say $\{0,1\}^*$). For any oracle machine M, Query(M,B,x) represents the set of all query words in the computation of $M^B(x)$. Write #B to denote the cardinality of a set B.

(1) $f\in PF_T(B)$ <=> some p-time deterministic oracle transducer M computes f with B as an oracle.

(2) $f\in PF_{tt}(B)$ <=> $f\in PF_T(B)$ via a transducer M which Query(M,B,x) is listable by some function $g\in PF_T$. Call g a query list.

(3) $f\in PF_{T[O(\log n)]}(B)$ <=> $f\in PF_T(B)$ via a transducer M, where $\#Query(T,B,x)\le c\log|x|+d$ holds for all x by some absolute constants $c,d\ge 0$.

(4) $f\in PF_{btt}(B)$ <=> $f\in PF_{tt}(B)$ via a transducer M, where for all x #Query(M,B,x) is bounded by some absolute constant.

We now introduce the concept of p-time accessibilities.

DEFINITION 1. Assume $r\in\{T,tt,T[O(\log n)],btt\}$ and $B\subseteq\Sigma^*$.

(1) A language A belongs to the *p-time r-accessible class with the oracle B*, $PA_r(B)$, if and only if there exist two defining P-predicates Q,R, a defining polynomial p and a function $f\in PF_r(B)$ such that

(i) $A=\{x\mid(\exists y,|y|=p(|x|))Q(x,y)\}=\{x\mid(\forall y,|y|=p(|x|)))R(x,y)\}$, and

(ii) $\forall x[|f(x)|=p(|x|)\wedge(Q(x,f(x))\vee\neg R(x,f(x)))]$.

(2) For a complexity class C, $PA_r(C)$ is the union of $PA_r(B)$ for every oracle set $B\in C$.

It should be noted that $PA_r(\emptyset)=PA_r(P)=P$ holds for every $r\in\{T,tt,T[O(\log n)],btt\}$, and also $PA_r(2^{\Sigma^*})\subseteq NP\cap coNP$ is shown, where 2^{Σ^*} denotes the power set of Σ^*.

PROPOSITION 2. $PA_T(NP)=PA_{tt}(\Delta_2^P)=NP\cap coNP$.

Schöning introduced the concept of polynomially helping the robust algorithms. We can see

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A note on polynomial-time accessibility to symmetric solutions for NP∩coNP-sets

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the relationship of two concepts, the p-time T-accessibility and the polynomially helping. $P_{help}(B)$ denotes the class of languages helped by B.

THEOREM 3. *$PA_T(B)=P_{help}(B)$.*

Some observations show that $PF_{T[O(\log n)]}(B)$-functions are no more powerful than PF_{tt}(REC-TALLY)-functions. Therefore we get:

THEOREM 4. *$PA_{T[O(\log n)]}(2^{\Sigma^*})=PA_{btt}(2^{\Sigma^*})=P$.*

Recently Ko showed Strong-P/poly is no-helper, in other words, PA_T(Strong-P/poly)=P. Assume *C* is either P, NP∩coNP or NP. His argument can be generalized for Strong-*C*/poly.

PROPOSITION 5. *PA_T(Strong-C/poly)=$PA_T(C)$, where C is either P, NP∩coNP or NP.*

Turn to the p-time tt-accessible classes which can collapse to P. We next see two collapsing results concerning the concepts of the polynomial terseness and the p-selectivity.

Let us begin with *(f,g)*-pterse sets, an extension of pterse sets. In the following theorem we use the counting argument directly.

THEOREM 6. *$PA_{tt}(B)=P$ unless B is (poly,logpoly)-pterse.*

Selman showed P_T(PSEL)=P_T(TALLY), while P_{tt}(PSEL)≠P_T(PSEL) is recently proved by Watanabe. This difference is clear in the case PA_r(PSEL). From Selman's result,

$$PA_T(PSEL)=PA_T(TALLY)=PA_T(P/poly)$$

holds, however, PA_{tt}(PSEL) collapses to P.

bPSEL denotes the family of all bounded-p-selective sets. We here notice that bPSEL is contained between PSEL and the class wPSEL of weakly p-selective sets. Hence it is infered that,for r∈{T,tt},

$$PA_r(PSEL)\subseteq PA_r(bPSEL)\subseteq PA_r(wPSEL).$$

Let us claim the collapse of PA_{tt}(bPSEL) to P.

THEOREM 7. *$PA_{tt}(bPSEL)=PA_{tt}(PSEL)=P$.*

We next devote our attention to the elucidation of structural complexities of the remaining accessible classes among well-known complexity subclasses of NP∩coNP.

APT is the family of languages A so that some almost polynomial time machine recognizing A.

PROPOSITION 8. *$PA_T(APT)=PA_{tt}(APT)=PA_r(REC\text{-}TALLY)=PA_r(REC\text{-}SPARSE)$, where r∈{T,tt}.*

It reminds us that ZPP is a natural probabilistic class contained in BPP∩(NP∩coNP). We can show the following theorem.

THEOREM 9. *$PA_T(BPP)\subseteq ZPP\subseteq PA_T(APT)$.*

Another well-known class belonging to NP∩coNP is the class FewP∩coFewP. For the case FewP∩coFewP, we can use Hemachandra's technique of hiding informations to a NP-set. Then we get:

THEOREM 10. *FewP∩coFewP⊆$PA_T(FewP)\cap PA_{tt}(NP)$.*

REFERENCES

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