A New Parametric Method for Localized Star Spanning Tree Generation

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Abstract: A typical network topological design problem is to determine link connections and their capacity to achieve high performance, low initial and operational costs, and high reliability under the given traffic and link length data between nodes. Because of the difficulties of this problem, approximate solutions such as probabilistic searches have long been studied. However, the real-world network topologies seem to be more type oriented than the above traditional computer based solutions. In fact, most real network topologies consist of a hierarchical combination of basic types such as the bus, the star, and the ring to avoid the difficulties of the design problem. In this paper, a new parametric method for localized spanning tree (ST) generation is proposed with good experimental results. The method performs node clustering and physical link generation in one step. This is realized by a new idea of the parameterized virtual node distance incorporating both the physical node distance and the traffic gravity between nodes with a parametric weight. A set of localized spanning trees can be generated on traditional MST algorithm, by changing the weight. As the main computational costs are the MST generation and the depth-first shortest-route search, which is not very expensive, so this is a high-speed approximate solver of the network topology design problem. To assist selecting a good solution, a link capacity determination function to achieve the given mean delay time and the monthly cost estimation function are incorporated. Approximate mathematical discussions to prove the existence of a minimum cost solution in the generated candidates is given also.

Keywords: computer network, topology design, parametric method, localized star spanning tree, clustering, type-respecting, optimization

1. Introduction

The goal of the network topological design problem (NTDP) is to obtain the optimal link structure between nodes to fulfill given conditions. Even though various NTDP formulations are possible, a typical one is shown below for the popular packet store-and-forward computer communication networks. We call this problem A.

Problem A

Input

(1) Node set N = {1, 2, …, n}.
(2) Traffic flow matrix between nodes FM = $\{f_{ij} \mid i, j \in N, i \neq j\}$, f_{ij}: traffic between nodes i and j (Kbps).
(3) Distance matrix between nodes DM = $\{d_{ij} \mid i, j \in N\}$, d_{ij}: distance between nodes i and j (Km) that is the length of link l_{ij}.
(4) Link cost function g(c, d) (K¥(JPY) / month), c: link capacity (Kbps), d: link length (Km).
(5) Target mean delay time t (sec / Kbit).

Output

(6) Physical link matrix LM = $\{c_{ij} \mid i \neq j, i, j \in N\}$, c_{ij}: physical link capacity between i and j (Kbps), $c_{ij} = 0$ (if i and j are disconnected).

Objective function

(7) Get LM that minimizes the total link cost LC = Σg(c, d) and fulfill the target mean delay time t to transfer the traffic of the flow matrix FM.

The NTDP formulation of problem A and its relatives, with their approximate solutions, was given by Kleinroch et al. as early as the ARPA network construction period in the USA [1]. The proposed formula to estimate the mean delay time induced by the flow matrix FM on the physical link matrix LM has long been known as the Kleinroch formula [1]. The formula has long been used in real designs. Although its limitation for burst traffic has been debated recently, it is still widely used as a first-choice tool for network performance evaluation.

The NTDP has diverged with the advancement of communication networks. For example, in the real-time control domain, the delay matrix that specifies the goal delay t_{ij} for each node pair is introduced instead of a unique mean delay t over the whole network [2].

As the difficulty of the NTDP has long been recognized [3], approximate solution methods have been a main research target. Especially, the number of probabilistic search methods has been increasing; some well-known examples are the genetic algorithm (GA) and the simulated annealing (SA), which exploits high-power computing to perform time-consuming generate-and-test computations [2][4][5]. Constraint sensitive formulations incorporating hop limit, node degree, capacity limit and the number of redundant paths are also investigated [5][6]. In [8], possible pre-computed routes between given nodes are used to give feasible solutions by using SA. In [6], small topologies satisfying constraints are prepared first, and their hierarchical combinations are generated by dynamic programming (DP).

Other approaches have been proposed to provide the best possible designs, such as pareto optimal designs using the multi-goal optimization method to trade goals of the mean delay t and the total link cost LC [4][7]. The final solution is selected by human judgment. In [8][9], a set of solutions for human final selection is provided by changing one of the competing criteria.

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As shown above, the mainstream NTDP studies for more than 40 years since Kleinroch have primarily focused on computer-based optimizations of problem A or similar problems.

However, from the experience of one of the authors, who has worked for more than 30 years in this field, the real-world network topologies seem to be more type oriented than the above traditional computer based solutions. In fact, most real network topologies consist of a hierarchical combination of basic types such as the bus, the star, and the ring, and the topologies provided by traditional computer based approaches have rarely been adapted.

In this paper, a new parametric method for localized spanning tree generation is proposed. It is a type-constrained solver of problem A and it automates the first and the second steps of the above human design process in one step. This is realized by a new idea of the virtual node distance incorporating both the physical node distance and the traffic gravity between nodes with a parametric weight. Under the parameterized virtual distance, a set of localized spanning trees, from the complete star with smallest hops and largest node degrees to the bus-like minimum distance one with large hops and small node degrees, can be generated on traditional MST algorithm.

2. Parameterized spanning tree generation method

2.1. Network topology characterization by mean link properties

2.1.1 Properties of typical network topologies

The typical types of network topologies are the star, the bus, and the ring. Examples of them with five fixed nodes are shown in Fig. 1. The first two types belong to the class of the spanning tree (ST), mainly discussed in this paper. The properties of these types are considered from the problem A perspective.

(1) Number of links

The number of links is n-1 for ST, the star and the bus in this case, and n for the ring, if the number of nodes is n.

(2) Communication hops h

The number of links between 2 nodes (hops) is 1 or 2 for the star. It is 1 to n-1 for the bus. In the ring, it is 1 to (n-1) / 2 under the routing policy for the minimum hops, which is a smaller amount than that in the bus. If the number of hops is equated with the computational complexity of the algorithm theory, the simple star has O(1), the hierarchical star has O(log(n)), and the bus and the ring have O(n) communication complexities. So, we can group them as {simple star, hierarchical star, {bus, ring}}. As the ring is closer to the bus, we concentrate on ST type topologies.

(3) Link length d and link capacity c

The star having less link selection freedom normally has a bigger d value than that of the bus and the ring, which have more link selection freedom. The mean link load becomes high for the topology with more hops because more traffic flow f_{ij} passes through the same link. Fig. 2 shows a typical example assuming that nodes are placed at the center and on the circle of radius r. The constant traffic flow 2f is assumed between each center node and the periphery nodes (f for each of the up and down link). In this case, (mean link length, mean load) becomes (r, 2f) for the star and nearly (r(2π+1) / (n-1), nf /2) for the bus. If n is big enough, the star has (r, 2f) and the bus has (0, ∞) properties.

As the link capacity should be set larger than the link load to realize average link delay time t of problem A (if equal, t=∞), the link capacity of the bus should be set bigger than that of the star.

2.1.2. Approximate ST characterization by two link properties

Generally, the link cost g(c, d) is an increasing function of the link capacity c and the link length d, and it has a contour curve convex towards the origin in the (c, d) plane, as shown by the dotted curve in Fig. 3.

The monthly cost of a network, LC = Σg(c, d), is the sum of each link cost g(c, d). As the number of links is equal to n-1 for all n-node spanning trees, LC is equivalent to the mean link cost multiplied by n-1, i.e., LC = (n-1)*m(g(c, d)), where m(X) is the average of X. So, for ST of n-nodes, it is sufficient to know m(g(c, d)) to get LC. But this is generally very difficult. However, when g(c, d) is concave in c and d, which is generally the case, we have m(g(c, d)) ≦ g(m(c), m(d)). So, we get LC = (n-1)*m(g(c, d)) ≦ (n-1)*g(m(c), m(d)). This means that, instead of knowing the intractable cost m(g(c, d)), a more tractable upper bound value of g(m(c), m(d)) can be used as an approximation. So it can be said that from the viewpoint of cost LC, ST of identical number of nodes can be approximately characterized by m(c) and m(d). Henceforth, we simply note m(c) as c, m(d) as d, and Fig. 3 is interpreted to represent the mean characteristics of a link of ST.

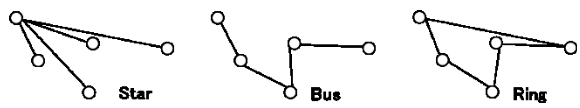

Figure 1 Typical network topology types

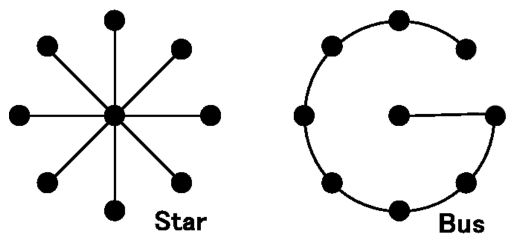

Figure 2 Star and bus on a circle

2.1.3. Viewing star, bus and their combination by link properties

From the discussions of 2.1.1 above, if delay $t = \infty$, the position of the star and the bus can be shown by the white circle and the white square, respectively, as shown in Fig. 3.

To realize a short delay t, we should give higher link capacity c. In this case their positions move upward in Fig. 3, because d is fixed for each topology. But, as shown in 2.1.1, the upward movement of the bus is bigger, i.e., requires higher link capacity, than that of the star. So, the delay contour become as shown in Fig. 3.

If the contour of the cost and the delay are as shown, the combined topology can have lower cost than the star or the bus. A hierarchical combination of the star and the bus, which is frequently found in networks in the real world, can be interpreted as its instance.

Summing up, this two dimensional link feature space contains infinitely many candidate ST solutions of problem A with varying link structure, mean delay time and the total cost.

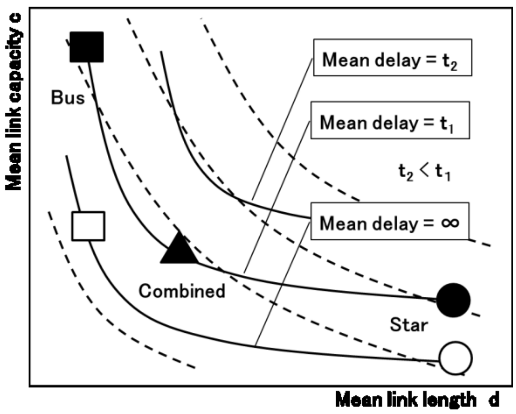

Figure 3 The link characteristics of the star and bus and their combination

2.2 Parametric method for localized spanning tree generation

2.2.1. Basic ideas

Based on the discussions in 2.1.3, we hope to design a method to systematically extract candidate ST solutions of problem A out of the link feature space of Fig. 3. Our basic idea is to construct a method to systematically traverse the space of Fig. 3 along the bold contours. The method is composed of following steps.

(1) Parameterized ST link generation
(2) Link capacity determination to realize the target mean delay
(3) LC estimation

For a given delay t, steps (1), (2) and (3) give cost-evaluated STs on a bold contour in Fig. 3 by using a weight parameter w, explained shortly.

As long as the delay time contour curve is monotone, as shown in Fig. 3, we can cover all the space of Fig. 3 by changing the values of w and t. For this to be true, it is hoped that both the mean link length and the mean link capacity are the reversely monotone functions of parameter w under a given t. Furthermore, in this case, we are more likely to find some cost-effective combined topology in the midway $0.0 < w < 1.0$ than in the terminal star and bus. In the following, we give such method side by side with mathematical backgrounds.

2.2.2. Parametric generation of spanning trees

(1) Parametric ST generation

An efficient method to get the spanning tree among n nodes is the minimum spanning tree (MST) algorithm. If the link weight is the link length, we get the minimum distance ST (MST). Generally, this is not the linear bus structure but a tree with the minimum sum of link lengths. Respecting this minimum link length property, we adopt the minimum distance MST instead of the bus as one of the extreme types in Fig. 3. Next, we hope to get the star, which is the other extreme type, by using the same MST algorithm under some appropriate link weighting scheme. Furthermore, we hope to get the combination of these two extreme types also.

For these purposes, we define the weight u_{ij} of link l_{ij} between node i and j as follows.

$$u_{ij} = (1-w)*d_{ij} - w*a_{ij} \quad (0 \leqq w \leqq 1.0), \qquad (1)$$

where $0 \leqq w \leqq 1.0$ is the weighting parameter, $0 \leqq d_{ij} \leqq 1.0$ is the normalized link length between nodes i, j and $0 \leqq a_{ij} \leqq 1.0$ is the normalized gravity between nodes i, j defined below. The term $(1-w)*d_{ij}$ gives high priotiry to shorter links, whereas the term $-w*a_{ij}$ gives high priority to links connecting heavy traffic nodes. Defining a_{ij} by (2), the largest traffic node tends to become the cluster center.

$$a_{ij} = \max(e(f_i), e(f_j)) / \max(a_{ij}), \qquad (2)$$

where $f_i = (f_{i*}+f_{*i}) / \max(f_i)$, f_{i*} is the total flow out of node i, and f_{*i} is the total flow incoming to node i. These are the i-th row sum, and the i-th column sum of the flow matrix FM of Problem A, respectively. Additionally, $0 \leqq f_i \leqq 1.0$ is the normalized sum of them and $e(f_i)$ is the contrast-enhancing sigmoid function. The denominator $\max(a_{ij})$ in (2) normalizes a_{ij} less than 1.0. As $a_{ij} = a_{ji}$ by (2), both links l_{ij} and l_{ji} become to carry the larger of the flows f_i and f_j of its terminal nodes. We represent the MST under the weight w of (1) as ST(w).

Under the definition of u_{ij} in equation (1) and a_{ij} in equation (2), following facts can be proved.

■ST(0.0) becomes the minimum distance spanning tree and ST(1.0) becomes the star.

■ Each mean link length d and mean link capacity c have a tendency to be a monotonically increasing and decreasing function of parameter w. That is, for all w, $\partial d / \partial w > 0$ and $\partial c / \partial w < 0$, respectively.

■ The proposed parametric process finds a unique minimum point w* of sup(LC) = (n-1) * g(c, d), which is the upper bounding function of the monthly cost LC.

2.2.3. Link capacity determination fulfilling the target mean delay time

The link capacities of the generated ST(w) fulfilling the target mean delay time t are determined as follows. We assume prevalent full-duplex communication links that physically realize the ST(w).

(1) Physical link load estimation

The flow matrix FM of Problem A is assigned on ST(w) assuming the minimum hop routing.

(2) Physical link capacity assumption

We temporarily set the capacity c_{ij} of link $l_{ij} \in$ ST(w) by using formula (4), where x > 1.0 is the temporal parameter.

$$c_{ij}(x) = c_{ji}(x) = \max(x*h_{ij}, x*h_{ji}) \quad (4)$$

(3) Mean delay time estimation

By using the Kleinroch's formula (5), we estimate the mean delay time t(x) under the temporal parameter x.

$$t(x) = \Sigma ((1.0/\gamma)*h_{ij} / (c_{ij}(x) - h_{ij})) \quad (5)$$

(4) **Fulfilling the target mean delay time**

By using (5), we search the x that fulfills the target mean delay time t by using the binary search method.

By the above processes, we obtain the link capacity matrix LM of the given ST(w) fulfilling the given target mean delay time t. This is a candidate solution of problem A under the parameters w and t.

2.2.4. Estimation of the monthly cost

We give c $\in$ LM(w, t) and d $\in$ DM of each link to g(c, d) and sums up them to get LC(w, t).

2.2.5. Best topology selection

By using the method under candidate mean delay list TL and w list WL, we can get a set of candidate solutions. We choose the best one out of them.

3. Experiment

3.1. Purpose of the experiment

The feasibilities of the following core functions of the proposed method are examined.

(F1) Parametric ST(w) generation from the star to the minimum distance MST under parameter w change.

(F2) Monotonic increase / decrease properties of the mean link length / capacity under the w increase and the existence of the minimum solution at some w*.

3.2. Experimental design

Input data to problem A are as follows.

(1) Nodes N

N = {1, 2, …, 64}. These are randomly placed in the rectangle of dimensions (500 Km, 300 Km).

(2) Traffic flow matrix FM

After random generation of the initial flow matrix FM = $\{f_{ij} \mid i, j \in N, i \neq j\}$ ($f_{ij} \leqq$ 1000 Kbps), flows of the randomly selected 80% nodes are reduced to 20% of each of their initial values. The purpose is to reflect the general observation that dominant flows tend to concentrate on a small portion of nodes.

(3) Physical distance matrix DM

DM = $\{d_{ij} \mid i, j \in N, i \neq j\}$ is generated, where d_{ij} is the physical distance between nodes.

(4) Link cost function g(c, d)

The following fitting function for the real monthly cost data of a communication service company was used.

$g(c, d) = g_1(c, 100\text{ Km}) * g_2(d, 100\text{ Km})$,

$g_1(c, 100\text{ Km}) = 56.37c^{0.358}$ (K¥),

$g_2(d, 100\text{ Km}) = (d/100)^{0.231}$.

(5) Target mean delay time

t = 0.001 (sec/Kbit).

(6) Parameter w set WL = {0.0, 0.1, …, 1.0}.

Generated ST(w)s are drawn to visually verify goal F1. For each weighting parameter w, the mean link length d, the mean link capacity c and the monthly cost LC are drawn to verify goal F2.

3.3. Experimental results

3.3.1. Generated spanning trees

Fig. 6 shows ST(w)s for w∈{0.0, 0.2, …, 1.0}. As expected, the parametric generation of spanning trees from the pure star to the MST is realized. In the middle part, hierarchical structures, each of which is the combination of the upper trunk links and the lower local stars, are observed.

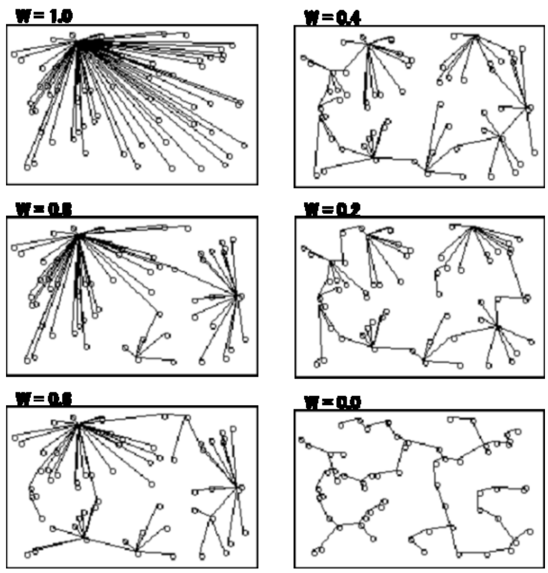

Figure 6 Parametric generation of ST(w)

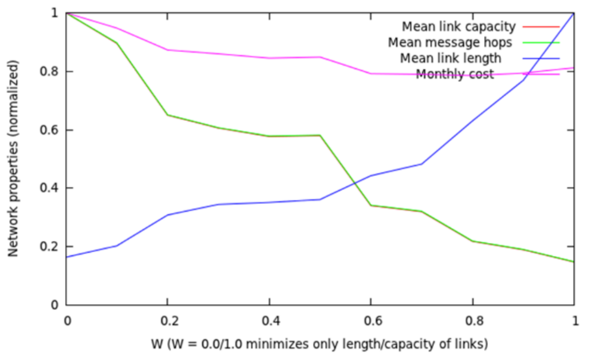

Figure 7 Link property dependency on w. Mean delay time is 0.001 sec/Kbits.

3.3.2. Mean link properties versus w

Fig. 7 shows the w-dependency of the mean link length, mean link capacity, and the monthly cost. As expected, the link length and the link capacity are the monotonically increasing and decreasing function of parameter w, respectively. The monthly cost curve is convex and has a minimum near w = 0.6, although the curve is not sharp.

4. Concluding Remarks

In this paper, a new parametric method for localized spanning tree (ST) generation is proposed.

The method performs node clustering and physical link generation in one step. This is realized by a new idea of the virtual node distance incorporating both the physical node distance and the traffic gravity between nodes with a parametric weight.

Under the parameterized virtual distance, a set of continuous localized spanning trees including the complete star and the minimum distance ST at both ends can be generated on traditional MST algorithm.

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