Inter-Contact Time Study for MANETs with Random Walk Mobility

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Abstract: In mobile ad hoc networks (MANETs), the inter-contact time, the time interval between two consecutive contacts of two mobile nodes, has been shown to have significant impact on network performances. Analyzing the inter-contact time in MANETs serves as the foundation for network performance evaluation and efficient network design. Despite lots of research efforts, accurate analytical results of the inter-contact time still remain elusive. As a step toward this direction, this paper provides theoretical analysis on the inter-contact time in a MANET with random walk mobility. Specifically, we first develop a powerful theoretical framework based on quasi-birth-and-death (QBD) process to characterize the mobility process of mobile nodes in the concerned MANET. With the help of this theoretical framework, we then derive accurate analytical results of the cumulative distribution function as well as mean and variance of the inter-contact time. Finally, we carry out numerical studies to investigate the impact of network parameters on inter-contact time performance.

1. Introduction

Mobile ad hoc networks (MANETs) represent a class of self-configuring and peer-to-peer networks with mobile nodes [1]. Due to their appealing distributed feature, MANETs find many critical applications, like disaster relief, environment monitoring, battle field communication, etc. Since pre-existing infrastructures are not available in MANETs, nodes in such networks mainly rely on node mobility and node contacts to implement communications among them (two nodes are called to contact each other when their distance is less than their transmission range). Thus, the inter-contact time between two mobile nodes, i.e. the time interval between two consecutive contacts of these two nodes, has significant impact on performances of network communications (e.g., throughput and packet delay) and serves as a fundamental quantity to study for network performance evaluation and efficient network design [2].

Available works on inter-contact time have reported either empirical results or analytical estimation results. Regarding empirical studies on inter-contact time, exponential inter-contact time distribution has been reported for MANETs after simulating random waypoint mobility and random direction mobility [3]; power-law inter-contact time distribution has been reported by analyzing real mobility traces [4, 5]. It is notable that all these works uses aggregate inter-contact time distribution (by aggregating the pairwise inter-contact time distribution of all node pairs in the network) to represent the pairwise inter-contact time distribution, which is hard to justify and has been revisited later in [6, 7]. However, empirical results only provide case study, they can not be applied to general network scenarios. Recently, general analytical results of estimations to inter-contact time have been explored in [8–10]. Upper bounds for the complementary cumulative distribution function of inter-contact time was established in [8]. Given two nodes rarely meet each other in the network, the inter-contact time distribution was approximated as an exponential distribution in [9]. An algorithm was also proposed in [10] to estimate the distribution of the inter-contact time.

However, accurate analytical results on inter-contact time in MANETs still remain elusive, which significantly hinders performance evaluation and protocol design in MANETs and thus greatly stunts the development of such networks. Indeed, inter-contact time modeling in MANETs is challenging, which is partially due to the complicated network dynamics from node mobility, but also due to the lack of efficient theoretical framework to capture these dynamics. As a step toward this direction, this paper provides accurate theoretical analysis on the inter-contact time in a MANET with random walk mobility. The contributions are summarized as follows.

- We first develop a powerful theoretical framework, based on quasi-birth-and-death (QBD) process theory, to capture the complicated network dynamics from node mobility in the considered MANET.
- With the help of the theoretical framework, we then derive the cumulative distribution function (CDF) as well as mean and variance of the inter-contact time.
- Finally, based on the derived theoretical results, we examine the impact of node mobility parameters and network size on the inter-contact time performance.

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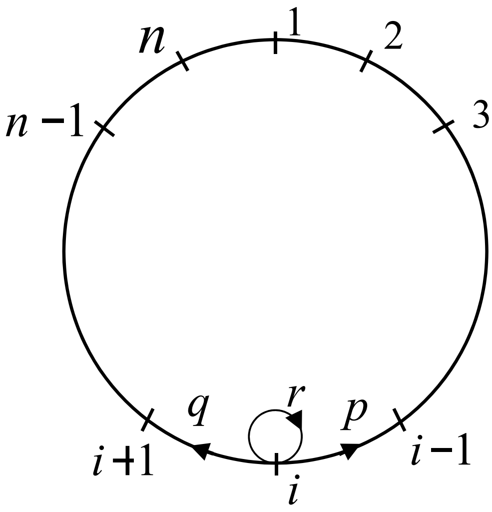


Fig. 1 A MANET of ring topology with n sites.

The reminder of this paper is organized as follows. In Section 2, we introduce preliminaries regarding network model, node mobility model and the definition of inter-contact time. We then develop in Section 3 a QBD-based theoretical framework to model the inter-contact time, based on which we derive the cumulative distribution function as well as mean and variance of inter-contact time. Numerical evaluation and corresponding discussions are given in Section 4. Finally, we conclude the paper in Section 5.

2. Preliminaries

Network model: As ring network topology is widely observed in daily life, like loop bus route [11] and ring road encircling a city [12], we consider a MANET of ring topology with n sites, as shown in Fig. 1. We assume network time is synchronized and slotted [13]. In the concerned MANET, there exist multiple mobile nodes moving from site to site and from time slot to time slot.

Random walk mobility: In the network, each node moves independently of other nodes following the random walk mobility model [14]. According to the mobility model, a node will stay in its current site with probability r as shown in Fig. 1, move counterclockwise to the next site with probability p and move clockwise to the next site with probability q at the beginning of each time slot, where $0 < r, p, q < 1$ and $r + p + q = 1$.

Node contact: Denote by $0 \leq d(t) \leq n-1$ the distance in terms of the number of sites a node, say X_1, has to traverse in the clockwise direction to reach another node X_2 at time slot t ($t = 0, 1, 2, \cdots$). If $d(t) = 0$, we say nodes X_1 and X_2 contact each other at slot t.

Inter-contact time: The inter-contact time T_I between two consecutive contacts between X_1 and X_2 is defined as follow.

$$T_I = \inf_{t>t_0} \{t - t_0 - 1 : d(t) = 0\} \tag{1}$$

given that $d(t_0) = 0$ and $d(t_0 + 1) > 0$.

3. Inter-Contact Time Analysis

In this section, we first develop a QBD-based theoretical framework to capture the distance evolving process of the two mobile nodes X_1 and X_2. With the help of this framework, we then derive analytical results on the CDF as well as mean and variance of the inter-contact time.

3.1 QBD-Based Theoretical Framework

As indicated in the definition (1), the inter-contact time T_I denotes the time interval between two consecutive contacts between X_1 and X_2. Thus, to analyze the inter-contact time, we need to first determine the distribution of the initial distance between the two nodes just after they lose contact and then model how the distance between them evolves with time.

Notice that under the random walk mobility model, there are four possible states of the initial distance between X_1 and X_2, namely, 1, 2, $n-2$ and $n-1$.

Lemma 1 Let τ_1, τ_2, τ_{n-2} and τ_{n-1} denote the possibilities that the initial distance between X_1 and X_2 just after they lose contact is 1, 2, $n-2$ and $n-1$, respectively. Then, we have

$$\tau_1 = \frac{rq + pr}{1 - r^2 - p^2 - q^2}, \tag{2}$$

$$\tau_2 = \frac{pq}{1 - r^2 - p^2 - q^2}, \tag{3}$$

$$\tau_{n-2} = \frac{qp}{1 - r^2 - p^2 - q^2}, \tag{4}$$

$$\tau_{n-1} = \frac{rp + qr}{1 - r^2 - p^2 - q^2}. \tag{5}$$

Thus, the probability vector $\boldsymbol{\tau}$ regarding the initial distance is given by

$$\boldsymbol{\tau} = [\tau_1 \quad \tau_2 \quad 0 \quad \cdots \quad 0 \quad \tau_{n-2} \quad \tau_{n-1}], \tag{6}$$

Proof: See Appendix A.1 for the proof.

Next, we model how the distance $d(t)$ between X_1 and X_2 evolves with time after they lose contact. Suppose the distance between X_1 and X_2 is at some state $d(t) > 0$ in the current time slot t, all possible distance between them in the next time slot (i.e., the one-step state transitions of their distance) are summarized in Fig. 2, where p_2^-, p_1^-, p_0, p_1^+ and p_2^+ denote probabilities defined as follows.

Lemma 2 For a time slot, let p_2^-, p_1^-, p_0, p_1^+ and p_2^+ be the probabilities that the distance between X_1 and X_2 changes by -2, -1, 0, $+1$ and $+2$, respectively. Then, we have

$$p_2^- = qp, \tag{7}$$

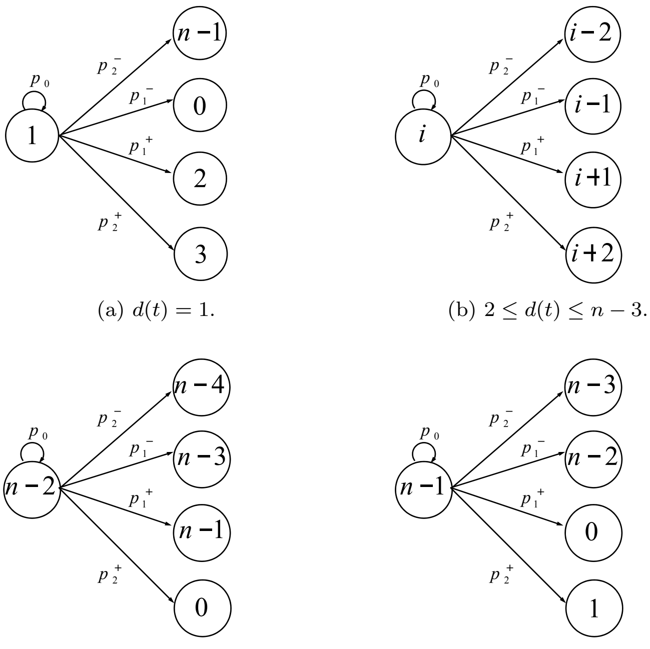
$$p_1^- = rp + qr, \tag{8}$$

$$p_0 = r^2 + p^2 + q^2, \tag{9}$$

$$p_1^+ = rq + pr, \tag{10}$$

$$p_2^+ = pq. \tag{11}$$

Proof: According to the definition of random walk mobility model in the concerned MANET, we can easily derive



(a) $d(t) = 1$.　(b) $2 \leq d(t) \leq n-3$.

(c) $d(t) = n-2$.　(d) $d(t) = n-1$.

Fig. 2 One-step state transitions of distance $d(t)$.

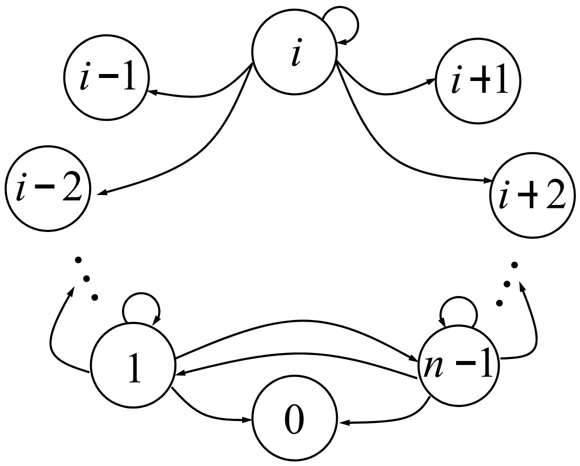


Fig. 3 State transition diagram for the distance between two mobile nodes X_1 and X_2. For simplicity, only transitions from a typical state i is illustrated, while other transitions are the same as that shown in Fig. 2.

these probabilities.

From Fig. 2 we can see that as time evolves, the state transitions of the distance between X_1 and X_2 form the transition diagram illustrated in Fig. 3, which indicates that the evolving process of their distance follows a one-dimensional QBD process [15]

$$\{d(t), t = 0, 1, 2, \cdots\} \tag{12}$$

on state space

$$\{0, 1, 2, \cdots, n-1\}, \tag{13}$$

where state 0 is an absorbing state [16].

3.2 CDF, Mean and Variance of Inter-Contact Time

If we arrange all the n states in Fig. 3 in a row and column, we can get the transition matrix $\mathbf{P}$ of the QBD process as follows.

$$\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ n-2 \\ n-1 \end{array} \begin{array}{c} \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & \cdots & n-2 & n-1 \end{array} \\ \begin{pmatrix} 1 & & & & & & & \\ p_1^- & p_0 & p_1^+ & p_2^+ & & & & p_2^- \\ p_2^- & p_1^- & p_0 & p_1^+ & p_2^+ & & & \\ & p_2^- & p_1^- & p_0 & p_1^+ & p_2^+ & & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots & \\ p_2^+ & & & & & & p_0 & p_1^+ \\ p_1^+ & p_2^+ & & & & & p_1^- & p_0 \end{pmatrix} \end{array} \tag{14}$$

By rearranging matrix $\mathbf{P}$, we have

$$\mathbf{P} = \begin{pmatrix} 1 & \mathbf{0} \\ \boldsymbol{c} & \mathbf{Q} \end{pmatrix} \tag{15}$$

where

$$\boldsymbol{c} = \begin{bmatrix} p_1^- & p_2^- & 0 & \cdots & 0 & p_2^+ & p_1^+ \end{bmatrix}^T, \tag{16}$$

$$\mathbf{Q} = \begin{pmatrix} p_0 & p_1^+ & p_2^+ & & & & p_2^- \\ p_1^- & p_0 & p_1^+ & p_2^+ & & & \\ p_2^- & p_1^- & p_0 & p_1^+ & p_2^+ & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & & & & p_0 & p_1^+ \\ p_2^+ & & & & & p_1^- & p_0 \end{pmatrix}. \tag{17}$$

Now, we are ready to derive the CDF, mean and variance of the inter-contact time.

Theorem 1 In a MANET of ring topology with n sites, where nodes move around following random walk mobility model, the CDF $Pr\{T_I \leq t\}$, mean $\overline{T_I}$ and variance $\sigma_{T_I}^2$ of inter-contact time T_I between a pair of two nodes are given by

$$Pr\{T_I \leq t\} = 1 - \boldsymbol{\tau}\mathbf{Q}^t\mathbf{1} \tag{18}$$

$$\overline{T_I} = \boldsymbol{\tau}(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1} \tag{19}$$

$$\sigma_{T_I}^2 = 2\boldsymbol{\tau}(\mathbf{I} - \mathbf{Q})^{-2}\mathbf{Q}\mathbf{1} + \overline{T_I} - (\overline{T_I})^2 \tag{20}$$

where $\mathbf{1}$ is a column vector of proper size with all elements being 1 and $\mathbf{I}$ is the identity matrix of proper size.

Proof: See Appendix A.2 for the proof.

4. Inter-Contact Time Evaluation

With the analytical results on the inter-contact time, we now explore how node mobility parameters r, p and the number of network sites n will affect inter-contact time T_I.

We first examine the impact of r and p on T_I, which is summarized in Fig. 4. From Fig. 4, we can see that for fixed p, as r increases, the mean value of inter-contact time always first decreases and then increases. This phenomenon can be explained as follows. For fixed p, an increase in r has two-fold effects on inter-contact time: on one hand, it decreases the clockwise moving probability q ($q = 1 - p - r$)

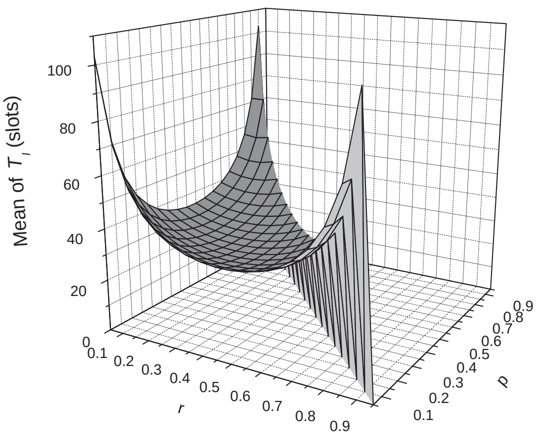


Fig. 4 The mean value of inter-contact time T_I vs. mobility parameters r and p.

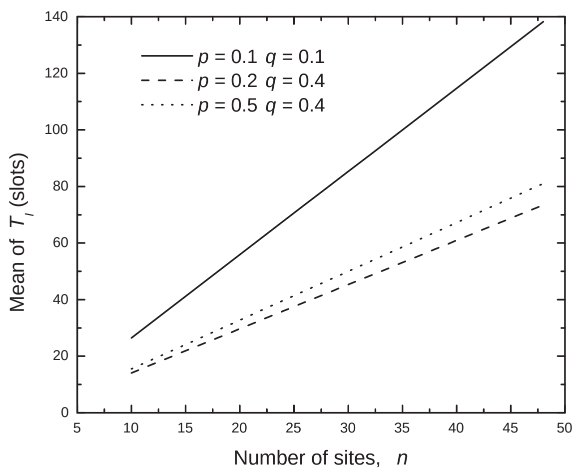


Fig. 5 The mean value of inter-contact time T_I vs. the number of network sites n.

and thus decreases node clockwise moving speed, making it easier for X_1 (resp. X_2) to catch up with X_2 (resp. X_1) in the clockwise direction and thus resulting in a decrease in the inter-contact time T_I; on the other hand, it increases the probability r of staying in the same site, making X_1 and X_2 move more slowly to catch up with each other and thus resulting in an increase in the inter-contact time. When the first effect dominates the second one, T_I decreases as r increases, while when the second effect dominates the first one, T_I increases as r increases. Another observation of the numerical results in Fig. 4 indicates that the inter-contact time can be minimized by a proper setting of p and r.

We then examine the impact of n on T_I, which is summarized in Fig. 5. As shown in Fig. 5 that for all the three network scenarios there, inter-contact time T_I always increases as n increases. This can be explained as follows. An increase in n leads to an increase in the network size, making X_1 and X_2 to traverse longer distance to reach each other, thus resulting in an increase in the inter-contact time T_I.

Table A·1

X_1	X_2	$d(t+1)$
M1	M1	0
M1	M2	n-1
M1	M3	1
M2	M1	1
M2	M2	0
M2	M3	2
M3	M1	n-1
M3	M2	n-2
M3	M3	0

5. Conclusion

In this paper, we analytically modeled the inter-contact time perfomrance for MANETs with random walk mobility. To capture the complicated network dynamics of node mobility, a powerful QBD-based theoretical framework was developed. With the help of this theoretical framework, the cumulative distribution function as well as mean and variance of inter-contact time was derived. Numerical study showed that the inter-contact time can be minimized by a proper setting of mobility parameters.

It is notable that we only analyzed the inter-contact time for MANETs with random walk mobility. Therefore, it will be interesting to extend our inter-contact time analysis to MANETs with more realistic mobility models, like Levy walk mobility [17]. Since the theoretical framework and analytical results on inter-contact time in this paper were developed for MANETs with nodes following the same mobility model, another future work is to explore inter-contact time for MANETs with nodes following heterogeneous mobility models.

Appendix

A.1 Proof of Lemma 1

Suppose X_1 and X_2 contact each other in the current time slot t, i.e., $d(t) = 0$, then all possible $d(t+1)$ in the next time slot under random walk mobility are listed in Table A·1, where we use M1, M2 and M3 to denote the movement of staying in its current site, moving counterclockwise and moving clockwise, respectively.

From Table A·1, we can calculate probability τ_1 as follows.

$$\tau_1 = \frac{Pr\{d(t+1)=1\}}{Pr\{d(t+1)\geq 1\}}, \tag{A.1.1}$$

where

$$Pr\{d(t+1)=1\} \tag{A.1.2}$$

$$= Pr\{X_1 \text{ takes move M1, } X_2 \text{ takes move M3}\} + Pr\{X_1 \text{ takes move M2, } X_2 \text{ takes move M1}\} \tag{A.1.3}$$

$$= rq + pr, \tag{A.1.4}$$

and

$$Pr\{d(t+1) \geq 1\} \quad \text{(A.1.5)}$$
$$=1 - Pr\{d(t+1) = 0\} \quad \text{(A.1.6)}$$
$$=1 - Pr\{X_1 \text{ takes move M1}, X_2 \text{ takes move M1}\}$$
$$- Pr\{X_1 \text{ takes move M2}, X_2 \text{ takes move M2}\}$$
$$- Pr\{X_1 \text{ takes move M3}, X_2 \text{ takes move M3}\} \quad \text{(A.1.7)}$$
$$=1 - r^2 - p^2 - q^2. \quad \text{(A.1.8)}$$

Thus, we have

$$\tau_1 = \frac{rq + pr}{1 - r^2 - p^2 - q^2}. \quad \text{(A.1.9)}$$

The calculations of τ_2, τ_{n-2} and τ_{n-1} follow the similar process and thus are omitted here.

A.2 Proof of Theorem 1

We see from Fig. 3 the evolving process of the distance between X_1 and X_2 follows a one-dimensional QBD process with initial probability vector $\boldsymbol{\tau}$ in (6) and transition matrix $\mathbf{P}$ in (15). This indicates that the distribution of inter-contact time T_I follows a Phase-type distribution [15] and its probability mass function $Pr\{T_I = t\}$ is given by

$$Pr\{T_I = t\} = \boldsymbol{\tau}\mathbf{Q}^{t-1}\boldsymbol{c} \quad \text{for} \quad t \geq 1. \quad \text{(A.2.1)}$$

Thus, we have

$$Pr\{T_I \leq t\} = \sum_{i=1}^{t} Pr\{T_I = i\} \quad \text{(A.2.2)}$$
$$= \boldsymbol{\tau}(\mathbf{I} + \mathbf{Q} + \cdots + \mathbf{Q}^{t-1})\boldsymbol{c}. \quad \text{(A.2.3)}$$

From the matrix $\mathbf{P}$ in (15), we know that

$$\boldsymbol{c} + \mathbf{Q}\mathbf{1} = \mathbf{1}. \quad \text{(A.2.4)}$$

After substituting (A.2.4) into (A.2.3), we have

$$Pr\{T_I \leq t\} = \boldsymbol{\tau}(\mathbf{I} + \mathbf{Q} + \cdots + \mathbf{Q}^{t-1})(\mathbf{1} - \mathbf{Q}\mathbf{1}) \quad \text{(A.2.5)}$$
$$= \boldsymbol{\tau}\Big((\mathbf{I} + \mathbf{Q} + \cdots + \mathbf{Q}^{t-1})\mathbf{1}$$
$$- (\mathbf{Q} + \mathbf{Q}^2 + \cdots + \mathbf{Q}^t)\mathbf{1}\Big) \quad \text{(A.2.6)}$$
$$= \boldsymbol{\tau}(\mathbf{1} - \mathbf{Q}^t\mathbf{1}) \quad \text{(A.2.7)}$$
$$= 1 - \boldsymbol{\tau}\mathbf{Q}^t\mathbf{1}. \quad \text{(A.2.8)}$$

Based on the probability mass function (A.2.1), we obtain the corresponding moment generating function

$$G(z) = \mathbb{E}\{z^{T_I}\} \quad \text{(A.2.9)}$$
$$= \sum_{t=1}^{\infty} z^t Pr\{T_I = t\} \quad \text{(A.2.10)}$$
$$= \sum_{t=1}^{\infty} z^t \boldsymbol{\tau}\mathbf{Q}^{t-1}\boldsymbol{c} \quad \text{(A.2.11)}$$
$$= z\boldsymbol{\tau}\left(\sum_{t=1}^{\infty} (z\mathbf{Q})^{t-1}\right)\boldsymbol{c} \quad \text{(A.2.12)}$$
$$= z\boldsymbol{\tau}(\mathbf{I} - z\mathbf{Q})^{-1}\boldsymbol{c} \quad \text{for} \quad |z| \leq 1 \quad \text{(A.2.13)}$$

Then, the mean value $\overline{T_I}$ of inter-contact time T_I can be calculated as

$$\overline{T_I} = \mathbb{E}\{T_I\} \quad \text{(A.2.14)}$$
$$= G^{'}(z)|_{z=1} \quad \text{(A.2.15)}$$
$$= \boldsymbol{\tau}(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1} \quad \text{(A.2.16)}$$

and the variance $\sigma^2_{T_I}$ can be calculated as

$$\sigma^2_{T_I} = \mathbb{E}\{T_I^2\} - (\overline{T_I})^2 \quad \text{(A.2.17)}$$
$$= \mathbb{E}\{T_I(T_I - 1)\} + \overline{T_I} - (\overline{T_I})^2 \quad \text{(A.2.18)}$$
$$= G^{''}(z)|_{z=1} + \overline{T_I} - (\overline{T_I})^2 \quad \text{(A.2.19)}$$
$$= 2\boldsymbol{\tau}(\mathbf{I} - \mathbf{Q})^{-2}\mathbf{Q}\mathbf{1} + \overline{T_I} - (\overline{T_I})^2 \quad \text{(A.2.20)}$$

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