Approximation of the Independent Feedback Vertex Set Problem

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Abstract: Given a graph G with n vertices, the independent feedback vertex set problem is to find a vertex subset F of G with the minimum number of vertices such that F is both an independent set and a feedback vertex set of G, if it exists. This problem is known to be NP-hard for bipartite planar graphs. In this paper, we study the approximability of the problem. We first show that, for any fixed $\varepsilon > 0$, unless P = NP, there exists no polynomial-time $n^{1-\varepsilon}$-approximation algorithm even for bipartite planar graphs. This gives a contrast to the existence of a polynomial-time 2-approximation algorithm for the original feedback vertex set problem on general graphs. We then give an $\alpha(\Delta - 1)/2$-approximation algorithm for bipartite graphs G of maximum degree Δ, which runs in $O(t(G) + \Delta n)$ time, under the assumption that there is an α-approximation algorithm for the original feedback vertex set problem on bipartite graphs which runs in $O(t(G))$ time.

Keywords: graph algorithm, approximation, independent feedback vertex set, bipartite graphs

1. Introduction

A *feedback vertex set* F of an undirected graph $G = (V, E)$ is a vertex subset of G such that the subgraph of G induced by $V \setminus F$ is a forest. (See Fig. 1(b) as an example.) For a given graph G, the *feedback vertex set problem* is to find a feedback vertex set of G with the minimum number of vertices. The feedback vertex set problem is one of the most classical NP-hard problems, and many algorithms have been developed from various viewpoints over the years.

Misra et al. [13] introduced an independence variant of the feedback vertex set problem. An *independent set* I of a graph G is a vertex subset of G such that the subgraph of G induced by I contains no edge. A vertex subset F' of G is said to be an *independent feedback vertex set* of G if it is both an independent set and a feedback vertex set of G. (See Fig. 1(c).) Note that an independent feedback vertex set of a graph does not always exist; for example, consider a complete graph with four or more vertices. For a given graph G, the *independent feedback vertex set problem* is to find an independent feedback vertex set of G with the minimum number of vertices, if it exists. For convenience, we sometimes call the feedback vertex set problem the *original problem*, and the independent feedback vertex set problem the *independence variant*.

1.1 Related results and known results

The original problem is APX-complete for general graphs [2]. This means that the problem is unlikely to have a polynomial-time approximation scheme (PTAS). Moreover, it remains NP-hard even for bipartite planar graphs of maximum degree four [14]. The original problem has been intensively studied from various viewpoints, such as of approximation [1], [2], [4], fixed-parameter tractability (FPT) [10], and tractability on special graph classes [8], [9], [15].

As Misra et al. pointed out in [13], by inserting a new vertex in every edge of a graph, the original problem can be reduced to the independence variant without changing the size of optimal solutions. This implies that the independence variant is APX-hard for bipartite graphs, and remains NP-hard even for bipartite planar graphs of maximum degree four. In the same paper, Misra et al. also developed a fixed-parameter algorithm whose running time is $O(5^k n^{O(1)})$, where n is the number of vertices in a graph and k is the solution size as the parameter. Recently, Li and Pilipczuk improved this running time to $O(3.619^k n^{O(1)})$ [11]. The independence variant has been also studied from the viewpoint of graph classes; for example, it is solvable in polynomial time for bounded treewidth graphs [16], chordal graphs [16], P_5-free graphs [7], and graphs of diameter two [6]. Interestingly, for the latter two graph classes, their polynomial-time solvabilities of the original problem remain open.

The independence variant is strongly related to the *near-bipartiteness problem* [5], [7], [17]. In the problem, for a given graph G, our task is to decide whether G has *at least one* independent feedback vertex set. Therefore, the intractability of the independence variant is inherited from the near-bipartiteness problem. The near-bipartiteness problem is known to be NP-complete even for graphs of maximum degree four [17], graphs of diameter at most three [5], and for line graphs of bipartite planar subcubic graphs [7]. Note that, since any bipartite graph has an independent feedback vertex set, the near-bipartiteness problem is triv-

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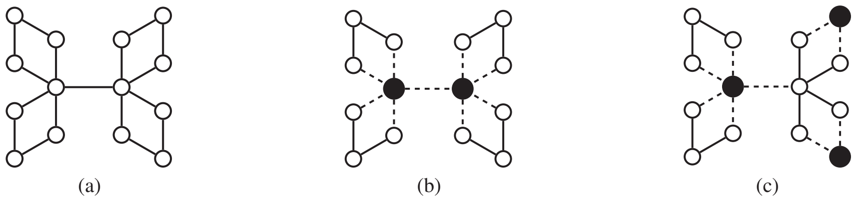


Fig. 1 (a) A graph G, (b) a minimum feedback vertex set of G, and (c) a minimum independent feedback vertex set of G, where each vertex in the feedback vertex sets is depicted by a black circle.

ially solvable for bipartite graphs.

1.2 Our contribution

In this paper, we study the approximability of the independent feedback vertex set problem for bipartite graphs.

We first show that, unless P = NP, the independence variant admits no polynomial-time approximation algorithm within a factor $n^{1-\varepsilon}$ for any fixed $\varepsilon > 0$, even on bipartite planar graphs, where n is the number of vertices in the graph. This gives a contrast to the existence of polynomial-time 2-approximation algorithms for the original problem on general graphs [2], [4]. One might think that this hardness result is straightforward, because the independence variant has the constraint of independent sets. In fact, the independent set problem, which finds a maximum-size independent set of a given graph, is also hard to approximate within a factor $n^{1-\varepsilon}$ in polynomial time, for any fixed $\varepsilon > 0$ [18]. However, since the independent set problem is a maximization problem whereas the independent feedback vertex set problem is a minimization problem, it is not straightforward to give our hardness result. We also point out that the independent set problem admits a PTAS for planar graphs [3], and is solvable in polynomial time for bipartite graphs (from König's theorem).

We then give an $\alpha(\Delta - 1)/2$-approximation algorithm for bipartite graphs G of n vertices and maximum degree Δ, which runs in $O(t(G) + \Delta n)$ time, under the assumption that there is an α-approximation algorithm for the original problem on bipartite graphs which runs in $O(t(G))$ time. (In this paper, we omit the details.) Notice that, from our inapproximability result, unless P = NP, there is no polynomial-time $\Delta^{1-\varepsilon}$-approximation algorithm for any fixed $\varepsilon > 0$. In this sense, our approximation factor is best possible with respect to the exponent of Δ.

2. Preliminaries

In this paper, we assume that graphs are undirected, unweighted, simple and connected. Let $G = (V, E)$ be a graph; we sometimes denote by $V(G)$ and $E(G)$ the vertex set and edge set of G, respectively. For a vertex subset V' of a graph $G = (V, E)$, let $G[V']$ be the subgraph of G induced by V'. For a subset $W \subseteq V$, we denote simply by $G - W$ the induced subgraph $G[V \setminus W]$.

For a graph G, a vertex subset I of G is called an *independent set of* G if $G[I]$ contains no edge, and a vertex subset F of G is called a *feedback vertex set of* G if $G - F$ is a forest. We sometimes say that a feedback vertex set F of G is *independent* if $G[F]$ forms an independent set of G. Let

$$\mathsf{OPT}(G) = \min\{|F| : F \text{ is an independent feedback vertex set of } G\};$$

and let $\mathsf{OPT}(G) = +\infty$ if G has no independent feedback vertex set. Given a graph G, the *independent feedback vertex set problem* is to find an independent feedback vertex set F of G such that $|F| = \mathsf{OPT}(G)$. Analogously, we define $\mathsf{OPT}_{\mathrm{FVS}}(G)$ for feedback vertex sets; notice that $\mathsf{OPT}_{\mathrm{FVS}}(G) < |V(G)| - 1$ always holds.

3. Inapproximability

As mentioned before, the independent feedback vertex set problem is APX-hard even for bipartite graphs. In this section, we give the following stronger result.

Theorem 1. *Let $\varepsilon > 0$ be any fixed constant. The independent feedback vertex set problem admits no polynomial-time approximation algorithm within a factor $n^{1-\varepsilon}$ for bipartite planar graphs of n vertices, unless* P = NP.

We prove the theorem in the remainder of this section, by giving a gap-producing reduction from the planar 3-satisfiability problem.

Recall that the 3-*satisfiability problem* (3-SAT, for short) is the problem of asking if there exists a satisfying assignment for a given 3-CNF formula ϕ. The *associated graph $G_\phi = (X \cup C, E)$ of ϕ* is a bipartite graph such that

(i) each vertex in X corresponds to a variable in ϕ, and each vertex in C corresponds to a clause of ϕ; and
(ii) two vertices $v \in X$ and $w \in C$ are joined by an edge in E if and only if the variable corresponding to v appears in the clause corresponding to w.

For example, we illustrate the associated graph of 3-CNF formula $\phi = (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (x_2 \vee \neg x_3 \vee \neg x_4)$ in Fig. 2(a). The formula ϕ is said to be *planar* if its associated graph is planar. The graph G_ϕ in Fig. 2(a) has a plane embedding as shown in Fig. 2(b), and hence the formula ϕ is planar. Given a planar 3-CNF formula ϕ, the *planar* 3-*satisfiability problem* (PLANAR 3-SAT, for short) is to determine whether there exists a satisfying assignment of ϕ. PLANAR 3-SAT is known to be NP-complete [12].

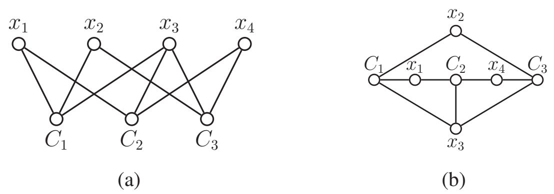


Fig. 2 (a) An associated graph G_ϕ of a 3-CNF formula $\phi = C_1 \wedge C_2 \wedge C_3$, where $C_1 = (x_1 \vee \neg x_2 \vee x_3), C_2 = (\neg x_1 \vee \neg x_3 \vee x_4)$ and $C_3 = (x_2 \vee \neg x_3 \vee \neg x_4)$. (b) A plane embedding of G_ϕ.

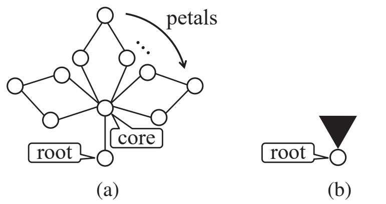


Fig. 3 (a) A forbidding gadget, where some petals are omitted. (b) The simplified illustration of a forbidding gadget, where the core vertex and all petals are simply illustrated as a black triangle.

3.1 Gadgets

In this subsection, we construct five kinds of gadgets for our reduction.

We first define a *forbidding gadget*, which consists of three parts: a *root vertex*, a *core vertex*, and *petals*. (See Fig. 3(a).) The root vertex will be identified with a vertex of another gadget defined later. Each petal is a cycle of four vertices, and the forbidding gadget has p petals that share the core vertex. We use the simplified illustration shown in Fig. 3(b) for the forbidding gadget. The forbidding gadget forbids adding the root vertex to a minimum independent feedback vertex set, and forces to choose the core vertex. To see this, notice that we cannot choose both root and core vertices at the same time, because they are adjacent. Therefore, if we choose the root vertex, we must choose at least p vertices additionally, each of which comes from a petal in the forbidding gadget. An independent feedback vertex set is said to be *proper* if it contains only the core vertex for every forbidding gadget.

Using the forbidding gadget, we will define the other gadgets. The *variable gadget* X_i is illustrated in Fig. 4(a), and corresponds to a variable x_i of a given 3-CNF formula ϕ. Every proper independent feedback vertex set must contain v or v'_i of X_i. We regard $v \in F$ as setting $x_i = \textsf{true}$, and say that X_i is the true-*state*. Conversely, we regard $v \notin F$ as setting $x_i = \textsf{false}$, and say that X_i is the false-*state*.

The *clause gadget* C_j is illustrated in Fig. 4(b). This gadget corresponds to a clause $(c_{j,1} \vee c_{j,2} \vee c_{j,3})$ in ϕ, and the three vertices $u_{j,1}, u_{j,2}$ and $u_{j,3}$ of C_j correspond to the three literals in the clause. Every proper independent feedback vertex set must contain at least one of $u_{j,1}$, $u_{j,2}$, and $u_{j,3}$.

The *positive* and *negative edge gadgets* are illustrated in Fig. 4(c) and (d), respectively. The purpose of these edge gadgets is to propagate the state of a variable gadget to a clause gadget; note that the negative edge gadget propagates the opposite state of the variable gadget. The positive edge gadget has only two proper independent feedback vertex sets, as shown in Fig. 5. Similarly, the negative edge gadget has only two proper independent feedback vertex sets $\{v_\ell\}$ and $\{u_\ell\}$ (together with all core vertices of forbidden gadgets). Observe that for every proper independent feedback vertex set F, $v_\ell \in F$ if and only if $u_\ell \in F$ for the positive edge gadget, and $v_\ell \in F$ if and only if $u_\ell \notin F$ for the negative edge gadget.

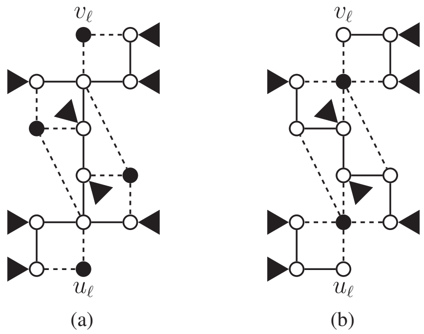


Fig. 5 The two proper independent feedback vertex sets of the positive edge gadget, formed by the black vertices together with all core vertices of forbidding gadgets.

3.2 Reduction

In this subsection, we construct the corresponding graph $G_{\phi,p}$ for the independent feedback vertex set problem, from a given instance ϕ of PLANAR 3-SAT.

Let $G_\phi = (X \cup C, E)$ be the associated graph of a given 3-CNF formula ϕ for PLANAR 3-SAT. We fix a plane embedding of G_ϕ arbitrarily. Then, we replace each vertex $x_i \in X$ with a variable gadget X_i, and each vertex $c_j \in C$ with a clause gadget C_j. Next, consider an edge $x_i c_j \in E$, and assume that $x_i c_j$ appears as the k-th edge, where $k \in \{1, 2, 3\}$, if we see the three edges incident to c_j in the clockwise direction. We replace the edge $x_i c_j \in E$ with a positive edge gadget if the corresponding variable of x_i appears as a positive literal; otherwise we replace $x_i c_j$ with a negative edge gadget. In either case, we identify the vertex v_ℓ in the edge gadget with v of X_i, and also identify the vertex u_ℓ with $u_{j,k}$ of C_j. (See also Fig. 6.) Let $G_{\phi,p}$ be the resulting graph, where p is the number of petals in each forbidding gadget. Since G_ϕ and all gadgets

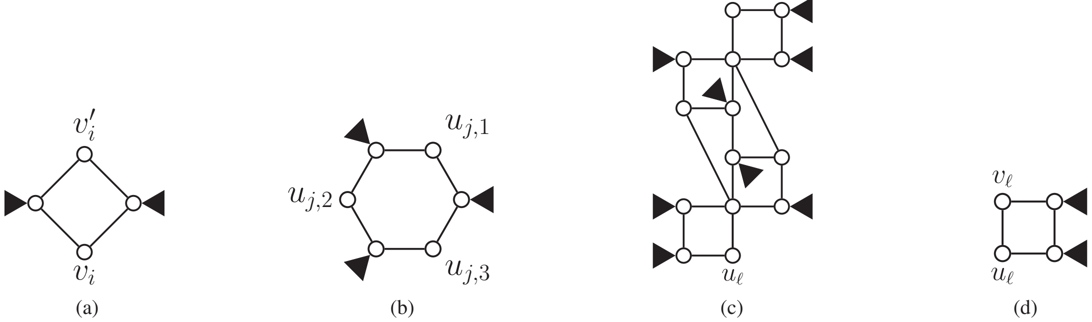
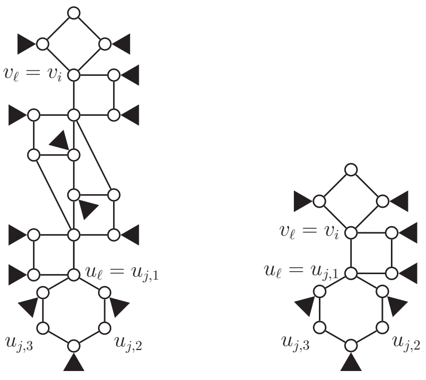


Fig. 4 (a) A variable gadget, (b) a clause gadget, (c) a positive edge gadget, and (d) a negative edge gadget.



(a) (b)

Fig. 6 This illustrates a replacement of the edge $x_i c_j$ which appears as the first edge of c_j with (a) a positive edge gadget, and (b) a negative edge gadget. In either case, we identify the vertex v_ℓ in the edge gadget with v of the variable gadget X_i, and also identify the vertex u_ℓ with $u_{j,1}$ of the clause gadget C_j.

are bipartite and planar, $G_{\phi,p}$ is also a bipartite planar graph. In addition, we can construct $G_{\phi,p}$ in polynomial time.

Let q_ϕ be an arbitrary integer such that $q_\phi \geq 81(s+t)$, where s and t are the numbers of variables and clauses in ϕ, respectively. We denote by $n_{\phi,p}$ the number of vertices in $G_{\phi,p}$. Then, we have the following lemma.

Lemma 1. *It holds that* $n_{\phi,p} < (p+1)q_\phi$.

Proof. For each forbidding gadget, the number of vertices is exactly $3p+1$ except for the root vertex. In addition,

- for each variable gadget X_i,
  $|V(X_i)| = 2 \cdot (3p+1) + 4 = 6(p+1)$;
- for each clause gadget C_j,
  $|V(C_j)| = 3 \cdot (3p+1) + 6 = 9(p+1)$; and
- for each edge gadget E_ℓ,
  $|V(E_\ell)| \leq 8 \cdot (3p+1) + 14 < 24(p+1)$.

Note that, since ϕ is 3-CNF formula, the number of edge gadgets is exactly $3t$. Therefore, we have the following inequality:

$$\begin{aligned} n_{\phi,p} &< 6(p+1)\cdot s + 9(p+1)\cdot t + 24(p+1)\cdot 3t \\ &= 3(p+1)(2s+27t) \\ &< (p+1)q_\phi. \end{aligned}$$

□

The following lemma is the key for the proof of Theorem 1.

Lemma 2. *For any integer* $p \geq q_\phi$*, the following* (I) *and* (II) *hold*:

(I) *if* $\mathsf{OPT}(G_{\phi,p}) < p+1$*, then* ϕ *has a satisfying assignment*; *and*

(II) *if* $\mathsf{OPT}(G_{\phi,p}) > q_\phi$*, then* ϕ *has no satisfying assignment.*

Proof. **[The proof of (I).]** We take a minimum independent feedback vertex set F of G arbitrarily. Then, F is proper, because the size of F is less than $p+1$ and thus F must contain only a core vertex for every forbidding gadget. Recall that the proper independent feedback vertex set F must contain v or v'_i of each variable gadget X_i, and contain at least one of $u_{j,1}$, $u_{j,2}$, and $u_{j,3}$ of each clause gadget C_j. In addition, each edge gadget propagates the state of a variable gadget to a clause gadget. Therefore, this implies that we can form a satisfying assignment of ϕ from F if we regard $v \in F$ as setting $x_i = \mathsf{true}$, and regard $v \notin F$ as setting $x_i = \mathsf{false}$.

[The proof of (II).] We proceed by contraposition, that is, we show that if ϕ has a satisfying assignment, then $\mathsf{OPT}(G_{\phi,p}) \leq q_\phi$. We will construct a proper independent feedback vertex set F of the graph $G_{\phi,p}$ from a satisfying assignment in accordance with the following steps.

Step 0. We prepare an empty set F.

Step 1. We add cores of all forbidden gadgets of $G_{\phi,p}$ to F.

Step 2. For $1 \leq i \leq s$, each variable gadget X_i and the corresponding variable x_i of ϕ, we add v of X_i to F if $x_i = \mathsf{true}$. Conversely, we add v'_i of X_i to F if $x_i = \mathsf{false}$.

Step 3. For each positive edge gadget E_ℓ and v of a variable gadget X_i identified with v_ℓ of E_ℓ, we add the black vertices shown in Fig. 5(a) to F if $v \in F$. Otherwise, we add the black vertices shown in Fig. 5(b) to F.

Step 4. For each negative edge gadget E_ℓ and v of a variable gadget X_i identified with v_ℓ of E_ℓ, we add u_ℓ to F if $v \notin F$. (In the case that $v \in F$, since the negative edge gadget is already acyclic, there is no need to add any vertices to F on this step.)

It is easy to see that the set F obtained from the above steps forms an independent set of $G_{\phi,p}$. We will show that F also forms a feedback vertex set of $G_{\phi,p}$. For every variable gadget X_i and every edge gadget E_ℓ, F removes all cycles in these gadgets. Moreover, by Steps 3 and 4, F is constructed so that $v_\ell \in F$ if and only if $u_\ell \in F$ for the positive edge gadget, and $v_\ell \in F$ if and only if $u_\ell \notin F$ for the negative edge gadget. Since F is obtained from a satisfying assignment of ϕ, F must contain at least one of $u_{j,1}$, $u_{j,2}$, and $u_{j,3}$ for each clause gadget C_j, that is, F removes all cycles in every clause gadget. Our remaining task is to verify that F also removes every cycle passing through several gadgets. It suffices to notice that such a cycle must contain the vertices included in F on Steps 3 and 4. (See also Fig. 5.)

Now, we will bound the size of F. We denote by F_i a set of vertices added to F during Step i. The size of F_1 equals to the number of forbidden gadgets in $G_{\phi,p}$ because, for each forbidden gadget, F contains only its core. Recall that, since ϕ is 3-CNF formula, the number of edge gadgets is exactly $3t$. It follows that $|F_1| \leq 2s + 8 \cdot 3t + 3t = 2s + 27t$. In addition, $|F_2| = s, |F_3| + |F_4| \leq 3 \cdot 3t = 9t$. Thus, we conclude $|F| = \sum_{i=1}^{4} |F_i| \leq 3s + 36t \leq q_\phi$. □

Note: The number of vertices calculated in the above proofs of Lemmas 1 and 2 is obviously overestimated in order to simplify the following proof.

We now prove Theorem 1. Assume for a contradiction that there exists a polynomial-time approximation algorithm within a factor $n^{1-\varepsilon}$ for some fixed $\varepsilon > 0$, where n is the number of vertices in a given graph. Observe that $\varepsilon \leq 1$ must hold. We denote by $\mathsf{APX}(G_{\phi,p})$ the size of a solution for $G_{\phi,p}$ produced by the approximation algorithm. Then, we have

$$\mathsf{OPT}(G_{\phi,p}) \leq \mathsf{APX}(G_{\phi,p}) \leq n_{\phi,p}^{1-\varepsilon} \cdot \mathsf{OPT}(G_{\phi,p}). \qquad (1)$$

We set

$$p = \left\lceil q_\phi^{(2-\varepsilon)/\varepsilon} \right\rceil - 1.$$

Consider the case where $\mathsf{APX}(G_{\phi,p}) < p+1$ holds. By (1) it holds in this case that $\mathsf{OPT}(G_{\phi,p}) < p+1$. Then, Lemma 2(I) says that ϕ has a satisfying assignment. Consider the other case, where $\mathsf{APX}(G_{\phi,p}) \geq p+1$ holds. By (1) it holds in this case that $n_{\phi,p}^{1-\varepsilon} \cdot \mathsf{OPT}(G_{\phi,p}) \geq p+1$. Then, by Lemma 1 we have

$$\mathsf{OPT}(G_{\phi,p}) \geq \frac{p+1}{n_{\phi,p}^{1-\varepsilon}} > \frac{p+1}{((p+1)q_\phi)^{1-\varepsilon}} = \frac{(p+1)^\varepsilon}{q_\phi^{1-\varepsilon}} \geq \frac{\left(q_\phi^{(2-\varepsilon)/\varepsilon}\right)^\varepsilon}{q_\phi^{1-\varepsilon}} = q_\phi.$$

Then, Lemma 2(II) says that ϕ has no satisfying assignment.

In this way, $\mathsf{APX}(G_{\phi,p}) < p+1$ if and only if ϕ has a satisfying assignment. Since we have assumed that $\mathsf{APX}(G_{\phi,p})$ can be computed in polynomial time, this means that we can solve PLANAR 3-SAT in polynomial time. This is a contradiction unless P = NP. This completes the proof of Theorem 1.

4. Conclusion

In this paper, we have shown that the independent feedback vertex set problem for bipartite planar graphs of n vertices admits no polynomial-time approximation algorithm within a factor $n^{1-\varepsilon}$, for any fixed $\varepsilon > 0$, unless P = NP. This gives a contrast to the fact that the original problem admits a polynomial-time 2-approximation algorithm for general graphs. We also have developed an $\alpha(\Delta-1)/2$-approximation algorithm for the independence variant on bipartite graphs of maximum degree Δ, where α is the approximation factor of an algorithm for the original problem on bipartite graphs.

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