Journal of Information Processing Vol.21 No.1 2–8 (Jan. 2013)

[DOI: 10.2197/ipsjjip.21.2]

**Regular Paper**

# Expected Price of Anarchy for the Dynamic Network Formation Game Model

Tetsuo Imai[1,a) ] Atsushi Tanaka[1,b)]

Received: January 27, 2012, Accepted: July 2, 2012

***Abstract:*** Recent studies revealed that some social and technological network formations can be represented by the network formation games played by selfish multiple agents. In general, the topologies formed by selfish multiple agents are worse than or equal to those formed by the centralized designer in the sense of social total welfare. Several works such as the price of anarchy are known as a measure for evaluating the inefficiency of solutions obtained by selfish multiple agents compared to the social optimal solution. In this paper, we introduce the expected price of anarchy which is proposed as a valid measure for evaluating the inefficiency of the dynamic network formation game whose solution space is divided into basins with multimodal sizes. Moreover, through some computer simulations we show that it can represent the average case behavior of inefficiency of dynamic network formation games which is missed by two previous measures.

***Keywords:*** Network formation game, Price of Anarchy, Complex Networks, Network dynamics

## 1. Introduction

Recent studies revealed that some social and technological network formations can be represented by the network formation games played by selfish multiple agents. Furthermore, there are some works introducing the dynamicity to the traditional model for representing and analyzing the dynamics of the network formation [2], [4].

In general, the topologies formed by selfish multiple agents are worse than or equal to those formed by the centralized designer in the sense of social total welfare. In the fields of computer science and game theory there are several previous works for evaluating the inefficiency of solutions obtained by decentralized solving compared to that by centralized solving, known as the name of *the price of anarchy* (*PoA*) and *the price of stability* (*PoS*). *The PoA* is defined as the ratio between the worst evaluation value of equilibrium of the game and that of an optimal outcome, *the PoS* is that for the best evaluation value. Since these works are based on either worst-case or best-case analysis, it may not be valid especially in the case where there are a large number of equilibria.

In this paper, a new measure *the expected price of anarchy* (*EPoA*) which is called as *the weighted price of anarchy* in the original paper Ref. [1] is introduced. This is a measure to evaluate the inefficiency of solutions of the dynamic network formation game model formulated by Imai et al. [2]. This new measure utilizes the property of their model that the solution space of the model is divided into basins corresponding to the solutions. These basin sizes represent proportional values to the probability of solutions in the assumption that the probabilities of initial states take equal values. The validity of the new measure stands on the "average-case" analysis using these basin sizes. We also compare the actual values of previous measures and that of our measures by some computer simulations, and show that the EPoA is a natural measure for the dynamic network formation games which may have many stable solutions.

The rest of the paper is organized as follows. Section 2 presents basics on the static network formation games established in the field of game theory and it follows the introduction of the dynamic network formation game model formulated by Imai et al. [2]. At the first part of Section 3, we describe details of previous measures for evaluating equilibria obtained by decentralized solving, and these problems are presented. The new measure *the EPoA* for the dynamic network formation games is introduced at the last parts of that section. Following Section 4, we describe some numerical simulations for comparing previous measures and our measure of inefficiency and show these results. In the last section, we present conclusions and future works.

## 2. Network Formation Game

### 2.1 Static Network Formation Game

In this subsection, we introduce some results of the network formation game. Myerson firstly suggested the game which represents the network formation [7], described as the link-announcement game [3]. In this paper, we refer to their model as the *static network formation game* to contrast it with the dynamic one described in the next subsection. It is formulated as follows.

Let $N$ be the set of players and $n$ be the size of $N$, and they can form links among any pair of players. The topology (which is same as *graph* in the graph theory) $g$ is defined by a combination of the set of agents $N$ and the set of links $L \subset N \times N$.

[1] Graduate School of the Science and Engineering, Yamagata University, Yonezawa, Yamagata 992–8510, Japan
[a)] imai@etas.yz.yamagata-u.ac.jp
[b)] tanaka@yamagata-u.ac.jp

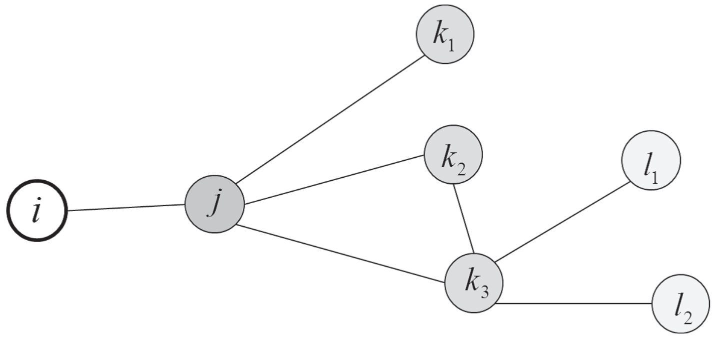



**Fig. 1** An example of payoff value defined by the payoff function (3). In this topology, a payoff value of a node $i$ is given by $u_i(g) = (\delta - c_{ij}) + 3\delta^2 + 2\delta^3$. The first term $(\delta - c_{ij})$ represents the benefit from node $j$ and the cost of a direct link to it, and the second term $3\delta^2$ represents benefit from nodes with distance 2, $k_1, k_2$ and $k_3$ and the last term $2\delta^3$ is derived by $l_1$ and $l_2$ with distance 3. Each distance is given by the shortest path hops between them.

Although generally a link is represented as $(i, j) \in L$, for simplicity we denote it as $ij$. In this paper, we consider only undirected topologies. The strategy space of player $i$ is $S_i = 2^{N \setminus \{i\}}$, where $N \setminus i = \{1, 2, \ldots, i-1, i+1, \ldots, n\}$ and $2^{N \setminus \{i\}}$ is the power set of $N \setminus \{i\}$. If $\boldsymbol{s} \in S_1 \times \cdots \times S_n$ is the profile of strategies played, then link $ij$ forms if and only if both $j \in s_i$ and $i \in s_j$. The outcome network $g(\boldsymbol{s})$ is represented by $g(\boldsymbol{s}) = \{ij | i \in s_j \text{ and } j \in s_i\}$.

Instead of the Nash equilibrium which is generally utilized as the concept of solution in the game theory, Jackson et al. [6] proposed the novel stability concept called *pairwise stability* which departs from the notion of Nash equilibrium because of the specialty of the network formation game. A topology $g$ is pairwise stable if

$$u_i(g) \geq u_i(g - ij), \quad u_j(g) \geq u_j(g - ij) \quad (\forall ij \in g) \tag{1}$$

and

$$u_i(g + ij) > u_i(g) \Rightarrow u_j(g + ij) < u_j(g) \quad (\forall ij \notin g) \tag{2}$$

where $u_i(g)$ is the payoff for player $i$ in topology $g$ and $g + ij$ is the topology which is obtained by adding a link $ij$ to topology $g$, $g - ij$ is the topology which is obtained by removing a link $ij$ from topology $g$. The former condition implies that no players raise their payoffs by removing a link which they are directly involved in. The latter condition implies that no two players can both benefit (at least one strictly) by adding a link between themselves.

They also proposed some concepts about efficiency and investigated the relationship between pairwise stable topologies and efficient topologies. In the paper we adopt the *strong efficiency* as the measure of efficiency of the topology. A topology $g$ is strongly efficient if $\sum_i u_i(g) \geq \sum_i u_i(g')$ for all $g' \in G$.

There are two problems in the static network formation game. Firstly, it is difficult to find pairwise stable topologies because the state space is critically huge and efficient methods for finding pairwise stable topologies have not been found yet. Secondly, there are many pairwise stable topologies in general, and we can not identify importานt ones among these topologies. These problems are especially serious in the case where the number of players is large, and are caused by the stability conditions which require only that no link changes occur in that topology.

### 2.2 Dynamic Network Formation Game

In this subsection, we describe the details of the dynamic network formation game model which is analyzed in the paper. It is formulated by Imai et al. [2] as a deterministic version of introducing dynamicity to the static network formation game model, based on the model of Jackson et al. [4]. They adopted the concept *defeat* and *improving path* defined by Jackson and Watts, and modified their previous dynamic network formation model to that of behaving deterministically by specifying the most payoff-improving transition among possible transitions.

The model is formulated as a kind of processes of a time series of simultaneous-move game as follows. At each discrete time step $t$, the game in a strategic form determined by the current state (same as topology) $g(t)$ is played and it determines the next state $g(t+1)$. As for the game which is played at each time step, *players* are agents who intend to improve their payoffs. The *strategy* of each agent $i$ is indicated as a vector $\boldsymbol{s}_i(t) = (s_{i1}(t), \ldots, s_{in}(t))$, where $s_{ij}(t) \in \{0, 1\}$. The player $i$ independently sets $s_{ij}(t)$ according to a change of its payoff with a change of link $ij$, $s_{ij}(t)$ is set to 1 if it is desirable to add (or maintain) the link with player $j$, otherwise $s_{ij}(t)$ is set to 0. $s_{ii}$ is always equal to 0. The *payoff* of the player $i$ is defined by the following distance-based payoff function,

$$u_i(g) = \sum_{j \neq i} \delta^{d_{ij}} - \sum_{j \in \{j | ij \in g\}} c_{ij} \tag{3}$$

where $0 \leq \delta \leq 1$ indicates the decay of the benefit from a connected agent with an increase of the distance, $d_{ij}$ is the distance between agent $i$ and $j$ (the number of hops from $i$ to $j$), and $c_{ij}$ is a link cost to add or maintain the link between $i$ and $j$. **Figure 1** shows an example of payoff value defined by the payoff function (3). The *outcome* $g(t+1)$ obtained by playing the game at time step $t$ is determined as follows. We describe about two concepts *adjacent* and *defeat*. Two topologies $g$ and $g'$ are *adjacent* if $g'$ differs in only one link from $g$, and a state $g'$ *defeats* an adjacent state $g$ if either

$$g' = g - ij \text{ and } (u_i(g') > u_i(g) \text{ or } u_j(g') > u_j(g)) \tag{4}$$

or

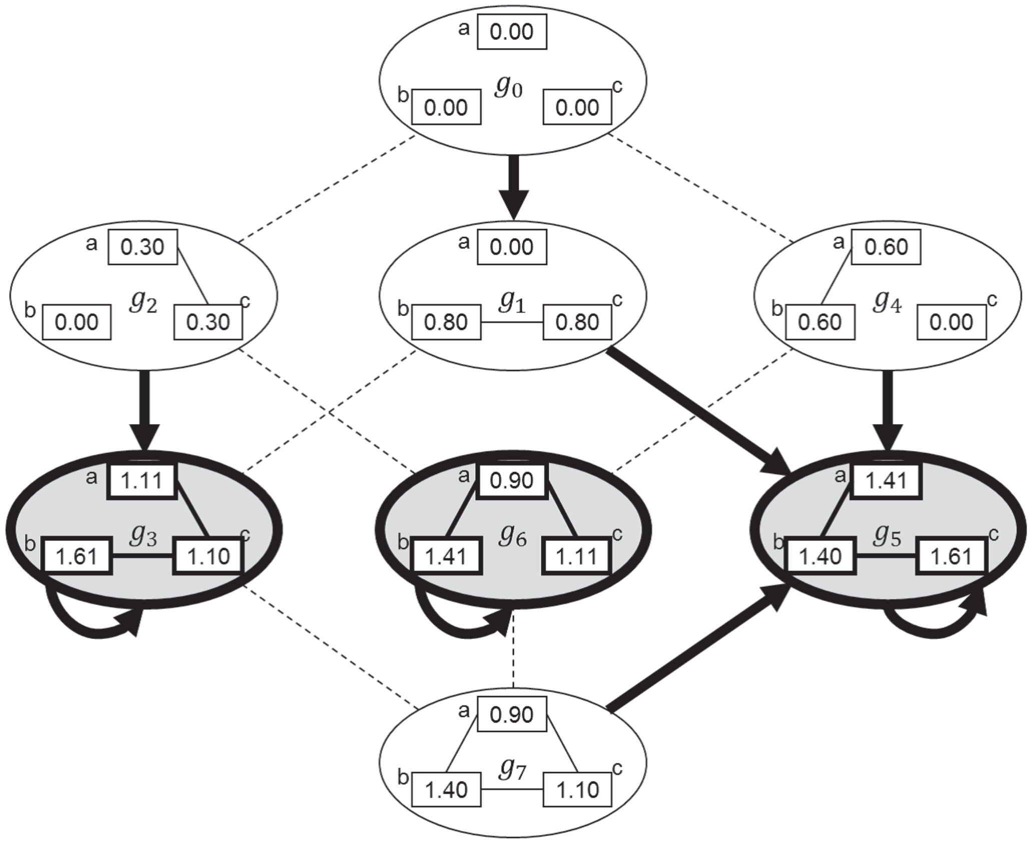



**Fig. 2** An example of state transitions of the dynamic network formation game model formulated by Imai and Tanaka [2]. In this example, there exists 3 nodes and the parameters of the payoff function (3) are as follows, $\delta = 0.9, c_{ab} = c_{ba} = 0.3, c_{bc} = c_{cb} = 0.1, c_{ca} = c_{ac} = 0.6$. There are $8 = 2^{{}_3C_2}$ states(topologies) which can be constructed by 3 nodes, and each of two adjacent states are linked by dashed lines. The respective values of payoffs are listed in the nodes. There are arrows from a state $g(t)$ to a state $g(t+1)$ which have a relation that it stays at the state $g(t)$ at time step $t$ and it moves to the state $g(t+1)$ at next time step $t+1$. These arrows indicate the most payoff improving transition according to the equation (8). There are 3 pairwise stable solutions in the case (and no improving cycles) $g_3$,$g_5$ and $g_6$. The basin sizes of the solutions are respectively 2, 5 and 1.

$$\begin{aligned} &g' = g + ij \text{ and} \\ &\quad\{(u_i(g') \geq u_i(g) \text{ and } u_j(g') \geq u_j(g)), \\ &\quad\ \text{except } (u_i(g') = u_i(g) \text{ and } u_j(g') = u_j(g))\}. \end{aligned} \tag{5}$$

$g(t+1)$ is specified deterministically among states which can defeat $g(t)$ and $g(t)$ itself. For a concrete description of $g(t+1)$, two definitions $\Delta u_{ij}(t+1)$ and *acceptable link set* $L_{acceptable}(g)$ are described as follows. Firstly, $\Delta u_{ij}(t+1)$ is defined as the amount of change of $i$'s payoff in the case that the change of link $ij$ occurs at time step $t$. It is formulated as follows,

$$\Delta u_{ij}(t+1) = \begin{cases} u_i(g(t)+ij) - u_i(g(t)), & \text{if } \ ij \notin g(t) \\ u_i(g(t)-ij) - u_i(g(t)), & \text{if } \ ij \in g(t). \end{cases} \tag{6}$$

Secondly, $L_{acceptable}(g) \subset L \subset N \times N$ is defined as the set of links which are acceptable for both player $i$ and player $j$. It is formally described as follows.

$$\begin{aligned} L_{acceptable}(g) = &\{ij | ij \notin g \text{ and } g + ij \text{ } \textit{defeats } g\} \\ &\cup \{ij | ij \in g \text{ and } g - ij \text{ } \textit{defeats } g\}. \end{aligned} \tag{7}$$

The link $ij$ which is changed during the game is described as

$$ij = \underset{ij \in L_{acceptable}(g(t))}{\operatorname{argmax}} \Delta u_{ij}(t+1) \tag{8}$$

and determines the outcome $g(t+1)$. If no links satisfy this condition then $g(t+1)$ is exactly the same as $g(t)$, and if there is more than one link satisfying that, the link involved by the agent who has the youngest ID is prior than others as a matter of convenience. Note that agents decide their strategies at each time step $t$ only to make their own payoffs of the next step $t+1$ be better off without any forecasts. The process starts from the initial state of topology $g_0$, and it continues until the state converges to a stable state or a part of a cycle which is described as the *solution* of the process. It is clear from definitions that a state is pairwise stable if and only if the solution of the process consists of one state. Solutions can be an *improving cycle* which consists of a sequence of adjacent states $\{g_1, g_2, \ldots, g_K\}$ such that each defeats the previous one and $g_1 = g_K$. The condition of existence of an improving cycle is analyzed by Jackson et al. [5]. **Figure 2** shows an example of the state transitions of the dynamic network formation game model.

We describe the solution space $G$. The size of $G$ is the number of capable states (topologies) constructed by $n$ agents, $|G| = 2^{{}_nC_2}$, therefore it rapidly increases by increasing the node size $n$. Let the number of initial states converging to a solution be a *basin*, it is the general term of the dynamic systems, the solution space is divided into some mutually exclusive and collectively exhaustive

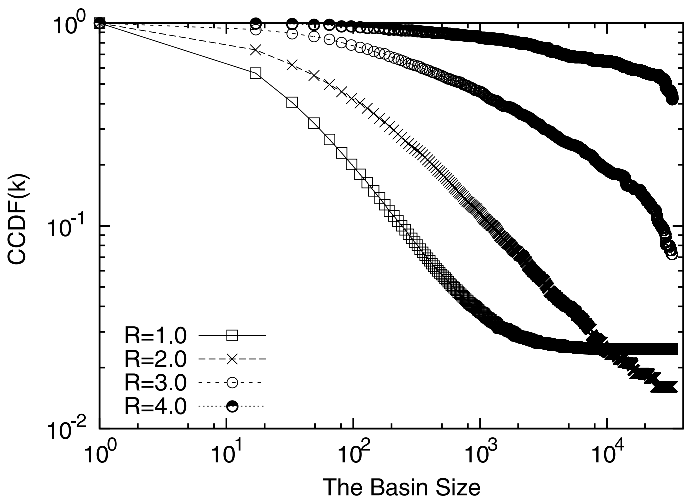



**Fig. 3** An example of divisions of the whole state space $G$ by basins with highly variant sizes. It shows results of 6 agents. $CCDF(k)$ is the Cumulative Complementary Distribution Function which indicates the rate of greater values than $k$. This figure is a double logarithmic chart. Parameters in the payoff function (3) are as follows. $\delta$ is fixed to 0.9, and $c_{ij}$ are symmetric and randomly sampled from a uniform distribution of the range $(0, R]$. All Results are the averages of 100 random parameters.

basins for each corresponding solution in the model. **Figure 3** shows an example of divisions of the whole state space $G$. It shows that basins might take greatly different sizes in the cases of some conditions of payoff functions. It seems natural to consider the basin size of a solution as the importance of the solution because the basin size of the solution is proportional to the probability of convergence to the solution in the assumption that all initial states $g_0$ have the same probability.

## 3. Evaluation of Inefficiency

In this section, we describe previous works for evaluating the inefficiency of solutions obtained by selfish multiple agents compared to the strongly efficient solution. In addition, we introduce a new measure *the expected price of anarchy* (*EPoA*) to evaluate the inefficiency of solutions for the dynamic network formation game which is described in the previous section.

### 3.1 Previous Measures of Inefficiency and Their Problems

In the field of computer science, it is a popular issue to know the relation between the solution obtained by centralized solving and that by decentralized solving [12]. The centralized solving is the one where each agent in the system is told exactly what to do and must do so, and the decentralized solving is the one where each agent tries to optimize its own payoffs selfishly, therefore the latter is game theoretic. Of course a centralized solving may be able to obtain more socially an optimal solution than that of a decentralized solving, how much more beneficial can it be?

*The price of anarchy* (*PoA*) which is formulated by Koutsoupias et al. [8] is the most popular measure of the inefficiency of decentralized solving. Precisely, the PoA of a game is defined as the ratio between the worst evaluation value of an equilibrium of the game and that of an optimal outcome. Since the original paper, the PoA has been studied in many settings like the traffic routing problem [9].

*The price of stability* (*PoS*) is another measure of the inefficiency of decentralized solving, which is the ratio between the best evaluation value of one of its equilibria and that of an optimal outcome [11]. It is designed to differentiate between games in which all equilibria are inefficient and those in which only some equilibrium is inefficient.

In this paper, we use these measures for evaluating the inefficiency of the dynamic network formation game by evaluating the solutions obtained by selfish multiple agents. In the case that the process of the dynamic network formation game converges to one pairwise stable state, we compare the optimal outcome to pairwise stable ones instead of comparing it to outcomes derived by Nash equilibria in the original definition.

The social evaluation function of a state (topology) is simply given by the sum of payoffs of agents, and the social evaluation function of a solution is given by averaged values of social evaluation functions of each component states of the solution. That is, in the case of solutions which consist of one pairwise stable states, the social evaluation value of the state indicates directly that of the solution.

Formally, using the social evaluation function $f_g(g) : G \to \mathbb{R}$ for a state $g$, the social evaluation function $f_s(s_j) : 2^G \to \mathbb{R}$ of the $j$-th solution $s_j$ consists of states $\{g_1, \ldots, g_{K_j}\}$ and is described as follows,

$$f_g(g) = \sum_i u_i(g) \tag{9}$$

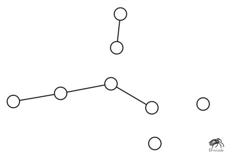
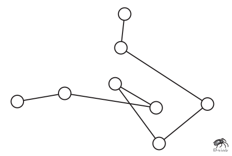
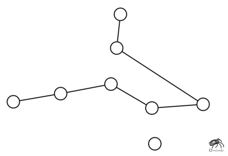
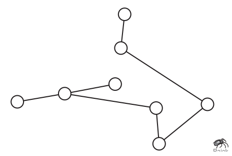
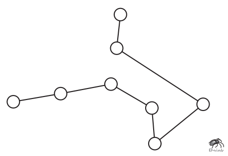
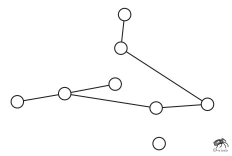
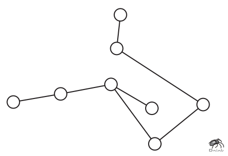
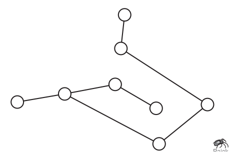
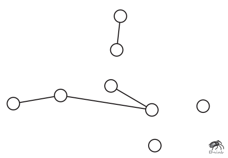
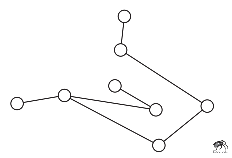
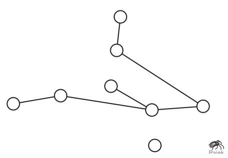
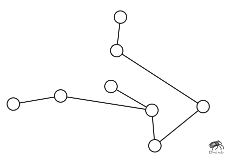
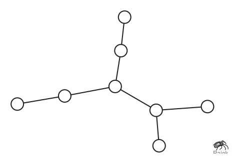
$$f_s(s_j) = \sum_{k=1}^{K_j} f_g(g_k)/K_j \tag{10}$$

where $i$ indicates the agent ID. The strongly efficient solution $s^*$ which consists of only the strongly efficient topology $g^*$ takes the maximal social evaluation value for all solutions.

Since the strongly efficient topology $g^*$ maximizes the social evaluation function for all states, we considered it as the "optimal" state. Using the value of $f_g(g^*)$, the PoA and the PoS for the dynamic network formation game are described as follows,

Journal of Information Processing Vol.21 No.1 2–8 (Jan. 2013)

**Table 1** An example of the expected price of anarchy of the dynamic network formation model by 8 agents. There are 12 pairwise stable solutions (no cycles). For each solution, the actual topology of each pairwise stable state is illustrated. The value of the social evaluation function which is given by the sum of payoffs of agents is described in the next column, the basin size which equals to the number of initial states converge to the solution by the process of this model is described in the last column. The worst and best value of the social evaluation among these solutions is respectively 5.81 and 27.11. The strongly efficient topology $g^*$ which is not included in these stable solutions is shown at the end of the enumeration, its value of social evaluation function is 30.22. The value of the price of anarchy is calculated as $30.22/5.81 \approx 5.20$, and that of the price of stability is $30.22/27.11 \approx 1.11$. The expected price of anarchy is approximately 2.03. All pictures of topologies are created using *Pajek* [13].

| Solution | Topology | Sum of Payoffs | Basin Size | Solution | Topology | Sum of Payoffs | Basin Size |
|---|---|---|---|---|---|---|---|
| $s_1$ |  | 7.83 | 106,966,207 | $s_8$ |  | 21.21 | 177,032 |
| $s_2$ |  | 21.82 | 69,057,682 | $s_9$ |  | 25.41 | 1,522,866 |
| $s_3$ |  | 27.11 | 6,745,020 | $s_{10}$ |  | 20.01 | 14,864,197 |
| $s_4$ |  | 24.29 | 841,521 | $s_{11}$ |  | 25.18 | 8,107,411 |
| $s_5$ |  | 5.81 | 23,915,112 | $s_{12}$ |  | 23.16 | 735,093 |
| $s_6$ |  | 20.63 | 31,149,583 | | | | |
| $s_7$ |  | 26.15 | 4,353,732 | $s^*$ |  | 30.22 | – |

$$(\text{PoA}) = \frac{f_g(g^*)}{\min_j f_s(s_j)} \tag{11}$$

$$(\text{PoS}) = \frac{f_g(g^*)}{\max_j f_s(s_j)}. \tag{12}$$

We describe the limitation of these previous measures. In general, there are multiple equilibria and these equilibria take respective evaluation values. Especially in the case that there are a huge number of equilibria and they take widely distributed evaluation values, neither the worst-case analysis of the PoA nor the best-case analysis of the PoS may be far from "average-case" behavior. There are some works to analyze a "typical" equilibrium by defining some kind of Nash equilibrium as a "typical" one. However, they have not been used successfully to study the inefficiency of equilibria [11] because, within the frame of the static game, it is difficult to define in a meaningful and analytically tractable way to differentiate some kind of "typical" Nash equilibrium from others.

### 3.2 The Expected Price of Anarchy

We introduce a new measure *the expected price of anarchy* (*EPoA*) for evaluating the inefficiency of the dynamic network formation game which has the property that each solution is weighted by its basin size. Precisely, it is the ratio between the weighted average evaluation value of solutions by their basin sizes and that of the strongly efficient solution,

$$(\text{EPoA}) = \frac{f_g(g^*)}{\sum_j w_j f_s(s_j)/|G|} \tag{13}$$

where $w_j$ is the basin size of the solution $s_j$ and $\sum_j w_j = |G|$. **Table 1** shows an example of the expected price of anarchy of the dynamic network formation model.

It is clear from the definitions that (PoS) $\leq$ (EPoA) $\leq$ (PoA), and all values are equal if and only if there is only one solution. This EPoA seems a natural candidate as a valid measure for evaluating the inefficiency of the dynamic network formation game in average means, because, as described above, the basin size of the solution is proportional to the probability of converging to the solution in the assumption that all initial states $g_0$ take the same probability.

Following are two notations about contributions of the paper. First is that we do not insist that we should use only the EPoA instead of the two previous measures, PoA and PoS. These are successively valid because they represent a particular boundary of possible behaviors of the system. We only insist that the EPoA is also valid as an additional information for evaluating the distribution of inefficiency of the solution. That is available in the case that we can evaluate importance of multiple stable solutions. Second is that the stable solutions (except cycle solutions) of the dynamic model are also the pairwise stable solutions of the static game. Although all pairwise stable solutions are indifferent in the frame of the static game, each solution has its basin size in the frame of the dynamic model, and these are additional properties of solutions. The main contribution of the paper is to propose a more precise method for evaluating the inefficiency of pairwise stable solutions of the static game. For this purpose, we use the additional property (i.e., basin size) about each solution obtained by the frame of the dynamic model.

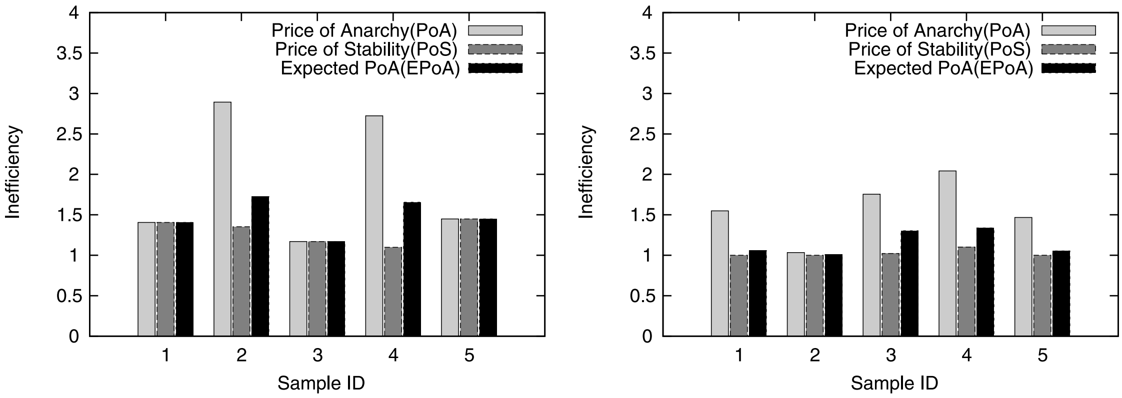
## 4. Numerical Results

In this section, we investigate the actual values of the measures described in the previous section by computer simulations.

**Figure 4** shows the results of computer simulations. In the simulation, the number of agents is 8 in all settings. Link cost parameters $c_{ij}$ in the payoff function (3) are symmetric and randomly sampled from a uniform distribution of the range $(0, R]$, *i.e.*, $0 < c_{ij} \leq R$. It shows the results for each case of $\delta = 0.5, 0.9$ and $R = 3.0$. The simulations are run with 5 random parameters, and all results in Fig. 4 are the averages of these trials.

The number of agents in the simulations is very small compared to the social and technological networks in the real world like the AS-level network of the Internet which is constructed by over 30,000 nodes [10], therefore there may be a lack of some important properties. The reasons of adopting the setting of 8 agents are as follows. Mainly it is caused by the limitation of the computing performance compared to the solution space described above. In addition, the payoff function (3) uses the distance (the number of hops of shortest path) from an agent to all other nodes and we use the simple Dijkstra algorithm to obtain it. Then the amount of computation for obtaining the value of the payoff function is too large in the case of many agents. The latter problem might be improved by applying techniques of parallel computing and more efficient algorithms for solving the all-pairs-shortest-path problem.

Although the simulations have limitations in the sense of the scale, we can find in these results that the values of our new measure EPoA are different both from that of PoA and of PoS. It is especially clear in the case of many stable solutions, for example the case of $\delta = 0.9, R = 3.0$. There are examples of represent-




(a) $\delta = 0.5, c_{ij} \in [0.0, 3.0)$ (b) $\delta = 0.9, c_{ij} \in [0.0, 3.0)$

**Fig. 4** Results of numerical simulations of 8 agents. All solutions which consist of one stable states(topologies), therefore these are pairwise stable solutions of the static network formation game. Link cost parameters $c_{ij}$ in the payoff function (3) are symmetric and randomly sampled from a uniform distribution of the range $(0.0, 3.0]$. All results in these figures are the averages of 5 trials. In the result (a) of $\delta = 0.5$, the number of solutions of each sample is 1,3,1,6 and 1 respectively. In the result (b) of $\delta = 0.9$, these are 418,6,199,721 and 186 respectively.

ing the average case behavior which is not covered with the two previous measures.

## 5. Conclusion

We have focused on the inefficiency of topologies formed by selfish multiple agents compared to that by a centralized designer in the sense of social total welfare. We have introduced the two previous measures for evaluating that, the former is *the price of anarchy* (*PoA*) which is defined as the ratio between the worst evaluation value of an equilibrium of the game and that of the optimal outcome, and the latter is *the price of stability* (*PoS*) which is the ratio of the best value and the optimal value. In addition we have pointed out their limitation that it may not be valid especially in the case that there are a large number of equilibria. We have introduced a new measure *the expected price of anarchy* (*EPoA*) to evaluate the inefficiency of solutions of the dynamic network formation game model formulated by Imai et al. [2]. *The EPoA* utilizes the property of their model whereby the solution space of the model is divided into basins corresponding to the solutions and these basin sizes are proportional to the probabilities of solutions in the natural assumption that the probability of initial states take the same value. Moreover, through some computer simulations we show that it can represent the average case behavior of inefficiency of dynamic network formation games which is not covered with the two previous measures.

We present two future works. It is important for the dynamic network formation game model to investigate the behavior of a larger number of agents through more large scale simulations, because some actual networks consist of a huge number of nodes (agents). Under the assumption that initial states are given by some probabilistic distribution, inefficiencies of solutions obtained by the dynamic network formation game model are random variables which obey some distribution function of solutions. Although it is guaranteed that the variance of the distribution function converges to a finite value because the number of instance of solutions is finite, concrete shapes of the distribution function of solutions are not revealed yet. In addition, the distribution of basin sizes which is shown by Fig. 3 implies that the distribution function of solutions might take multimodal shapes. Then it might be needed some additional verification of the validity of the representativeness of an expected value.

**Acknowledgments** Authors would like to greatly thank Prof. David Kinny, an associate professor in the Department of Social Informatics Graduate School of Informatics, Kyoto University. He has given us many insightful comments about estimating basin sizes and calculating shortest paths.

### References


[1] Imai, T. and Tanaka, A.: Weighted Price of Anarchy for the Multi-Agent based Dynamic Network Formation Game, *Proc. 2nd International Joint Agent Workshop & Symposium* (*iJAWS2011*), 2011-18 (2011).

[2] Imai, T. and Tanaka, A.: A Game Theoretic Model for AS Topology Formation with the Scale-Free Property, *IEICE Trans. Inf. Syst.*, Vol.E93.D, No.11, pp.3051–3058 (2010).

[3] Jackson, M.O.: *Social and economic networks*, Princeton University Press (2008).

[4] Jackson, M.O. and Watts, A.: The Evolution of Social and Economic Networks, *Journal of Economic Theory*, Vol.106, No.2, pp.265–295 (online) (2002), available from ⟨http://ideas.repec.org/a/eee/jetheo/v106y2002i2p265-295.html⟩.

[5] Jackson, M.O. and Watts, A.: The existence of pairwise stable networks, *Seoul Journal of Economics*, Vol.14, pp.299–321 (2001).

[6] Jackson, M.O. and Wolinsky, A.: A Strategic Model of Social and Economic Networks, *Journal of Economic Theory*, Vol.71, No.1, pp.44–74 (online) (1996), available from ⟨http://ideas.repec.org/a/eee/jetheo/v71y1996i1p44-74.html⟩.

[7] Myerson, R.B.: Graphs and Cooperation in Games, Discussion Papers 246, Northwestern University, Center for Mathematical Studies in Economics and Management Science (1976).

[8] Koutsoupias, E. and Papadimitriou, C.: Worst-case equilibria, *Proc. 16th Annual Symposium on Theoretical Aspects of Computer Science*, pp.404–413 (1999).

[9] Roughgarden, T. and Tardos, E.: How bad is selfish routing?, *J. ACM*, Vol.49, No.2, pp.236–259 (2002).

[10] Siganos, G.: Power-Laws and the AS-level Internet Topology, *IEEE/ACM, Trans. Networking*, Vol.11, No.4, pp.514–524 (online) (2003), available from ⟨http://ci.nii.ac.jp/naid/80016100323/⟩.

[11] Nisan, N.: *Algorithmic game theory*, Cambridge University Press (2007).

[12] Halpern, J.Y.: Computer Science and Game Theory: A Brief Survey, *CoRR*, Vol. abs/cs/0703148 (2007).

[13] Batagelj, V. and Mrvar, A.: Pajek - Program for Large Network Analysis. available from ⟨http://vlado.fmf.uni-lj.si/pub/networks/pajek/⟩.


**Tetsuo Imai** received his B.E. and M.E. degrees from Hokkaido University, Japan in 2000, 2002 and his Ph.D. degree from Yamagata University in 2012. In 2002, he joined NEC Corporation, Kawasaki, Japan. Since October 2012, he has been a Research Fellow in Yamagata University, Japan. His research interests include computer communication networks, complex networks and complex systems. He is a member of IEICE, IEEE and APS.

**Atsushi Tanaka** received his B.S., M.E, D.E degrees from Tohoku University, Sendai, Japan in 1986, 1988 and 1991 respectively. From 1991 to 1998, he was an Assistant Professor in the department of Electrical and Information Engineering, Faculty of Engineering, Yamagata University. From 1998 to 2003, he was an Associate Professor in the Computing Service Center, Yamagata University. Since April 2003, he has been an Associate Professor in Graduate School of Science and Engineering, Yamagata University. His research interests include complex systems and pattern formation. He is a member of IPSJ, IEICE, JPS and JSIAM.