IPSJ Transactions on Computer Vision and Applications Vol.5 30–34 (July 2013)

[DOI: 10.2197/ipsjtcva.5.30]

Express Paper

Scale-invariant Edge Detection Using Spectral Theory

Gou Koutaki[1,a)] Keiichi Uchimura[2,b)]

Received: March 11, 2013, Accepted: April 24, 2013, Released: July 29, 2013

Abstract: In this paper, we propose a method of scale-invariant edge detection that represents edge images as polynomials in a scale parameter using spectral decomposition (generalized PCA), in order to obtain an optimal local scale. As this proposed method is successfully able to estimate the local scale of each pixel, accurate scale-invariant edge amplitudes and directions can be obtained. Our experimental results show that the proposed method detects rough edge contours in indistinct parts and detailed contours in the clarified parts of test images.

Keywords: scale space, spectral theory, edge detection

1. Introduction

Edge detection is the basic technique for object recognition and low-level feature extraction in computer vision. In previous edge detection research, local mask filters to detect the gradient magnitude of an input image using Roberts or Sobel operators [7] were proposed. Marr and Hildreth [5] introduced the use of a smoothing filter, following which the application of Laplacian of Gaussian (LoG) filters was proposed, while Canny [1] developed a framework optimal filter theory.

Meanwhile, scale-space image theory, in which a Gaussian filter with set scale parameters is used to generate a series of blurred images, has become the primary computer vision image processing technique [3]. Lindeberg proposed the use of edge detection in the context of scale-spaces [4]; however, the size of detected edge contours depends strongly on the scale parameter, with large parameters resulting in rough contours and smaller parameters detecting smoother contours. Therefore, a variety of scale parameter sets should be used for edge detection in a given input image, with scale resolution generally improving in proportion to the number of scale-space parameter sets used. However, computation time also increases in proportion to the number of scale parameters used.

Recently, the application of spectral theory to scale-space compression was proposed [2]. Spectral theory is a generalized form of principal component analysis (PCA) that can be used to efficiently compress Gaussian scale-spaces and is applicable to scale-space image processing involving infinitely large parameter sets.

In this paper, we introduce the use of spectral theory for edge detection in scale-space and demonstrate a novel application of scale-invariant edge detection.

1 Priority Organization for Innovation and Excellence, Kumamoto University, Kumamoto 860–8555, Japan
2 Graduate School of Science and Technology, Kumamoto University, Kumamoto 860–8555, Japan
a) koutaki@cs.kumamoto-u.ac.jp
b) uchimura@cs.kumamoto-u.ac.jp

2. Edge Detection

2.1 Classic Edge Detection

A local change of image intensity (edge) for an input 2D-image $f(x, y)$ can be defined as:

$$\nabla f(x, y) = \left(\frac{\partial f(x, y)}{\partial x}, \frac{\partial f(x, y)}{\partial y}\right) \equiv \left(f_x, f_y\right), \tag{1}$$

where f_x and f_y are the x- and y-derivatives, respectively, of the image function. The edge amplitude $Amp(x, y)$ and edge direction $Dir(x, y)$ of each pixel (x, y) can then be respectively defined as:

$$Amp(x, y) = \sqrt{f_x^2 + f_y^2},$$
$$Dir(x, y) = \tan^{-1} \frac{f_y}{f_x}$$

In image recognition, edge pixels (edge contours) are detected as local features or namely as neighborhood maxima of edge amplitude $Amp(x, y)$ with given values of edge direction $Dir(x, y)$, often by means of eight-direction quantization.

2.2 Edge Detection on the Scale-space

An additional scale parameter s is used for edge detection in scale-space. The process involves rendering an input image $f(x, y)$ in scale space using

$$L(x, y; s) = g(x, y; s) * f, \tag{2}$$

where $*$ is convolution operator and $g(x, y; s)$ is a gaussian kernel defined as:

$$g(x, y; s) = \frac{1}{2\pi s} e^{-\frac{x^2+y^2}{2s}}. \tag{3}$$

A scale normalized differential operator, such as $\partial^s \equiv s\partial$, can be used as the edge operator on the scale-space. From this, the x- and y-derivative images on the scale-space can be respectively defined as:

$$L_x(x, y; s) = \frac{s\partial g(x, y; s)}{\partial x} * f = \left(-\frac{x}{2\pi s^3} e^{-\frac{x^2+y^2}{2s}}\right) * f,$$
$$L_y(x, y; s) = \frac{s\partial g(x, y; s)}{\partial y} * f = \left(-\frac{y}{2\pi s^3} e^{-\frac{x^2+y^2}{2s}}\right) * f.$$

Similarly, the edge amplitude and direction on the scale-space can be respectively defined as

$$Amp(x, y; s) = \sqrt{L_x^2 + L_y^2},$$
$$Dir(x, y; s) = \tan^{-1} \frac{L_y}{L_x}.$$

Note that if a large value of the scale parameter s is used in these equations, then rough edge contours will be detected, while a smaller scale parameter will result in more detailed edge contours.

3. Proposed Method

As described in above section, the edge smoothness in a given scale-space depends on the scale parameter s. Accordingly, a local scale $s^*(x, y)$ can be defined for each pixel (x, y) and be used to detect edge amplitude and direction, allowing for the definition of scale-invariant edges:

$$s^*(x, y) = \arg\max_s Amp(x, y; s),$$
$$Amp^*(x, y) = Amp(x, y; s^*),$$
$$Dir^*(x, y) = Dir(x, y; s^*).$$

The nonlinearity of $Amp(x, y; s)$ combined with the continuity of the scale parameter s complicates the task of finding an optimal value of s^*; however, if the differential operator G_x in Eq. (4) is represented as the sum of a series of polynomials in s:

$$G_x(x, y; s) \equiv -\frac{x}{2\pi s^3} e^{-\frac{x^2+y^2}{2s}} = s^0 q_0(x, y) + s^1 \cdot q_1(x, y) + \cdots + s^N \cdot q_N(x, y), \quad (4)$$

then $Amp(x, y; s)$ can also be represented using polynomials in s, making it simple to obtain an exact optimal local scale s^* in Eq. (4).

To develop an exact polynomial representation of G_x in terms of s in the case where there is a finite number of scale parameters (i.e., $s^1, s^2, \cdots, s^N$), it is possible to use a subspace method [9] to solve an $N \times N$ matrix-based eigenproblem in order to express the original operator G_x as a linear combinations of eigenvectors and eigenvalues:

$$\mathbf{C}\varphi = \lambda\varphi. \quad (5)$$

The matrix $\mathbf{C}$ is a covariance matrix with its i-th row and j-th column elements defined by

$$C_{ij} = \left\langle G_x(x, y; s_i), G_x(x, y; s_j)\right\rangle \equiv \iint G_x(x, y; s_i) G_x(x, y; s_j) dxdy. \quad (6)$$

However, because the scale parameter s is continuous, it is difficult to apply this matrix-based PCA to scale-space compression. In the case where $N \to \infty$, it is necessary to expand the eigenproblem. In mathematical function analysis, this approach is known as spectral theory [8]. By applying spectral theory to Eq. (5), the matrix eigenproblem can be transformed into the following Fredholm integral equation:

$$\int K(s, t)\varphi(s) ds = \lambda\varphi(t). \quad (7)$$

where $K(t, s)$ is the integral kernel corresponding to a covariance matrix of Eq. (5), and $K(t, s)$ is defined as:

$$K(s, t) = \iint G_x(x, y; s) G_x(x, y; t) dxdy = \frac{1}{2\pi\left(\frac{1}{s^2} + \frac{1}{t^2}\right)(st)^3}. \quad (8)$$

If this integral kernel is non-zero, symmetric, and finite, Eq. (7) has a unique solution. Nevertheless, the integral equation remains difficult to solve exactly without using a set of specific integral kernels. Therefore, we propose a solution by using a polynomial approximation:

$$\varphi_i(s) = s^0 a_i^0 + s^1 a_{i,1} + s^2 a_{i,2} + \cdots + s^N a_{i,N} = \left(1, s, s^2, \cdots, s^N\right) \cdot \boldsymbol{a_i}. \quad (9)$$

By multiplying both sides of Eq. (7) with the polynomials $1, s, s^2, \cdots s^N$ and then integrating, Eq. (7) is transformed into the following generalized eigenproblem of an $(N+1)\times(N+1)$ matrix:

$$\mathbf{K}\boldsymbol{a} = \lambda\mathbf{S}\boldsymbol{a}. \quad (10)$$

The elements of $\mathbf{K}$ here are defined as:

$$K_{i+1\,j+1} = \frac{1}{2\pi} \iint \frac{s^j t^i}{\left(\frac{1}{s^2} + \frac{1}{t^2}\right)(st)^3} dsdt, \quad (11)$$

$$S_{i+1\,j+1} = \int s^{i+j} ds = \frac{s^{1+i+j}}{1 + i + j}. \quad (12)$$

By solving the above eigenproblem, we can obtain the eigensolutions with which G_x can be represented using the following Fourier series:

$$G_x(x, y; s) = \sum_{i=0}^{N} \langle G_x(x, y; s), \varphi_i(s)\rangle \varphi_i(s) \equiv F_{x,i}(x, y) \cdot \varphi_i(s), \quad (13)$$

in which $F_{x,i}(x, y)$ (the eigenimage) is defined as:

$$F_{x,i}(x, y) = \int_{s1}^{s2} G_x(x, y; s)\varphi_i(s) ds = -\sum_{n=0}^{N} \frac{-a_{i,n} x}{2^{3/2}\pi r} \left(\frac{r}{2^{1/2}}\right)^{n-1} \Gamma\left(\frac{2-n}{2}, \frac{r^2}{2s_1^2}, \frac{r^2}{2s_2^2}\right), \quad (14)$$

where $r = \sqrt{x^2 + y^2}$, and Γ is a complete gamma function defined as:

$$\Gamma(a, t_1, t_2) = \int_{t_1}^{t_2} t^{a-1} \exp(-t) dt \quad (15)$$

that can be calculated accurately using a continued fraction expansion [6].

In the same way, the eigenimage of the y-differential operator G_y can be defined as:

$$F_{y,i}(x,y) = \int_{s1}^{s2} G_y(x,y;s)\varphi_i(s)\,ds$$
$$= -\sum_{n=0}^{N} \frac{-a_{i,n}y}{2^{3/2}\pi r}\left(\frac{r}{2^{1/2}}\right)^{n-1}\Gamma\left(\frac{2-n}{2}, \frac{r^2}{2s_1^2}, \frac{r^2}{2s_2^2}\right). \qquad (16)$$

Finally, the x-and y-derivative images on the scale-space, L_x and L_y, can be respectively represented in polynomials of s as:

$$L_x(x,y;s) = \sum_{i=0}^{N} (F_{x,i}(x,y) * f) \times \left(a_{i,0} + a_{i,1}s + \cdots a_{i,N}s^N\right), \qquad (17)$$
$$L_y(x,y;s) = \sum_{i=0}^{N} \left(F_{y,i}(x,y) * f\right) \times \left(a_{i,0} + a_{i,1}s + \cdots a_{i,N}s^N\right). \qquad (18)$$

As discussed in Section 2.2 above, the local scale s^*, the scale-invariant edge amplitude $Amp(x,y;s^*)$, and the edge direction $Dir(x,y;s^*)$ can then be derived.

4. Numerical Examples

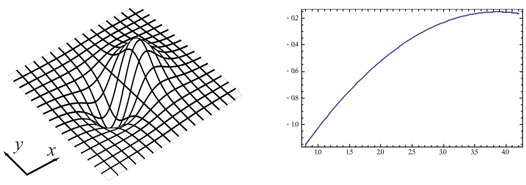
In this section, we show numerical examples of eigensolutions of Eq. (7). In order to approximate the eigenfunction of Eq. (9), we use second-order polynomials ($N = 2$) and set the integral range of the scale parameter s to $s_1 = 0.8, s_2 = 4.2$. Based on this, we can solve the 3×3 matrix-generalized eigenproblem of Eq. (10).

The solutions $a_{i,j}$ and eigenvalues λ_i for $N = 2$ are shown in **Table 1**, from which it can be seen that $\lambda_2 = 0.0007$ is only 2 [%] of $\lambda_0 = 0.0309$.

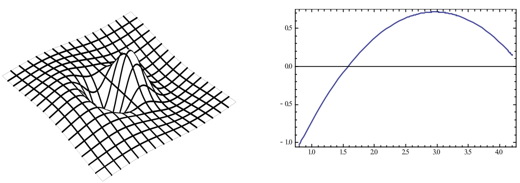
Based on this rapid decrease, it is apparent that the original Gaussian derivative function can be approximated by using a polynomial series of relatively small degree. The eigenimages for $N = 2$ are shown in **Fig. 1**. The left part of the figure $F_{x,i}$ shows the eigenimages on the xy-plane, while the right side shows the eigensolution φ_i. Note that the eigenimage becomes an odd function and the number of wave-like mountains increases as the degree increases.

Table 1 Solutions for $N = 2$.

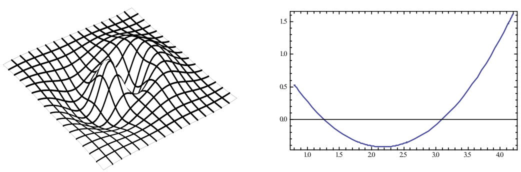
i	$a_{i,0}$	$a_{i,1}$	$a_{i,2}$	λ_i
0	−1.73812	0.82005	−0.10597	0.0309
1	−2.54528	2.19890	−0.37072	0.0070
2	−1.94300	−2.16838	0.49750	0.0007



(a) 1st Eigen image (left) and eigen function (right)



(b) 2nd Eigen image (left) and eigen function (right)



(c) 3rd Eigen image (left) and eigen function (right)

Fig. 1 Eigenimages and eigenfunctions for G_x.

5. Experimental Results

We performed edge detection on two images, **Fig. 2** (a) and **Fig. 3** (a). Figure 2 (a) is a 210×210 pixel, 8-bit gray-scale input image used to obtain experimental results in which the illumination change on a section of skin surface is loose and the hair has many edges. Figures 2 (b) and (c) show the results of edge detection using fixed scale parameters $s = 1.2$ and $s = 3.5$, respectively. From left to right, the figures show the x-derivative image, the y-derivative image, the edge amplitude, and the edge contour. It can be seen that scale factor $s = 1.2$ successfully extracts the edge of the hair, but over-edge detection occurs on the skin, while using $s = 3.5$ leads to many details on the edge of the hair being missed.

On the other hand, using the proposed method allows for detailed detection of the edges of the hair while suppressing edges in the skin (Fig. 2 (d)). The left side of Fig. 2 (d) shows the estimated local scale s^* using pseudocoloring.

Figure 3 shows the results for the second image, Fig. 3 (a), a 454×308 pixel input. The doll shown to the left has many sharp edges while the shadowing on the right is indistinct. Figures 3 (b) and (c) show edge contours detected using fixed parameters $s = 1.2$ and $s = 3.5$, respectively; for $s = 1.2$, the edge contours on the shadowed section on the right have been divided, and for $s = 3.5$, the edge contours on the hand and arm of the doll are broken off.

By contrast, the proposed method estimates local scales appropriately, with both the small scales on the doll and the large scales on the shadow detected correctly (Fig. 3 (d)).

6. Conclusion

In this paper, we propose a method of scale-invariant edge detection that represents edge images as polynomials in a scale parameter s using spectral decomposition, a generalized PCA, in order to obtain an optimal local scale.

As this proposed method is successfully able to estimate the local scale of each pixel, accurate scale-invariant edge amplitudes and directions can be obtained. Our experimental results show that the proposed method detects rough edge contours in indistinct parts and detailed contours in the clarified parts of test images.

In our future research, we plan to evaluate the proposed method quantitatively in terms of linearity of estimated scale in a scale-adjusted input image.

Acknowledgments This work was supported by Grant-in-Aid for Young Scientists (B) No.25730116 from MEXT Japan.

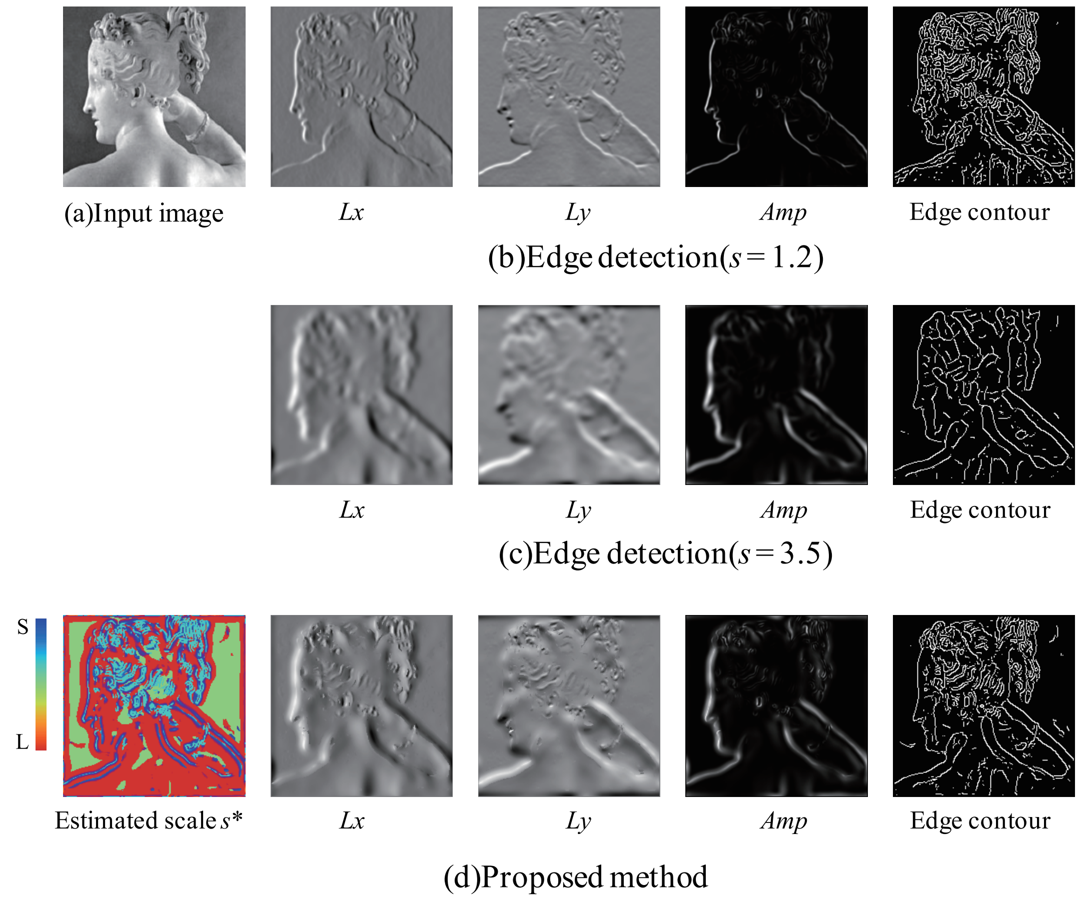


Fig. 2 Results of edge detection for venus.

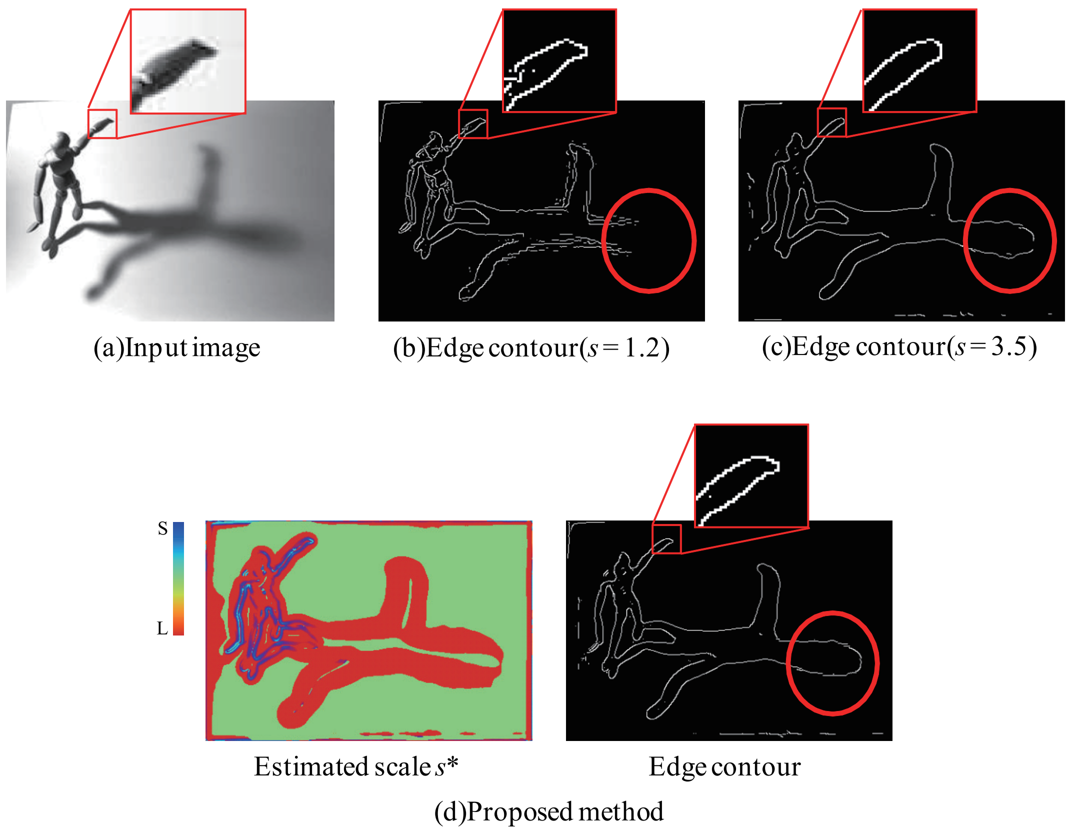


Fig. 3 Results of edge detection for doll.

References

[1] Canny, J.: A Computational Approach to Edge Detection, *IEEE Trans. Pattern Anal. Mach. Intell.*, Vol.8, No.6, pp.679–698 (1986).

[2] Koutaki, G. and Uchimura, K.: Application to Pattern Matching Using Spectrum Theory (in Japanese), *The 15th Meeting on Image Recognition and Understanding* (*MIRU*), IEICE (2012).

[3] Lindeberg, T.: *Scale-Space Theory in Computer Vision*, Kluwer Academic Publishers, Norwell, MA, USA (1994).

[4] Lindeberg, T.: Edge Detection and Ridge Detection with Automatic Scale Selection, *International Journal of Computer Vision*, Vol.30, pp.465–470 (1996).

[5] Marr, D. and Hildreth, E.: Theory of Edge Detection, *Proc. Royal Society of London. Series B, Biological Sciences*, Vol.207, No.1167, pp.187–217 (1980).

[6] Press, W.H., Teukolsky, S.A., Vetterling, W.T. and Flannery, B.P.: *Numerical Recipes 3rd Edition: The Art of Scientific Computing*, Cambridge University Press, New York, NY, USA (2007).

[7] Roberts, L.G.: *Machine Perception of Three-dimensional Solids*, MIT Press (1965).

[8] Shigeru, M.: *Introduction to Integral Equations in Japanese*, Asakura Press (1968).

[9] Turk, M. and Pentland, A.: Eigenfaces for recognition, *J. Cognitive Neuroscience*, Vol.3, No.1, pp.71–86 (1991).

(Communicated by *Shinichiro Omachi*)