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# 均衡型 $(C_5, C_{14})$-Foil デザインと関連デザイン

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グラフ理論において、グラフの分解問題は主要な研究テーマである。$C_5$ を５点を通るサイクル、$C_{14}$ を１４点を通るサイクルとする。１点を共有する辺素な $t$ 個の $C_5$ と $t$ 個の $C_{14}$ からなるグラフを $(C_5, C_{14})$-2$t$-foil という。本研究では、完全グラフ $K_n$ を 均衡的に $(C_5, C_{14})$-2$t$-foil 部分グラフに分解する均衡型 $(C_5, C_{14})$-foil デザインについて述べる。さらに、均衡型 $C_{19}$-foil デザイン、均衡型 $C_{38}$-foil デザイン、均衡型 $C_{57}$-foil デザイン、均衡型 $C_{76}$-foil デザイン、均衡型 $C_{95}$-foil デザイン、均衡型 $C_{114}$-foil デザイン、均衡型 $C_{133}$-foil デザイン、均衡型 $C_{152}$-foil デザイン、均衡型 $C_{171}$-foil デザイン、均衡型 $C_{190}$-foil デザインについて述べる。

# Balanced $(C_5, C_{14})$-Foil Designs and Related Designs

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In graph theory, the decomposition problem of graphs is a very important topic. Various type of decompositions of many graphs can be seen in the literature of graph theory. This paper gives balanced $(C_5, C_{14})$-foil designs, balanced $C_{19}$-foil designs, and balanced $C_{38}$-foil designs, and balanced $C_{57}$-foil designs, and balanced $C_{76}$-foil designs, and balanced $C_{95}$-foil designs, and balanced $C_{114}$-foil designs, and balanced $C_{133}$-foil designs, and balanced $C_{152}$-foil designs, and balanced $C_{171}$-foil designs, and balanced $C_{190}$-foil designs.

## 1. Balanced $(C_5, C_{14})$-Foil Designs

Let $K_n$ denote *the complete graph* of $n$ vertices. Let $C_5$ and $C_{14}$ be *the* 5-*cycle* and *the* 14-*cycle*, respectively. *The* $(C_5, C_{14})$-2$t$-*foil* is a graph of $t$ edge-disjoint $C_5$'s and $t$ edge-disjoint $C_{14}$'s with a common vertex and the common vertex is called *the center of the* $(C_5, C_{14})$-2$t$-*foil*. When $K_n$ is decomposed into edge-disjoint sum of $(C_5, C_{14})$-2$t$-foils and every vertex of $K_n$ appears in the same number of $(C_5, C_{14})$-2$t$-foils, we say that $K_n$ *has a balanced* $(C_5, C_{14})$-2$t$-*foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced* $(C_5, C_{14})$-*foil design*.

**Theorem 1.** $K_n$ has a balanced $(C_5, C_{14})$-2$t$-foil design if and only if $n \equiv 1 \pmod{38t}$.

**Proof.** **(Necessity)** Suppose that $K_n$ has a balanced $(C_5, C_{14})$-2$t$-foil decomposition. Let $b$ be the number of $(C_5, C_{14})$-2$t$-foils and $r$ be the replication number. Then $b = n(n-1)/38t$ and $r = (17t+1)(n-1)/38t$. Among $r$ $(C_5, C_{14})$-2$t$-foils having a vertex $v$ of $K_n$, let $r_1$ and $r_2$ be the numbers of $(C_5, C_{14})$-2$t$-foils in which $v$ is the center and $v$ is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to $v$, $4tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/38t$ and $r_2 = 17(n-1)/38$. Therefore, $n \equiv 1 \pmod{38t}$ is necessary.

**(Sufficiency)** Put $n = 38st + 1$ and $T = st$. Then $n = 38T + 1$. Construct a $(C_5, C_{14})$-2$T$-foil as follows:

$\{(38T+1, T, 16T, 36T+1, 16T+1), (38T+1, 10T+1, 11T+2, 17T+2, 24T+3, 29T+3, 2T+3, 18T+3, 5T+3, 34T+3, 30T+3, 28T+2, 21T+2, 13T+1)\} \cup$

$\{(38T+1, T-1, 16T-2, 36T, 16T+2), (38T+1, 10T+2, 11T+4, 17T+3, 24T+5, 29T+4, 2T+5, 18T+4, 5T+5, 34T+4, 30T+5, 28T+3, 21T+4, 13T+2)\} \cup$

$\{(38T+1, T-2, 16T-4, 36T-1, 16T+3), (38T+1, 10T+3, 11T+6, 17T+4, 24T+7, 29T+5, 2T+7, 18T+5, 5T+7, 34T+5, 30T+7, 28T+4, 21T+6, 13T+3)\} \cup$

$\ldots \cup$

$\{(38T+1, 2, 14T+4, 35T+3, 17T-1), (38T+1, 11T-1, 13T-2, 18T, 26T-1, 30T+1, 4T-1, 19T+1, 7T-1, 35T+1, 32T-1, 29T, 23T-2, 14T-1)\} \cup$

$\{(38T+1, 1, 14T+2, 35T+2, 17T), (38T+1, 11T, 13T, 18T+1, 26T+1, 30T+2, 4T+1, 19T+2, 7T+1, 4T(35T+2), 32T+1, 29T+1, 23T, 14T)\}$.

Decompose the $(C_5, C_{14})$-2$T$-foil into $s$ $(C_5, C_{14})$-2$t$-foils. Then these starters comprise a balanced $(C_5, C_{14})$-2$t$-foil decomposition of $K_n$.

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**Example 1.1. Balanced $(C_5, C_{14})$-2-foil design of $K_{39}$.**
$\{(39, 1, 16, 37, 17), (39, 11, 13, 19, 27, 32, 5, 21, 8, 4, 33, 30, 23, 14)\}$.
This starter comprises a balanced $(C_5, C_{14})$-2-foil decomposition of $K_{39}$.

**Example 1.2. Balanced $(C_5, C_{14})$-4-foil design of $K_{77}$.**
$\{(77, 2, 32, 73, 33), (77, 21, 24, 36, 51, 61, 7, 39, 13, 71, 63, 58, 44, 27)\} \cup$
$\{(77, 1, 30, 72, 34), (77, 22, 26, 37, 53, 62, 9, 40, 15, 8, 65, 59, 46, 28)\}$.
This starter comprises a balanced $(C_5, C_{14})$-4-foil decomposition of $K_{77}$.

**Example 1.3. Balanced $(C_5, C_{14})$-6-foil design of $K_{115}$.**
$\{(115, 3, 48, 109, 49), (115, 31, 35, 53, 75, 90, 9, 57, 18, 105, 93, 86, 65, 40)\} \cup$
$\{(115, 2, 46, 108, 50), (115, 32, 37, 54, 77, 91, 11, 58, 20, 106, 95, 87, 67, 41)\} \cup$
$\{(115, 1, 44, 107, 51), (115, 33, 39, 55, 79, 92, 13, 59, 22, 12, 97, 88, 69, 42)\}$.
This starter comprises a balanced $(C_5, C_{14})$-6-foil decomposition of $K_{115}$.

**Example 1.4. Balanced $(C_5, C_{14})$-8-foil design of $K_{153}$.**
$\{(153, 4, 64, 145, 65), (153, 41, 46, 70, 99, 119, 11, 75, 23, 139, 123, 114, 86, 53)\} \cup$
$\{(153, 3, 62, 144, 66), (153, 42, 48, 71, 101, 120, 13, 76, 25, 140, 125, 115, 88, 54)\} \cup$
$\{(153, 2, 60, 143, 67), (153, 43, 50, 72, 103, 121, 15, 77, 27, 141, 127, 116, 90, 55)\} \cup$
$\{(153, 1, 58, 142, 68), (153, 44, 52, 73, 105, 122, 17, 78, 29, 16, 129, 117, 92, 56)\}$.
This starter comprises a balanced $(C_5, C_{14})$-8-foil decomposition of $K_{153}$.

**Example 1.5. Balanced $(C_5, C_{14})$-10-foil design of $K_{191}$.**
$\{(191, 5, 80, 181, 81), (191, 51, 57, 87, 123, 148, 13, 93, 28, 173, 153, 142, 107, 66)\} \cup$
$\{(191, 4, 78, 180, 82), (191, 52, 59, 88, 125, 149, 15, 94, 30, 174, 155, 143, 109, 67)\} \cup$
$\{(191, 3, 76, 179, 83), (191, 53, 61, 89, 127, 150, 17, 95, 32, 175, 157, 144, 111, 68)\} \cup$
$\{(191, 2, 74, 178, 84), (191, 54, 63, 90, 129, 151, 19, 96, 34, 176, 159, 145, 113, 69)\} \cup$
$\{(191, 1, 72, 177, 85), (191, 55, 65, 91, 131, 152, 21, 97, 36, 20, 161, 146, 115, 70)\}$.
This starter comprises a balanced $(C_5, C_{14})$-10-foil decomposition of $K_{191}$.

**Example 1.6. Balanced $(C_5, C_{14})$-12-foil design of $K_{229}$.**
$\{(229, 6, 96, 217, 97), (229, 61, 68, 104, 147, 177, 15, 111, 33, 207, 183, 170, 128, 79)\} \cup$
$\{(229, 5, 94, 216, 98), (229, 62, 70, 105, 149, 178, 17, 112, 35, 208, 185, 171, 130, 80)\} \cup$
$\{(229, 4, 92, 215, 99), (229, 63, 72, 106, 151, 179, 19, 113, 37, 209, 187, 172, 132, 81)\} \cup$
$\{(229, 3, 90, 214, 100), (229, 64, 74, 107, 153, 180, 21, 114, 39, 210, 189, 173, 134, 82)\} \cup$
$\{(229, 2, 88, 213, 101), (229, 65, 76, 108, 155, 181, 23, 115, 41, 211, 191, 174, 136, 83)\} \cup$
$\{(229, 1, 86, 212, 102), (229, 66, 78, 109, 157, 182, 25, 116, 43, 24, 193, 175, 138, 84)\}$.
This starter comprises a balanced $(C_5, C_{14})$-12-foil decomposition of $K_{229}$.

**Example 1.7. Balanced $(C_5, C_{14})$-14-foil design of $K_{267}$.**
$\{(267, 7, 112, 253, 113), (267, 71, 79, 121, 171, 206, 17, 129, 38, 241, 213, 198, 149, 92)\} \cup$
$\{(267, 6, 110, 252, 114), (267, 72, 81, 122, 173, 207, 19, 130, 40, 242, 215, 199, 151, 93)\} \cup$
$\{(267, 5, 108, 251, 115), (267, 73, 83, 123, 175, 208, 21, 131, 42, 243, 217, 200, 153, 94)\} \cup$
$\{(267, 4, 106, 250, 116), (267, 74, 85, 124, 177, 209, 23, 132, 44, 244, 219, 201, 155, 95)\} \cup$
$\{(267, 3, 104, 249, 117), (267, 75, 87, 125, 179, 210, 25, 133, 46, 245, 221, 202, 157, 96)\} \cup$
$\{(267, 2, 102, 248, 118), (267, 76, 89, 126, 181, 211, 27, 134, 48, 246, 223, 203, 159, 97)\} \cup$
$\{(267, 1, 100, 247, 119), (267, 77, 91, 127, 183, 212, 29, 135, 50, 28, 225, 204, 161, 98)\}$.
This starter comprises a balanced $(C_5, C_{14})$-14-foil decomposition of $K_{267}$.

## 2. Balanced $C_{19}$-Foil Designs

Let $C_{19}$ be *the cycle* on 19 vertices. *The $C_{19}$-$t$-foil* is a graph of $t$ edge-disjoint $C_{19}$'s with a common vertex and the common vertex is called *the center of the $C_{19}$-$t$-foil*. When $K_n$ is decomposed into edge-disjoint sum of $C_{19}$-$t$-foils and every vertex of $K_n$ appears in the same number of $C_{19}$-$t$-foils, it is called that *$K_n$ has a balanced $C_{19}$-$t$-foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced $C_{19}$-foil design*.

**Theorem 2.** $K_n$ has a balanced $C_{19}$-$t$-foil design if and only if $n \equiv 1 \pmod{38t}$.

**Proof. (Necessity)** Suppose that $K_n$ has a balanced $C_{19}$-$t$-foil decomposition. Let $b$ be the number of $C_{19}$-$t$-foils and $r$ be the replication number. Then $b = n(n-1)/38t$ and

$r = (18t+1)(n-1)/38t$. Among $r$ $C_{19}$-$t$-foils having a vertex $v$ of $K_n$, let $r_1$ and $r_2$ be the numbers of $C_{19}$-$t$-foils in which $v$ is the center and $v$ is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to $v$, $2tr_1 + 2r_2 = n-1$. From these relations, $r_1 = (n-1)/38t$ and $r_2 = 18(n-1)/38$. Therefore, $n \equiv 1 \pmod{38t}$ is necessary.

**(Sufficiency)** Put $n = 38st+1, T = st$. Then $n = 38T+1$. Construct a $C_{19}$-$T$-foil as follows:

{ $(38T+1, T, 16T, 36T+1, 16T+1, 26T+2, 10T+1, 11T+2, 17T+2, 24T+3, 29T+3, 2T+3, 18T+3, 5T+3, 34T+3, 30T+3, 28T+2, 21T+2, 13T+1)$,

$(38T+1, T-1, 16T-2, 36T, 16T+2, 26T+4, 10T+2, 11T+4, 17T+3, 24T+5, 29T+4, 2T+5, 18T+4, 5T+5, 34T+4, 30T+5, 28T+3, 21T+4, 13T+2)$,

$(38T+1, T-2, 16T-4, 36T-1, 16T+3, 26T+6, 10T+3, 11T+6, 17T+4, 24T+7, 29T+5, 2T+7, 18T+5, 5T+7, 34T+5, 30T+7, 28T+4, 21T+6, 13T+3)$,

...,

$(38T+1, 2, 14T+4, 35T+3, 17T-1, 28T-2, 11T-1, 13T-2, 18T, 26T-1, 30T+1, 4T-1, 19T+1, 7T-1, 35T+1, 32T-1, 29T, 23T-2, 14T-1)$,

$(38T+1, 1, 14T+2, 35T+2, 17T, 28T, 11T, 13T, 18T+1, 26T+1, 30T+2, 4T+1, 19T+2, 7T+1, 4T(35T+2), 32T+1, 29T+1, 23T, 14T)$ }.

Decompose this $C_{19}$-$T$-foil into $s$ $C_{19}$-$t$-foils. Then these starters comprise a balanced $C_{19}$-$t$-foil decomposition of $K_n$.

**Example 2.1. Balanced $C_{19}$ design of $K_{39}$.**

{(39, 1, 16, 37, 17, 28, 11, 13, 19, 27, 32, 5, 21, 8, 4, 33, 30, 23, 14)}.

This stater comprises a balanced $C_{19}$-decomposition of $K_{39}$.

**Example 2.2. Balanced $C_{19}$-2-foil design of $K_{77}$.**

{(77, 2, 32, 73, 33, 54, 21, 24, 36, 51, 61, 7, 39, 13, 71, 63, 58, 44, 27),

(77, 1, 30, 72, 34, 56, 22, 26, 37, 53, 62, 9, 40, 15, 8, 65, 59, 46, 28)}.

This stater comprises a balanced $C_{19}$-2-foil decomposition of $K_{77}$.

**Example 2.3. Balanced $C_{19}$-3-foil design of $K_{115}$.**

{(115, 3, 48, 109, 49, 80, 31, 35, 53, 75, 90, 9, 57, 18, 105, 93, 86, 65, 40),

(115, 2, 46, 108, 50, 82, 32, 37, 54, 77, 91, 11, 58, 20, 106, 95, 87, 67, 41),

(115, 1, 44, 107, 51, 84, 33, 39, 55, 79, 92, 13, 59, 22, 12, 97, 88, 69, 42)}.

This stater comprises a balanced $C_{19}$-3-foil decomposition of $K_{115}$.

**Example 2.4. Balanced $C_{19}$-4-foil design of $K_{153}$.**

{(153, 4, 64, 145, 65, 106, 41, 46, 70, 99, 119, 11, 75, 23, 139, 123, 114, 86, 53),

(153, 3, 62, 144, 66, 108, 42, 48, 71, 101, 120, 13, 76, 25, 140, 125, 115, 88, 54),

(153, 2, 60, 143, 67, 110, 43, 50, 72, 103, 121, 15, 77, 27, 141, 127, 116, 90, 55),

(153, 1, 58, 142, 68, 112, 44, 52, 73, 105, 122, 17, 78, 29, 16, 129, 117, 92, 56)}.

This stater comprises a balanced $C_{19}$-4-foil decomposition of $K_{153}$.

**Example 2.5. Balanced $C_{19}$-5-foil design of $K_{191}$.**

{(191, 5, 80, 181, 81, 132, 51, 57, 87, 123, 148, 13, 93, 28, 173, 153, 142, 107, 66),

(191, 4, 78, 180, 82, 134, 52, 59, 88, 125, 149, 15, 94, 30, 174, 155, 143, 109, 67),

(191, 3, 76, 179, 83, 136, 53, 61, 89, 127, 150, 17, 95, 32, 175, 157, 144, 111, 68),

(191, 2, 74, 178, 84, 138, 54, 63, 90, 129, 151, 19, 96, 34, 176, 159, 145, 113, 69),

(191, 1, 72, 177, 85, 140, 55, 65, 91, 131, 152, 21, 97, 36, 20, 161, 146, 115, 70)}.

This stater comprises a balanced $C_{19}$-5-foil decomposition of $K_{191}$.

**Example 2.6. Balanced $C_{19}$-6-foil design of $K_{229}$.**

{(229, 6, 96, 217, 97, 158, 61, 68, 104, 147, 177, 15, 111, 33, 207, 183, 170, 128, 79),

(229, 5, 94, 216, 98, 160, 62, 70, 105, 149, 178, 17, 112, 35, 208, 185, 171, 130, 80),

(229, 4, 92, 215, 99, 162, 63, 72, 106, 151, 179, 19, 113, 37, 209, 187, 172, 132, 81),

(229, 3, 90, 214, 100, 164, 64, 74, 107, 153, 180, 21, 114, 39, 210, 189, 173, 134, 82),

(229, 2, 88, 213, 101, 166, 65, 76, 108, 155, 181, 23, 115, 41, 211, 191, 174, 136, 83),

(229, 1, 86, 212, 102, 168, 66, 78, 109, 157, 182, 25, 116, 43, 24, 193, 175, 138, 84)}.

This stater comprises a balanced $C_{19}$-6-foil decomposition of $K_{229}$.

## 3. Balanced $C_{19m}$-Foil Designs

Let $C_{19m}$ be *the cycle* on $19m$ vertices. *The $C_{19m}$-$t$-foil* is a graph of $t$ edge-disjoint $C_{19m}$'s with a common vertex and the common vertex is called *the center of the $C_{19m}$-$t$-foil*. When $K_n$ is decomposed into edge-disjoint sum of $C_{19m}$-$t$-foils and every vertex of $K_n$ appears in the same number of $C_{19m}$-$t$-foils, it is called that $K_n$ *has a balanced $C_{19m}$-$t$-foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced $C_{19m}$-foil design*.

**Theorem 3.** $K_n$ has a balanced $C_{38}$-$t$-foil design if and only if $n \equiv 1 \pmod{76t}$.

**Example 3.1. Balanced $C_{38}$ design of $K_{77}$.**
{(77, 2, 32, 73, 33, 54, 21, 24, 36, 51, 61, 7, 39, 13, 71, 63, 58, 44, 27, 55,
28, 46, 59, 65, 8, 15, 40, 9, 62, 53, 37, 26, 22, 56, 34, 72, 30, 1)}.
This stater comprises a balanced $C_{38}$ decomposition of $K_{77}$.

**Example 3.2. Balanced $C_{38}$-2-foil design of $K_{153}$.**
{(153, 4, 64, 145, 65, 106, 41, 46, 70, 99, 119, 11, 75, 23, 139, 123, 114, 86, 53, 107,
54, 88, 115, 125, 140, 25, 76, 13, 120, 101, 71, 48, 42, 108, 66, 144, 62, 3),
(153, 2, 60, 143, 67, 110, 43, 50, 72, 103, 121, 15, 77, 27, 141, 127, 116, 90, 55, 111,
56, 92, 117, 129, 16, 29, 78, 17, 122, 105, 73, 52, 44, 112, 68, 142, 58, 1)}.
This stater comprises a balanced $C_{38}$-2-foil decomposition of $K_{153}$.

**Example 3.3. Balanced $C_{38}$-3-foil design of $K_{229}$.**
{(229, 6, 96, 217, 97, 158, 61, 68, 104, 147, 177, 15, 111, 33, 207, 183, 170, 128, 79, 159,
80, 130, 171, 185, 208, 35, 112, 17, 178, 149, 105, 70, 62, 160, 98, 216, 94, 5),
(229, 4, 92, 215, 99, 162, 63, 72, 106, 151, 179, 19, 113, 37, 209, 187, 172, 132, 81, 163,
82, 134, 173, 189, 210, 39, 114, 21, 180, 153, 107, 74, 64, 164, 100, 214, 90, 3),
(229, 2, 88, 213, 101, 166, 65, 76, 108, 155, 181, 23, 115, 41, 211, 191, 174, 136, 83, 167,
84, 138, 175, 193, 24, 43, 116, 25, 182, 157, 109, 78, 66, 168, 102, 212, 86, 1)}.
This stater comprises a balanced $C_{38}$-3-foil decomposition of $K_{229}$.

**Example 3.4. Balanced $C_{38}$-4-foil design of $K_{305}$.**
{(305, 8, 128, 289, 129, 210, 81, 90, 138, 195, 235, 19, 147, 43, 275, 243, 226, 170, 105, 211,
106, 172, 227, 245, 276, 45, 148, 21, 236, 197, 139, 92, 82, 212, 130, 288, 126, 7),
(305, 6, 124, 287, 131, 214, 83, 94, 140, 199, 237, 23, 149, 47, 277, 247, 228, 174, 107, 215,
108, 176, 229, 249, 278, 49, 150, 25, 238, 201, 141, 96, 84, 216, 132, 286, 122, 5),
(305, 4, 120, 285, 133, 218, 85, 98, 142, 203, 239, 27, 151, 51, 279, 251, 230, 178, 109, 219,
110, 180, 231, 253, 280, 53, 152, 29, 240, 205, 143, 100, 86, 220, 134, 284, 118, 3),
(305, 2, 116, 283, 135, 222, 87, 102, 144, 207, 241, 31, 153, 55, 281, 255, 232, 182, 111, 223,
112, 184, 233, 257, 32, 57, 154, 33, 242, 209, 145, 104, 88, 224, 136, 282, 114, 1)}.
This stater comprises a balanced $C_{38}$-4-foil decomposition of $K_{305}$.

**Theorem 4.** $K_n$ has a balanced $C_{57}$-$t$-foil design if and only if $n \equiv 1 \pmod{114t}$.

**Example 4.1. Balanced $C_{57}$ design of $K_{115}$.**
{(115, 3, 48, 109, 49, 80, 31, 35, 53, 75, 90, 9, 57, 18, 105, 93, 86, 65, 40, 81,
41, 67, 87, 95, 106, 20, 58, 11, 91, 77, 54, 37, 32, 82, 50, 108, 46, 2, 45,
43, 44, 107, 51, 84, 33, 39, 55, 79, 92, 13, 59, 22, 12, 97, 88, 69, 42)}.
This stater comprises a balanced $C_{57}$-decomposition of $K_{115}$.

**Example 4.2. Balanced $C_{57}$-2-foil design of $K_{229}$.**
{(229, 6, 96, 217, 97, 158, 61, 68, 104, 147, 177, 15, 111, 33, 207, 183, 170, 128, 79, 159,
80, 130, 171, 185, 208, 35, 112, 17, 178, 149, 105, 70, 62, 160, 98, 216, 94, 89, 93,
4, 92, 215, 99, 162, 63, 72, 106, 151, 179, 19, 113, 37, 209, 187, 172, 132, 81),
(229, 3, 90, 214, 100, 164, 64, 74, 107, 153, 180, 21, 114, 39, 210, 189, 173, 134, 82, 165,
83, 136, 174, 191, 211, 41, 115, 23, 181, 155, 108, 76, 65, 166, 101, 213, 88, 2, 87,
85, 86, 212, 102, 168, 66, 78, 109, 157, 182, 25, 116, 43, 24, 193, 175, 138, 84)}.
This stater comprises a balanced $C_{57}$-2-foil decomposition of $K_{229}$.

**Theorem 5.** $K_n$ has a balanced $C_{76}$-$t$-foil design if and only if $n \equiv 1 \pmod{152t}$.

**Example 5.1. Balanced $C_{76}$ design of $K_{153}$.**
{(153, 4, 64, 145, 65, 106, 41, 46, 70, 99, 119, 11, 75, 23, 139, 123, 114, 86, 53, 107,
54, 88, 115, 125, 140, 25, 76, 13, 120, 101, 71, 48, 42, 108, 66, 144, 62, 59, 61,
2, 60, 143, 67, 110, 43, 50, 72, 103, 121, 15, 77, 27, 141, 127, 116, 90, 55, 111,
56, 92, 117, 129, 16, 29, 78, 17, 122, 105, 73, 52, 44, 112, 68, 142, 58, 1)}.
This stater comprises a balanced $C_{76}$-decomposition of $K_{153}$.

**Example 5.2. Balanced $C_{76}$-2-foil design of $K_{305}$.**
{(305, 8, 128, 289, 129, 210, 81, 90, 138, 195, 235, 19, 147, 43, 275, 243, 226, 170, 105, 211,
106, 172, 227, 245, 276, 45, 148, 21, 236, 197, 139, 92, 82, 212, 130, 288, 126, 119, 125,
6, 124, 287, 131, 214, 83, 94, 140, 199, 237, 23, 149, 47, 277, 247, 228, 174, 107, 215,
108, 176, 229, 249, 278, 49, 150, 25, 238, 201, 141, 96, 84, 216, 132, 286, 122, 5),
(305, 4, 120, 285, 133, 218, 85, 98, 142, 203, 239, 27, 151, 51, 279, 251, 230, 178, 109, 219,
110, 180, 231, 253, 280, 53, 152, 29, 240, 205, 143, 100, 86, 220, 134, 284, 118, 115, 117,
2, 116, 283, 135, 222, 87, 102, 144, 207, 241, 31, 153, 55, 281, 255, 232, 182, 111, 223,
112, 184, 233, 257, 32, 57, 154, 33, 242, 209, 145, 104, 88, 224, 136, 282, 114, 1)}.
This stater comprises a balanced $C_{76}$-2-foil decomposition of $K_{305}$.

**Theorem 6.** $K_n$ has a balanced $C_{95}$-$t$-foil design if and only if $n \equiv 1 \pmod{190t}$.

**Example 6.1. Balanced $C_{95}$ design of $K_{191}$.**
{(191, 5, 80, 181, 81, 132, 51, 57, 87, 123, 148, 13, 93, 28, 173, 153, 142, 107, 66, 133,
67, 109, 143, 155, 174, 30, 94, 15, 149, 125, 88, 59, 52, 134, 82, 180, 78, 4, 77,
73, 76, 179, 83, 136, 53, 61, 89, 127, 150, 17, 95, 32, 175, 157, 144, 111, 68, 137,
69, 113, 145, 159, 176, 34, 96, 19, 151, 129, 90, 63, 54, 138, 84, 178, 74, 2, 3,
1, 72, 177, 85, 140, 55, 65, 91, 131, 152, 21, 97, 36, 20, 161, 146, 115, 70)}.
This stater comprises a balanced $C_{95}$-decomposition of $K_{191}$.

**Example 6.2. Balanced $C_{95}$-2-foil design of $K_{381}$.**
{(381, 10, 160, 361, 161, 262, 101, 112, 172, 243, 293, 23, 183, 53, 343, 303, 282, 212, 131, 263,
132, 214, 283, 305, 344, 55, 184, 25, 294, 245, 173, 114, 102, 264, 162, 360, 158, 149, 157,
8, 156, 359, 163, 266, 103, 116, 174, 247, 295, 27, 185, 57, 345, 307, 284, 216, 133, 267,
134, 218, 285, 309, 346, 59, 186, 29, 296, 249, 175, 118, 104, 268, 164, 358, 154, 147, 153,
6, 152, 357, 165, 270, 105, 120, 176, 251, 297, 31, 187, 61, 347, 311, 286, 220, 135),
(381, 5, 150, 356, 166, 272, 106, 122, 177, 253, 298, 33, 188, 63, 348, 313, 287, 222, 136, 273,
137, 224, 288, 315, 349, 65, 189, 35, 299, 255, 178, 124, 107, 274, 167, 355, 148, 4, 7,
3, 146, 354, 168, 276, 108, 126, 179, 257, 300, 37, 190, 67, 350, 317, 289, 226, 138, 277,
139, 228, 290, 319, 351, 69, 191, 39, 301, 259, 180, 128, 109, 278, 169, 353, 144, 2, 143,
141, 142, 352, 170, 280, 110, 130, 181, 261, 302, 41, 192, 71, 40, 321, 291, 230, 140)}.
This stater comprises a balanced $C_{95}$-2-foil decomposition of $K_{381}$.

**Theorem 7.** $K_n$ has a balanced $C_{114}$-$t$-foil design if and only if $n \equiv 1 \pmod{228t}$.

**Example 7.1. Balanced $C_{114}$ design of $K_{229}$.**
{(229, 6, 96, 217, 97, 158, 61, 68, 104, 147, 177, 15, 111, 33, 207, 183, 170, 128, 79, 159,
80, 130, 171, 185, 208, 35, 112, 17, 178, 149, 105, 70, 62, 160, 98, 216, 94, 89, 93,
4, 92, 215, 99, 162, 63, 72, 106, 151, 179, 19, 113, 37, 209, 187, 172, 132, 81, 163,
82, 134, 173, 189, 210, 39, 114, 21, 180, 153, 107, 74, 64, 164, 100, 214, 90, 3, 5,
2, 88, 213, 101, 166, 65, 76, 108, 155, 181, 23, 115, 41, 211, 191, 174, 136, 83, 167,
84, 138, 175, 193, 24, 43, 116, 25, 182, 157, 109, 78, 66, 168, 102, 212, 86, 1)}.
This stater comprises a balanced $C_{114}$-decomposition of $K_{229}$.

**Theorem 8.** $K_n$ has a balanced $C_{133}$-$t$-foil design if and only if $n \equiv 1 \pmod{266t}$.

**Example 8.1. Balanced $C_{133}$ design of $K_{267}$.**
{(267, 7, 112, 253, 113, 184, 71, 79, 121, 171, 206, 17, 129, 38, 241, 213, 198, 149, 92, 185,
93, 151, 199, 215, 242, 40, 130, 19, 207, 173, 122, 81, 72, 186, 114, 252, 110, 6, 109,
103, 108, 251, 115, 188, 73, 83, 123, 175, 208, 21, 131, 42, 243, 217, 200, 153, 94, 189,
95, 155, 201, 219, 244, 44, 132, 23, 209, 177, 124, 85, 74, 190, 116, 250, 106, 4, 105,
101, 104, 249, 117, 192, 75, 87, 125, 179, 210, 25, 133, 46, 245, 221, 202, 157, 96, 193,
97, 159, 203, 223, 246, 48, 134, 27, 211, 181, 126, 89, 76, 194, 118, 248, 102, 2, 3,
1, 100, 247, 119, 196, 77, 91, 127, 183, 212, 29, 135, 50, 28, 225, 204, 161, 98)}.

This stater comprises a balanced $C_{133}$-decomposition of $K_{267}$.

**Theorem 9.** $K_n$ has a balanced $C_{152}$-$t$-foil design if and only if $n \equiv 1 \pmod{304t}$.

**Example 9.1. Balanced $C_{152}$ design of $K_{305}$.**
{(305, 8, 128, 289, 129, 210, 81, 90, 138, 195, 235, 19, 147, 43, 275, 243, 226, 170, 105, 211,
106, 172, 227, 245, 276, 45, 148, 21, 236, 197, 139, 92, 82, 212, 130, 288, 126, 119, 125,
6, 124, 287, 131, 214, 83, 94, 140, 199, 237, 23, 149, 47, 277, 247, 228, 174, 107, 215,
108, 176, 229, 249, 278, 49, 150, 25, 238, 201, 141, 96, 84, 216, 132, 286, 122, 117, 121,
4, 120, 285, 133, 218, 85, 98, 142, 203, 239, 27, 151, 51, 279, 251, 230, 178, 109, 219,
110, 180, 231, 253, 280, 53, 152, 29, 240, 205, 143, 100, 86, 220, 134, 284, 118, 3, 5,
2, 116, 283, 135, 222, 87, 102, 144, 207, 241, 31, 153, 55, 281, 255, 232, 182, 111, 223,
112, 184, 233, 257, 32, 57, 154, 33, 242, 209, 145, 104, 88, 224, 136, 282, 114, 1)}.
This stater comprises a balanced $C_{152}$-decomposition of $K_{305}$.

**Theorem 10.** $K_n$ has a balanced $C_{171}$-$t$-foil design if and only if $n \equiv 1 \pmod{342t}$.

**Example 10.1. Balanced $C_{171}$ design of $K_{343}$.**
{(343, 9, 144, 325, 145, 236, 91, 101, 155, 219, 264, 21, 165, 48, 309, 273, 254, 191, 118, 237,
119, 193, 255, 275, 310, 50, 166, 23, 265, 221, 156, 103, 92, 238, 146, 324, 142, 8, 141,
133, 140, 323, 147, 240, 93, 105, 157, 223, 266, 25, 167, 52, 311, 277, 256, 195, 120, 241,
121, 197, 257, 279, 312, 54, 168, 27, 267, 225, 158, 107, 94, 242, 148, 322, 138, 6, 137,
131, 136, 321, 149, 244, 95, 109, 159, 227, 268, 29, 169, 56, 313, 281, 258, 199, 122, 245,
123, 201, 259, 283, 314, 58, 170, 31, 269, 229, 160, 111, 96, 246, 150, 320, 134, 4, 7,
3, 132, 319, 151, 248, 97, 113, 161, 231, 270, 33, 171, 60, 315, 285, 260, 203, 124, 249,
125, 205, 261, 287, 316, 62, 172, 35, 271, 233, 162, 115, 98, 250, 152, 318, 130, 2, 129,
127, 128, 317, 153, 252, 99, 117, 163, 235, 272, 37, 173, 64, 36, 289, 262, 207, 126)}.
This stater comprises a balanced $C_{171}$-decomposition of $K_{343}$.

**Theorem 11.** $K_n$ has a balanced $C_{190}$-$t$-foil design if and only if $n \equiv 1 \pmod{380t}$.

**Example 11.1. Balanced $C_{190}$ design of $K_{381}$.**
{(381, 10, 160, 361, 161, 262, 101, 112, 172, 243, 293, 23, 183, 53, 343, 303, 282, 212, 131, 263,
132, 214, 283, 305, 344, 55, 184, 25, 294, 245, 173, 114, 102, 264, 162, 360, 158, 149, 157,
8, 156, 359, 163, 266, 103, 116, 174, 247, 295, 27, 185, 57, 345, 307, 284, 216, 133, 267,
134, 218, 285, 309, 346, 59, 186, 29, 296, 249, 175, 118, 104, 268, 164, 358, 154, 147, 153,
6, 152, 357, 165, 270, 105, 120, 176, 251, 297, 31, 187, 61, 347, 311, 286, 220, 135, 271,
136, 222, 287, 313, 348, 63, 188, 33, 298, 253, 177, 122, 106, 272, 166, 356, 150, 5, 9,
4, 148, 355, 167, 274, 107, 124, 178, 255, 299, 35, 189, 65, 349, 315, 288, 224, 137, 275,
138, 226, 289, 317, 350, 67, 190, 37, 300, 257, 179, 126, 108, 276, 168, 354, 146, 143, 145,
2, 144, 353, 169, 278, 109, 128, 180, 259, 301, 39, 191, 69, 351, 319, 290, 228, 139, 279,
140, 230, 291, 321, 40, 71, 192, 41, 302, 261, 181, 130, 110, 280, 170, 352, 142, 1)}.
This stater comprises a balanced $C_{190}$-decomposition of $K_{381}$.

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