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均衡型 (C_5, C_8)-Foil デザインと関連デザイン

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グラフ理論において、グラフの分解問題は主要な研究テーマである。C_5 を 5 点を通るサイクル、C_8 を 8 点を通るサイクルとする。1 点を共有する辺素な t 個の C_5 と t 個の C_8 からなるグラフを (C_5, C_8)-$2t$-foil という。本研究では、完全グラフ K_n を 均衡的に (C_5, C_8)-$2t$-foil 部分グラフに分解する均衡型 (C_5, C_8)-foil デザインについて述べる。さらに、均衡型 C_{13}-foil デザイン、均衡型 C_{26}-foil デザイン、均衡型 C_{39}-foil デザイン、均衡型 C_{52}-foil デザイン、均衡型 C_{65}-foil デザイン、均衡型 C_{78}-foil デザイン、均衡型 C_{91}-foil デザイン、均衡型 C_{104}-foil デザイン、均衡型 C_{117}-foil デザイン、均衡型 C_{130}-foil デザインについて述べる。

Balanced (C_5, C_8)-Foil Designs and Related Designs

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In graph theory, the decomposition problem of graphs is a very important topic. Various type of decompositions of many graphs can be seen in the literature of graph theory. This paper gives balanced (C_5, C_8)-foil designs, balanced C_{13}-foil designs, and balanced C_{26}-foil designs, and balanced C_{39}-foil designs, and balanced C_{52}-foil designs, and balanced C_{65}-foil designs, and balanced C_{78}-foil designs, and balanced C_{91}-foil designs, and balanced C_{104}-foil designs, and balanced C_{117}-foil designs, and balanced C_{130}-foil designs.

1. Balanced (C_5, C_8)-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_5 and C_8 be *the* 5-*cycle* and *the* 8-*cycle*, respectively. *The* (C_5, C_8)-$2t$-*foil* is a graph of t edge-disjoint C_5's and t edge-disjoint C_8's with a common vertex and the common vertex is called *the center of the* (C_5, C_8)-$2t$-*foil*. When K_n is decomposed into edge-disjoint sum of (C_5, C_8)-$2t$-foils and every vertex of K_n appears in the same number of (C_5, C_8)-$2t$-foils, we say that K_n *has a balanced* (C_5, C_8)-$2t$-*foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced* (C_5, C_8)-*foil design*.

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Theorem 1. K_n has a balanced (C_5, C_8)-$2t$-foil design if and only if $n \equiv 1 \pmod{26t}$.

Proof. (Necessity) Suppose that K_n has a balanced (C_5, C_8)-$2t$-foil decomposition. Let b be the number of (C_5, C_8)-$2t$-foils and r be the replication number. Then $b = n(n-1)/26t$ and $r = (11t+1)(n-1)/26t$. Among r (C_5, C_8)-$2t$-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_5, C_8)-$2t$-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/26t$ and $r_2 = 11(n-1)/26$. Therefore, $n \equiv 1 \pmod{26t}$ is necessary.
(Sufficiency) Put $n = 26st + 1$ and $T = st$. Then $n = 26T + 1$. Construct a (C_5, C_8)-$2T$-foil as follows:
$\{(26T+1, T, 12T, 23T+1, 14T), (26T+1, T+1, 5T+2, 24T+2, 3T+2, 23T+2, 20T+2, 17T+1)\} \cup$
$\{(26T+1, T-1, 12T-2, 23T, 14T-2), (26T+1, T+2, 5T+4, 24T+3, 3T+4, 23T+3, 20T+4, 17T+2)\} \cup$
$\{(26T+1, T-2, 12T-4, 23T-1, 14T-4), (26T+1, T+3, 5T+6, 24T+4, 3T+6, 23T+4, 20T+6, 17T+3)\} \cup$
$\ldots \cup$
$\{(26T+1, 1, 10T+2, 22T+2, 12T+2), (26T+1, 2T, 7T, 25T+1, 5T, 24T+1, 22T, 18T)\}$.
Decompose the (C_5, C_8)-$2T$-foil into s (C_5, C_8)-$2t$-foils. Then these starters comprise a balanced (C_5, C_8)-$2t$-foil decomposition of K_n.

Example 1.1. Balanced (C_5, C_8)-2-foil design of K_{27}.
$\{(27, 1, 12, 24, 14), (27, 2, 7, 26, 5, 25, 22, 18)\}$.
This starter comprises a balanced (C_5, C_8)-2-foil decomposition of K_{27}.

Example 1.2. Balanced (C_5, C_8)-4-foil design of K_{53}.
$\{(53, 2, 24, 47, 28), (53, 3, 12, 50, 8, 48, 42, 35)\} \cup$
$\{(53, 1, 22, 46, 26), (53, 4, 14, 51, 10, 49, 44, 36)\}$.
This starter comprises a balanced (C_5, C_8)-4-foil decomposition of K_{53}.

Example 1.3. Balanced (C_5, C_8)-6-foil design of K_{79}.
$\{(79, 3, 36, 70, 42), (79, 4, 17, 74, 11, 71, 62, 52)\} \cup$
$\{(79, 2, 34, 69, 40), (79, 5, 19, 75, 13, 72, 64, 53)\} \cup$
$\{(79, 1, 32, 68, 38), (79, 6, 21, 76, 15, 73, 66, 54)\}$.
This starter comprises a balanced (C_5, C_8)-6-foil decomposition of K_{79}.

Example 1.4. Balanced (C_5, C_8)-8-foil design of K_{105}.
$\{(105, 4, 48, 93, 56), (105, 5, 22, 98, 14, 94, 82, 69)\} \cup$
$\{(105, 3, 46, 92, 54), (105, 6, 24, 99, 16, 95, 84, 70)\} \cup$
$\{(105, 2, 44, 91, 52), (105, 7, 26, 100, 18, 96, 86, 71)\} \cup$
$\{(105, 1, 42, 90, 50), (105, 8, 28, 101, 20, 97, 88, 72)\}$.
This starter comprises a balanced (C_5, C_8)-8-foil decomposition of K_{105}.

Example 1.5. Balanced (C_5, C_8)-10-foil design of K_{131}.
$\{(131, 5, 60, 116, 70), (131, 6, 27, 122, 17, 117, 102, 86)\} \cup$
$\{(131, 4, 58, 115, 68), (131, 7, 29, 123, 19, 118, 104, 87)\} \cup$
$\{(131, 3, 56, 114, 66), (131, 8, 31, 124, 21, 119, 106, 88)\} \cup$
$\{(131, 2, 54, 113, 64), (131, 9, 33, 125, 23, 120, 108, 89)\} \cup$
$\{(131, 1, 52, 112, 62), (131, 10, 35, 126, 25, 121, 110, 90)\}$.
This starter comprises a balanced (C_5, C_8)-10-foil decomposition of K_{131}.

Example 1.6. Balanced (C_5, C_8)-12-foil design of K_{157}.
$\{(157, 6, 72, 139, 84), (157, 7, 32, 146, 20, 140, 122, 103)\} \cup$
$\{(157, 5, 70, 138, 82), (157, 8, 34, 147, 22, 141, 124, 104)\} \cup$
$\{(157, 4, 68, 137, 80), (157, 9, 36, 148, 24, 142, 126, 105)\} \cup$
$\{(157, 3, 66, 136, 78), (157, 10, 38, 149, 26, 143, 128, 106)\} \cup$
$\{(157, 2, 64, 135, 76), (157, 11, 40, 150, 28, 144, 130, 107)\} \cup$
$\{(157, 1, 62, 134, 74), (157, 12, 42, 151, 30, 145, 132, 108)\}$.
This starter comprises a balanced (C_5, C_8)-12-foil decomposition of K_{157}.

2. Balanced C_{13}-Foil Designs

Let C_{13} be *the cycle* on 13 vertices. *The C_{13}-t-foil* is a graph of t edge-disjoint C_{13}'s with a common vertex and the common vertex is called *the center of the C_{13}-t-foil*. When K_n is decomposed into edge-disjoint sum of C_{13}-t-foils and every vertex of K_n appears in the same number of C_{13}-t-foils, it is called that *K_n has a balanced C_{13}-t-foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced C_{13}-foil design*.

Theorem 2. K_n has a balanced C_{13}-t-foil design if and only if $n \equiv 1 \pmod{26t}$.

Proof. (Necessity) Suppose that K_n has a balanced C_{13}-t-foil decomposition. Let b be the number of C_{13}-t-foils and r be the replication number. Then $b = n(n-1)/26t$ and $r = (12t+1)(n-1)/26t$. Among r C_{13}-t-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of C_{13}-t-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $2tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/26t$ and $r_2 = 12(n-1)/26$. Therefore, $n \equiv 1 \pmod{26t}$ is necessary.
(Sufficiency) Put $n = 26st + 1, T = st$. Then $n = 26T + 1$. Construct a C_{13}-T-foil as follows:
$\{\ (26T+1, T, 12T, 23T+1, 14T, 15T+1, T+1, 5T+2, 24T+2, 3T+2, 23T+2, 20T+2, 17T+1),$
$(26T+1, T-1, 12T-2, 23T, 14T-2, 15T, T+2, 5T+4, 24T+3, 3T+4, 23T+3, 20T+4, 17T+2),$
$(26T+1, T-2, 12T-4, 23T-1, 14T-4, 15T-1, T+3, 5T+6, 24T+4, 3T+6, 23T+4, 20T+6, 17T+3),$

...,
$(26T+1, 1, 10T+2, 22T+2, 12T+2, 14T+2, 2T, 7T, 25T+1, 5T, 24T+1, 22T, 18T)$ }.
Decompose this C_{13}-T-foil into s C_{13}-t-foils. Then these starters comprise a balanced C_{13}-t-foil decomposition of K_n.

Example 2.1. Balanced C_{13} design of K_{27}.
$\{(27, 1, 12, 24, 14, 16, 2, 7, 26, 5, 25, 22, 18)\}$.
This stater comprises a balanced C_{13}-decomposition of K_{27}.

Example 2.2. Balanced C_{13}-2-foil design of K_{53}.
$\{(53, 2, 24, 47, 28, 31, 3, 12, 50, 8, 48, 42, 35)$,
$(53, 1, 22, 46, 26, 30, 4, 14, 51, 10, 49, 44, 36)\}$.
This stater comprises a balanced C_{13}-2-foil decomposition of K_{53}.

Example 2.3. Balanced C_{13}-3-foil design of K_{79}.
$\{(79, 3, 36, 70, 42, 46, 4, 17, 74, 11, 71, 62, 52)$,
$(79, 2, 34, 69, 40, 45, 5, 19, 75, 13, 72, 64, 53)$,
$(79, 1, 32, 68, 38, 44, 6, 21, 76, 15, 73, 66, 54)\}$.
This stater comprises a balanced C_{13}-3-foil decomposition of K_{79}.

Example 2.4. Balanced C_{13}-4-foil design of K_{105}.
$\{(105, 4, 48, 93, 56, 61, 5, 22, 98, 14, 94, 82, 69)$,
$(105, 3, 46, 92, 54, 60, 6, 24, 99, 16, 95, 84, 70)$,
$(105, 2, 44, 91, 52, 59, 7, 26, 100, 18, 96, 86, 71)$,
$(105, 1, 42, 90, 50, 58, 8, 28, 101, 20, 97, 88, 72)\}$.
This stater comprises a balanced C_{13}-4-foil decomposition of K_{105}.

Example 2.5. Balanced C_{13}-5-foil design of K_{131}.
$\{(131, 5, 60, 116, 70, 76, 6, 27, 122, 17, 117, 102, 86)$,
$(131, 4, 58, 115, 68, 75, 7, 29, 123, 19, 118, 104, 87)$,
$(131, 3, 56, 114, 66, 74, 8, 31, 124, 21, 119, 106, 88)$,
$(131, 2, 54, 113, 64, 73, 9, 33, 125, 23, 120, 108, 89)$,
$(131, 1, 52, 112, 62, 72, 10, 35, 126, 25, 121, 110, 90)\}$.
This stater comprises a balanced C_{13}-5-foil decomposition of K_{131}.

Example 2.6. Balanced C_{13}-6-foil design of K_{157}.
$\{(157, 6, 72, 139, 84, 91, 7, 32, 146, 20, 140, 122, 103)$,
$(157, 5, 70, 138, 82, 90, 8, 34, 147, 22, 141, 124, 104)$,
$(157, 4, 68, 137, 80, 89, 9, 36, 148, 24, 142, 126, 105)$,
$(157, 3, 66, 136, 78, 88, 10, 38, 149, 26, 143, 128, 106)$,
$(157, 2, 64, 135, 76, 87, 11, 40, 150, 28, 144, 130, 107)$,
$(157, 1, 62, 134, 74, 86, 12, 42, 151, 30, 145, 132, 108)\}$.
This stater comprises a balanced C_{13}-6-foil decomposition of K_{157}.

3. Balanced C_{13m}-Foil Designs

Let C_{13m} be *the cycle* on $13m$ vertices. *The C_{13m}-t-foil* is a graph of t edge-disjoint C_{13m}'s with a common vertex and the common vertex is called *the center of the C_{13m}-t-foil*. When K_n is decomposed into edge-disjoint sum of C_{13m}-t-foils and every vertex of K_n appears in the same number of C_{13m}-t-foils, it is called that K_n *has a balanced C_{13m}-t-foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced C_{13m}-foil design*.

Theorem 3. K_n has a balanced C_{26}-t-foil design if and only if $n \equiv 1 \pmod{52t}$.

Example 3.1. Balanced C_{26} design of K_{53}.
$\{(53, 2, 24, 47, 28, 31, 3, 12, 50, 8, 48, 42, 35, 18, 36, 44, 49, 10, 51, 14, 4, 30, 26, 46, 22, 1)\}$.
This stater comprises a balanced C_{26}-decomposition of K_{53}.

Example 3.2. Balanced C_{26}-2-foil design of K_{105}.
$\{(105, 4, 48, 93, 56, 61, 5, 22, 98, 14, 94, 82, 69, 34, 70, 84, 95, 16, 99, 24, 6, 60, 54, 92, 46, 3)$,
$(105, 2, 44, 91, 52, 59, 7, 26, 100, 18, 96, 86, 71, 38, 72, 88, 97, 20, 101, 28, 8, 58, 50, 90, 42, 1)\}$.

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This stater comprises a balanced C_{26}-2-foil decomposition of K_{105}.

Example 3.3. Balanced C_{26}-3-foil design of K_{157}.
$\{(157, 6, 72, 139, 84, 91, 7, 32, 146, 20, 140, 122, 103, 50, 104, 124, 141, 22, 147, 34, 8, 90, 82, 138, 70, 5),$
$(157, 4, 68, 137, 80, 89, 9, 36, 148, 24, 142, 126, 105, 54, 106, 128, 143, 26, 149, 38, 10, 88, 78, 136, 66, 3),$
$(157, 2, 64, 135, 76, 87, 11, 40, 150, 28, 144, 130, 107, 58, 108, 132, 145, 30, 151, 42, 12, 86, 74, 134, 62, 1)\}.$
This stater comprises a balanced C_{26}-3-foil decomposition of K_{157}.

Example 3.4. Balanced C_{26}-4-foil design of K_{209}.
$\{(209, 8, 96, 185, 112, 121, 9, 42, 194, 26, 186, 162, 137, 66, 138, 164, 187, 28, 195, 44, 10, 120, 110, 184, 94, 7),$
$(209, 6, 92, 183, 108, 119, 11, 46, 196, 30, 188, 166, 139, 70, 140, 168, 189, 32, 197, 48, 12, 118, 106, 182, 90, 5),$
$(209, 4, 88, 181, 104, 117, 13, 50, 198, 34, 190, 170, 141, 74, 142, 172, 191, 36, 199, 52, 14, 116, 102, 180, 86, 3),$
$(209, 2, 84, 179, 100, 115, 15, 54, 200, 38, 192, 174, 143, 78, 144, 176, 193, 40, 201, 56, 16, 114, 98, 178, 82, 1)\}.$
This stater comprises a balanced C_{26}-4-foil decomposition of K_{209}.

Example 3.5. Balanced C_{26}-5-foil design of K_{261}.
$\{(261, 10, 120, 231, 140, 151, 11, 52, 242, 32, 232, 202, 171, 82, 172, 204, 233, 34, 243, 54, 12, 150, 138, 230, 118, 9),$
$(261, 8, 116, 229, 136, 149, 13, 56, 244, 36, 234, 206, 173, 86, 174, 208, 235, 38, 245, 58, 14, 148, 134, 228, 114, 7),$
$(261, 6, 112, 227, 132, 147, 15, 60, 246, 40, 236, 210, 175, 90, 176, 212, 237, 42, 247, 62, 16, 146, 130, 226, 110, 5),$
$(261, 4, 108, 225, 128, 145, 17, 64, 248, 44, 238, 214, 177, 94, 178, 216, 239, 46, 249, 66, 18, 144, 126, 224, 106, 3),$
$(261, 2, 104, 223, 124, 143, 19, 68, 250, 48, 240, 218, 179, 98, 180, 220, 241, 50, 251, 70, 20, 142, 122, 222, 102, 1)\}.$
This stater comprises a balanced C_{26}-5-foil decomposition of K_{261}.

Theorem 4. K_n has a balanced C_{39}-t-foil design if and only if $n \equiv 1 \pmod{78t}$.

Example 4.1. Balanced C_{39} design of K_{79}.
$\{(79, 3, 36, 70, 42, 46, 4, 17, 74, 11, 71, 62, 52, 26, 53, 64, 72, 13, 75, 19, 5, 45, 40, 69, 34, 2, 33, 31, 32, 68, 38, 44, 6, 21, 76, 15, 73, 66, 54)\}.$
This stater comprises a balanced C_{39}-decomposition of K_{79}.

Example 4.2. Balanced C_{39}-2-foil design of K_{157}.
$\{(157, 6, 72, 139, 84, 91, 7, 32, 146, 20, 140, 122, 103, 50, 104, 124, 141, 22, 147, 34, 8, 90, 82, 138, 70, 65, 69, 4, 68, 137, 80, 89, 9, 36, 148, 24, 142, 126, 105),$
$(157, 3, 66, 136, 78, 88, 10, 38, 149, 26, 143, 128, 106, 56, 107, 130, 144, 28, 150, 40, 11, 87, 76, 135, 64, 2, 63, 61, 62, 134, 74, 86, 12, 42, 151, 30, 145, 132, 108)\}.$
This stater comprises a balanced C_{39}-2-foil decomposition of K_{157}.

Example 4.3. Balanced C_{39}-3-foil design of K_{235}.
$\{(235, 9, 108, 208, 126, 136, 10, 47, 218, 29, 209, 154, 74, 155, 210, 31, 219, 49, 11, 135, 124, 207, 106, 8, 105, 97, 104, 206, 122, 134, 12, 51, 220, 33, 211, 156),$
$(235, 6, 102, 205, 120, 133, 13, 53, 221, 35, 212, 157, 80, 158, 213, 37, 222, 55, 14, 132, 118, 204, 100, 95, 99, 4, 98, 203, 116, 131, 15, 57, 223, 39, 214, 159),$
$(235, 3, 96, 202, 114, 130, 16, 59, 224, 41, 215, 160, 86, 161, 216, 43, 225, 61, 17, 129, 112, 201, 94, 2, 93, 91, 92, 200, 110, 128, 18, 63, 226, 45, 217, 162)\}.$
This stater comprises a balanced C_{39}-3-foil decomposition of K_{235}.

Theorem 5. K_n has a balanced C_{52}-t-foil design if and only if $n \equiv 1 \pmod{104t}$.

Example 5.1. Balanced C_{52} design of K_{105}.
$\{(105, 4, 48, 93, 56, 61, 5, 22, 98, 14, 94, 82, 69, 34, 70, 84, 95, 16, 99, 24, 6, 60, 54, 92, 46, 43, 45,$

$2, 44, 91, 52, 59, 7, 26, 100, 18, 96, 86, 71, 38, 72, 88, 97, 20, 101, 28, 8, 58, 50, 90, 42, 1)\}$.
This stater comprises a balanced C_{52}-decomposition of K_{105}.

Example 5.2. Balanced C_{52}-2-foil design of K_{209}.
$\{(209, 8, 96, 185, 112, 121, 9, 42, 194, 26, 186, 162, 137, 66, 138, 164, 187, 28, 195, 44, 10, 120, 110, 184, 94, 87, 93, 6, 92, 183, 108, 119, 11, 46, 196, 30, 188, 166, 139, 70, 140, 168, 189, 32, 197, 48, 12, 118, 106, 182, 90, 5),$
$(209, 4, 88, 181, 104, 117, 13, 50, 198, 34, 190, 170, 141, 74, 142, 172, 191, 36, 199, 52, 14, 116, 102, 180, 86, 83, 85, 2, 84, 179, 100, 115, 15, 54, 200, 38, 192, 174, 143, 78, 144, 176, 193, 40, 201, 56, 16, 114, 98, 178, 82, 1)\}$.
This stater comprises a balanced C_{52}-2-foil decomposition of K_{209}.

Theorem 6. K_n has a balanced C_{65}-t-foil design if and only if $n \equiv 1 \pmod{130t}$.

Example 6.1. Balanced C_{65} design of K_{131}.
$\{(131, 5, 60, 116, 70, 76, 6, 27, 122, 17, 117, 102, 86, 42, 87, 104, 118, 19, 123, 29, 7, 75, 68, 115, 58, 4, 57, 53, 56, 114, 66, 74, 8, 31, 124, 21, 119, 106, 88, 46, 89, 108, 120, 23, 125, 33, 9, 73, 64, 113, 54, 2, 3, 1, 52, 112, 62, 72, 10, 35, 126, 25, 121, 110, 90)\}$.
This stater comprises a balanced C_{65}-decomposition of K_{131}.

Example 6.2. Balanced C_{65}-2-foil design of K_{261}.
$\{(261, 10, 120, 231, 140, 151, 11, 52, 242, 32, 232, 202, 171, 82, 172, 204, 233, 34, 243, 54, 12, 150, 138, 230, 118, 109, 117, 8, 116, 229, 136, 149, 13, 56, 244, 36, 234, 206, 173, 86, 174, 208, 235, 38, 245, 58, 14, 148, 134, 228, 114, 107, 113, 6, 112, 227, 132, 147, 15, 60, 246, 40, 236, 210, 175),$
$(261, 5, 110, 226, 130, 146, 16, 62, 247, 42, 237, 212, 176, 92, 177, 214, 238, 44, 248, 64, 17, 145, 128, 225, 108, 4, 7, 3, 106, 224, 126, 144, 18, 66, 249, 46, 239, 216, 178, 96, 179, 218, 240, 48, 250, 68, 19, 143, 124, 223, 104, 2, 103, 101, 102, 222, 122, 142, 20, 70, 251, 50, 241, 220, 180)\}$.
This stater comprises a balanced C_{65}-2-foil decomposition of K_{261}.

Theorem 7. K_n has a balanced C_{78}-t-foil design if and only if $n \equiv 1 \pmod{156t}$.

Example 7.1. Balanced C_{78} design of K_{157}.
$\{(157, 6, 72, 139, 84, 91, 7, 32, 146, 20, 140, 122, 103, 50, 104, 124, 141, 22, 147, 34, 8, 90, 82, 138, 70, 65, 69, 4, 68, 137, 80, 89, 9, 36, 148, 24, 142, 126, 105, 54, 106, 128, 143, 26, 149, 38, 10, 88, 78, 136, 66, 3, 5, 2, 64, 135, 76, 87, 11, 40, 150, 28, 144, 130, 107, 58, 108, 132, 145, 30, 151, 42, 12, 86, 74, 134, 62, 1)\}$.
This stater comprises a balanced C_{78}-decomposition of K_{157}.

Example 7.2. Balanced C_{78}-2-foil design of K_{313}.
$\{(313, 12, 144, 277, 168, 181, 13, 62, 290, 38, 278, 242, 205, 98, 206, 244, 279, 40, 291, 64, 14, 180, 166, 276, 142, 131, 141, 10, 140, 275, 164, 179, 15, 66, 292, 42, 280, 246, 207, 102, 208, 248, 281, 44, 293, 68, 16, 178, 162, 274, 138, 129, 137, 8, 136, 273, 160, 177, 17, 70, 294, 46, 282, 250, 209, 106, 210, 252, 283, 48, 295, 72, 18, 176, 158, 272, 134, 7),$
$(313, 6, 132, 271, 156, 175, 19, 74, 296, 50, 284, 254, 211, 110, 212, 256, 285, 52, 297, 76, 20, 174, 154, 270, 130, 5, 9, 4, 128, 269, 152, 173, 21, 78, 298, 54, 286, 258, 213, 114, 214, 260, 287, 56, 299, 80, 22, 172, 150, 268, 126, 123, 125, 2, 124, 267, 148, 171, 23, 82, 300, 58, 288, 262, 215, 118, 216, 264, 289, 60, 301, 84, 24, 170, 146, 266, 122, 1)\}$.
This stater comprises a balanced C_{78}-2-foil decomposition of K_{313}.

Theorem 8. K_n has a balanced C_{91}-t-foil design if and only if $n \equiv 1 \pmod{182t}$.

Example 8.1. Balanced C_{91} design of K_{183}.
$\{(183, 7, 84, 162, 98, 106, 8, 37, 170, 23, 163, 142, 120, 58, 121, 144, 164, 25, 171, 39, 9, 105, 96, 161, 82, 6, 81, 75, 80, 160, 94, 104, 10, 41, 172, 27, 165, 146, 122, 62, 123, 148, 166, 29, 173, 43, 11, 103, 92, 159, 78, 4, 77, 73, 76, 158, 90, 102, 12, 45, 174, 31, 167, 150, 124, 66, 125, 152, 168, 33, 175, 47, 13, 101, 88, 157, 74, 2, 3, 1, 72, 156, 86, 100, 14, 49, 176, 35, 169, 154, 126)\}$.
This stater comprises a balanced C_{91}-decomposition of K_{183}.

Theorem 9. K_n has a balanced C_{104}-t-foil design if and only if $n \equiv 1 \pmod{208t}$.

Example 9.1. Balanced C_{104} design of K_{209}.
$\{(209, 8, 96, 185, 112, 121, 9, 42, 194, 26, 186, 162, 137, 66, 138, 164, 187, 28, 195, 44, 10, 120,$

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110, 184, 94, 87, 93, 6, 92, 183, 108, 119, 11, 46, 196, 30, 188, 166, 139, 70, 140, 168, 189, 32, 197, 48, 12, 118, 106, 182, 90, 85, 89, 4, 88, 181, 104, 117, 13, 50, 198, 34, 190, 170, 141, 74, 142, 172, 191, 36, 199, 52, 14, 116, 102, 180, 86, 3, 5, 2, 84, 179, 100, 115, 15, 54, 200, 38, 192, 174, 143, 78, 144, 176, 193, 40, 201, 56, 16, 114, 98, 178, 82, 1)}.
This stater comprises a balanced C_{104}-decomposition of K_{209}.

Theorem 10. K_n has a balanced C_{117}-t-foil design if and only if $n \equiv 1 \pmod{234t}$.

Example 10.1. Balanced C_{117} design of K_{235}.
{(235, 9, 108, 208, 126, 136, 10, 47, 218, 29, 209, 154, 74, 155, 210, 31, 219, 49, 11, 135, 124, 207, 106, 8, 105, 97, 104, 206, 122, 134, 12, 51, 220, 33, 211, 156, 78, 157, 212, 35, 221, 53, 13, 133, 120, 205, 102, 6, 101, 95, 100, 204, 118, 132, 14, 55, 222, 37, 213, 158, 82, 159, 214, 39, 223, 57, 15, 131, 116, 203, 98, 4, 7, 3, 96, 202, 114, 130, 16, 59, 224, 41, 215, 160, 86, 161, 216, 43, 225, 61, 17, 129, 112, 201, 94, 2, 93, 91, 92, 200, 110, 128, 18, 63, 226, 45, 217, 162)}.
This stater comprises a balanced C_{117}-decomposition of K_{235}.

Theorem 11. K_n has a balanced C_{130}-t-foil design if and only if $n \equiv 1 \pmod{260t}$.

Example 11.1. Balanced C_{130} design of K_{261}.
{(261, 10, 120, 231, 140, 151, 11, 52, 242, 32, 232, 202, 171, 82, 172, 204, 233, 34, 243, 54, 12, 150, 138, 230, 118, 109, 117, 8, 116, 229, 136, 149, 13, 56, 244, 36, 234, 206, 173, 86, 174, 208, 235, 38, 245, 58, 14, 148, 134, 228, 114, 107, 113, 6, 112, 227, 132, 147, 15, 60, 246, 40, 236, 210, 175, 90, 176, 212, 237, 42, 247, 62, 16, 146, 130, 226, 110, 5, 9, 4, 108, 225, 128, 145, 17, 64, 248, 44, 238, 214, 177, 94, 178, 216, 239, 46, 249, 66, 18, 144, 126, 224, 106, 103, 105, 2, 104, 223, 124, 143, 19, 68, 250, 48, 240, 218, 179, 98, 180, 220, 241, 50, 251, 70, 20, 142, 122, 222, 102, 1)}.
This stater comprises a balanced C_{130}-decomposition of K_{261}.

参 考 文 献

1) Ushio, K. and Fujimoto, H.: Balanced bowtie and trefoil decomposition of complete tripartite multigraphs, *IEICE Trans. Fundamentals*, Vol. E84-A, No. 3, pp. 839–844 (2001).
2) Ushio, K. and Fujimoto, H.: Balanced foil decomposition of complete graphs, *IEICE Trans. Fundamentals*, Vol.E84-A, No.12, pp.3132–3137 (2001).
3) Ushio, K. and Fujimoto, H.: Balanced bowtie decomposition of complete multigraphs, *IEICE Trans. Fundamentals*, Vol.E86-A, No.9, pp.2360–2365 (2003).
4) Ushio, K. and Fujimoto, H.: Balanced bowtie decomposition of symmetric complete multi-digraphs, *IEICE Trans. Fundamentals*, Vol.E87-A, No.10, pp.2769–2773 (2004).
5) Ushio, K. and Fujimoto, H.: Balanced quatrefoil decomposition of complete multigraphs, *IEICE Trans. Information and Systems*, Vol.E88-D, No.1, pp.19–22 (2005).
6) Ushio, K. and Fujimoto, H.: Balanced C_4-bowtie decomposition of complete multigraphs, *IEICE Trans. Fundamentals*, Vol.E88-A, No.5, pp.1148–1154 (2005).
7) Ushio, K. and Fujimoto, H.: Balanced C_4-trefoil decomposition of complete multigraphs, *IEICE Trans. Fundamentals*, Vol.E89-A, No.5, pp.1173–1180 (2006).