Vol.2011-AL-137 No.10
2011/11/18

情報処理学会研究報告
IPSJ SIG Technical Report

均衡型 (C_5, C_6)-Foil デザインと関連デザイン

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グラフ理論において、グラフの分解問題は主要な研究テーマである。C_5 を 5 点を通るサイクル、C_6 を 6 点を通るサイクルとする。1 点を共有する辺素な t 個の C_5 と t 個の C_6 からなるグラフを (C_5, C_6)-2t-foil という。本研究では、完全グラフ K_n を 均衡的に (C_5, C_6)-2t-foil 部分グラフに分解する均衡型 (C_5, C_6)-foil デザインについて述べる。さらに、均衡型 C_{11}-foil デザイン、均衡型 (C_{10}, C_{12})-foil デザイン、均衡型 C_{22}-foil デザイン、均衡型 C_{33}-foil デザイン、均衡型 C_{44}-foil デザイン、均衡型 C_{55}-foil デザイン、均衡型 C_{66}-foil デザイン、均衡型 C_{77}-foil デザイン、均衡型 C_{88}-foil デザイン、均衡型 C_{99}-foil デザイン、均衡型 C_{110}-foil デザイン、について述べる。

Balanced (C_5, C_6)-Foil Designs and Related Designs

Kazuhiko Ushio

In graph theory, the decomposition problem of graphs is a very important topic. Various type of decompositions of many graphs can be seen in the literature of graph theory. This paper gives balanced (C_5, C_6)-foil designs, balanced C_{11}-foil designs, balanced (C_{10}, C_{12})-foil designs, balanced C_{22}-foil designs, balanced C_{33}-foil designs, balanced C_{44}-foil designs, balanced C_{55}-foil designs, balanced C_{66}-foil designs, balanced C_{77}-foil designs, balanced C_{88}-foil designs, balanced C_{99}-foil designs, balanced C_{110}-foil designs.

1. Balanced (C_5, C_6)-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_5 and C_6 be *the* 5*-cycle* and *the* 6*-cycle*, respectively. *The* (C_5, C_6)*-2t-foil* is a graph of t edge-disjoint C_5's and t edge-disjoint C_6's with a common vertex and the common vertex is called *the center of the* (C_5, C_6)*-2t-foil*. When K_n is decomposed into edge-disjoint sum of (C_5, C_6)-2t-foils and every vertex of K_n appears in the same number of (C_5, C_6)-2t-foils, we say that K_n *has a balanced* (C_5, C_6)*-2t-foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced* (C_5, C_6)*-2t-foil design*.

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Theorem 1. K_n has a balanced (C_5, C_6)-2t-foil design if and only if $n \equiv 1 \pmod{22t}$.

Proof. (Necessity) Suppose that K_n has a balanced (C_5, C_6)-2t-foil decomposition. Let b be the number of (C_5, C_6)-2t-foils and r be the replication number. Then $b = n(n-1)/22t$ and $r = (9t+1)(n-1)/22t$. Among r (C_5, C_6)-2t-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_5, C_6)-2t-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n-1$. From these relations, $r_1 = (n-1)/22t$ and $r_2 = 9(n-1)/22$. Therefore, $n \equiv 1 \pmod{22t}$ is necessary.
(Sufficiency) Put $n = 22st+1$ and $T = st$. Then $n = 22T+1$. Construct a (C_5, C_6)-2T-foil as follows:
$\{(22T+1, T, 15T, 21T+1, 9T+1), (22T+1, T+1, 4T+2, 12T+2, 6T+2, 2T+1)\} \cup$
$\{(22T+1, T-1, 15T-2, 21T, 9T+2), (22T+1, T+2, 4T+4, 12T+3, 6T+4, 2T+2)\} \cup$
$\{(22T+1, T-2, 15T-4, 21T-1, 9T+3), (22T+1, T+3, 4T+6, 12T+4, 6T+6, 2T+3)\} \cup$
... $\cup$
$\{(22T+1, 3, 13T+6, 20T+4, 10T-2), (22T+1, 2T-2, 6T-4, 13T-1, 8T-4, 3T-2)\} \cup$
$\{(22T+1, 2, 13T+4, 20T+3, 10T-1), (22T+1, 2T-1, 6T-2, 13T, 8T-2, 3T-1)\} \cup$
$\{(22T+1, 1, 13T+2, 20T+2, 10T), (22T+1, 2T, 6T, 13T+1, 8T, 3T)\}$.
($11T$ edges, $11T$ all lengths)
Decompose the (C_5, C_6)-2T-foil into s (C_5, C_6)-2t-foils. Then these starters comprise a balanced (C_5, C_6)-2t-foil decomposition of K_n.

Example 1.1. Balanced (C_5, C_6)-2-foil design of K_{23}.
$\{(23, 1, 15, 22, 10), (23, 2, 6, 14, 8, 3)\}$.

(11 edges, 11 all lengths)

This starter comprises a balanced (C_5, C_6)-2-foil decomposition of K_{23}.

Example 1.2. Balanced (C_5, C_6)-4-foil design of K_{45}.

$\{(45, 2, 30, 43, 19), (45, 3, 10, 26, 14, 5)\} \cup$

$\{(45, 1, 28, 42, 20), (45, 4, 12, 27, 16, 6)\}$.

(22 edges, 22 all lengths)

This starter comprises a balanced (C_5, C_{66})-4-foil decomposition of K_{45}.

Example 1.3. Balanced (C_5, C_6)-6-foil design of K_{67}.

$\{(67, 3, 45, 64, 28), (67, 4, 14, 38, 20, 7)\} \cup$

$\{(67, 2, 43, 63, 29), (67, 5, 16, 39, 22, 8)\} \cup$

$\{(67, 1, 41, 62, 30), (67, 6, 18, 40, 24, 9)\}$.

(33 edges, 33 all lengths)

This starter comprises a balanced (C_5, C_6)-6-foil decomposition of K_{67}.

Example 1.4. Balanced (C_5, C_6)-8-foil design of K_{89}.

$\{(89, 4, 60, 85, 37), (89, 5, 18, 50, 26, 9)\} \cup$

$\{(89, 3, 58, 84, 38), (89, 6, 20, 51, 28, 10)\} \cup$

$\{(89, 2, 56, 83, 39), (89, 7, 22, 52, 30, 11)\} \cup$

$\{(89, 1, 54, 82, 40), (89, 8, 14, 53, 32, 12)\}$.

(44 edges, 44 all lengths)

This starter comprises a balanced (C_5, C_6)-8-foil decomposition of K_{89}.

Example 1.5. Balanced (C_5, C_6)-10-foil design of K_{111}.

$\{(111, 5, 75, 106, 46), (111, 6, 22, 62, 32, 11)\} \cup$

$\{(111, 4, 73, 105, 47), (111, 7, 24, 63, 34, 12)\} \cup$

$\{(111, 3, 71, 104, 48), (111, 8, 26, 64, 36, 13)\} \cup$

$\{(111, 2, 69, 103, 49), (111, 9, 28, 65, 38, 14)\} \cup$

$\{(111, 1, 67, 102, 50), (111, 10, 30, 66, 40, 15)\}$.

(55 edges, 55 all lengths)

This starter comprises a balanced (C_5, C_6)-10-foil decomposition of K_{111}.

Example 1.6. Balanced (C_5, C_6)-12-foil design of K_{133}.

$\{(133, 6, 90, 127, 55), (133, 7, 26, 74, 38, 13)\} \cup$

$\{(133, 5, 88, 126, 56), (133, 8, 28, 75, 40, 14)\} \cup$

$\{(133, 4, 86, 125, 57), (133, 9, 30, 76, 42, 15)\} \cup$

$\{(133, 3, 84, 124, 58), (133, 10, 32, 77, 44, 16)\} \cup$

$\{(133, 2, 82, 123, 59), (133, 11, 34, 78, 46, 17)\} \cup$

$\{(133, 1, 80, 122, 60), (133, 12, 36, 79, 48, 18)\}$.

(66 edges, 66 all lengths)

This starter comprises a balanced (C_5, C_6)-12-foil decomposition of K_{133}.

Example 1.7. Balanced (C_5, C_6)-14-foil design of K_{155}.

$\{(155, 7, 105, 148, 64), (155, 8, 30, 86, 44, 15)\} \cup$

$\{(155, 6, 103, 147, 65), (155, 9, 32, 87, 46, 16)\} \cup$

$\{(155, 5, 101, 146, 66), (155, 10, 34, 88, 48, 17)\} \cup$

$\{(155, 4, 99, 145, 67), (155, 11, 36, 89, 50, 18)\} \cup$

$\{(155, 3, 97, 144, 68), (155, 12, 38, 90, 52, 19)\} \cup$

$\{(155, 2, 95, 143, 69), (155, 13, 40, 91, 54, 20)\} \cup$

$\{(155, 1, 93, 142, 70), (155, 14, 42, 92, 56, 21)\}$.

(77 edges, 77 all lengths)

This starter comprises a balanced (C_5, C_6)-14-foil decomposition of K_{155}.

2. Balanced C_{11}-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_{11} be *the* 11-*cycle*. *The* C_{11}-*t*-*foil* is a graph of t edge-disjoint C_{11}'s with a common vertex and the common vertex is called *the center of the* C_{11}-*t*-*foil*. When K_n is decomposed into edge-disjoint sum of C_{11}-t-foils and every vertex of K_n appears in the same number of C_{11}-t-foils, it is called that K_n *has a balanced* C_{11}-*t*-*foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced* C_{11}-*t*-*foil design*.

Theorem 2. K_n has a balanced C_{11}-t-foil design if and only if $n \equiv 1 \pmod{22t}$.

Proof. (Necessity) Suppose that K_n has a balanced C_{11}-t-foil decomposition. Let b be the number of C_{11}-t-foils and r be the replication number. Then $b = n(n-1)/22t$ and $r = (10t+1)(n-1)/22t$. Among r C_{11}-t-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of C_{11}-t-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $2tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/22t$ and $r_2 = 10(n-1)/22$. Therefore, $n \equiv 1 \pmod{22t}$ is necessary.

(Sufficiency) Put $n = 22st + 1, T = st$. Then $n = 22T + 1$. Construct a C_{11}-T-foil as follows:

$\{ (22T+1, T, 15T, 21T+1, 9T+1, 10T+2, T+1, 4T+2, 12T+2, 6T+2, 2T+1),$
$(22T+1, T-1, 15T-2, 21T, 9T+2, 10T+4, T+2, 4T+4, 12T+3, 6T+4, 2T+2),$
$(22T+1, T-2, 15T-4, 21T-1, 9T+3, 10T+6, T+3, 4T+6, 12T+4, 6T+6, 2T+3),$
...,
$(22T+1, 3, 13T+6, 20T+4, 10T-2, 12T-4, 2T-2, 6T-4, 13T-1, 8T-4, 3T-2),$
$(22T+1, 2, 13T+4, 20T+3, 10T-1, 12T-2, 2T-1, 6T-2, 13T, 8T-2, 3T-1),$
$(22T+1, 1, 13T+2, 20T+2, 10T, 12T, 2T, 6T, 13T+1, 8T, 3T) \}.$
($11T$ edges, $11T$ all lengths)

Decompose this C_{11}-T-foil into s C_{11}-t-foils. Then these starters comprise a balanced C_{11}-t-foil decomposition of K_n.

Example 2.1. Balanced C_{11} design of K_{23}.
$\{(23, 1, 15, 22, 10, 12, 2, 6, 14, 8, 3)\}$.
(11 edges, 11 all lengths)
This stater comprises a balanced C_{11}-decomposition of K_{23}.

Example 2.2. Balanced C_{11}-2-foil design of K_{45}.
$\{(45, 2, 30, 43, 19, 22, 3, 10, 26, 14, 5)$,
$(45, 1, 28, 42, 20, 24, 4, 12, 27, 16, 6)\}$.
(22 edges, 22 all lengths)
This stater comprises a balanced C_{11}-2-foil decomposition of K_{45}.

Example 2.3. Balanced C_{11}-3-foil design of K_{67}.
$\{(67, 3, 45, 64, 28, 32, 4, 14, 38, 20, 7)$,
$(67, 2, 43, 63, 29, 34, 5, 16, 39, 22, 8)$,
$(67, 1, 41, 62, 30, 36, 6, 18, 40, 24, 9)\}$.
(33 edges, 33 all lengths)
This stater comprises a balanced C_{11}-3-foil decomposition of K_{67}.

Example 2.4. Balanced C_{11}-4-foil design of K_{89}.
$\{(89, 4, 60, 85, 37, 42, 5, 18, 50, 26, 9)$,
$(89, 3, 58, 84, 38, 44, 6, 20, 51, 28, 10)$,
$(89, 2, 56, 83, 39, 46, 7, 22, 52, 30, 11)$,
$(89, 1, 54, 82, 40, 48, 8, 24, 53, 32, 12)\}$.
(44 edges, 44 all lengths)
This stater comprises a balanced C_{11}-4-foil decomposition of K_{89}.

Example 2.5. Balanced C_{11}-5-foil design of K_{111}.
$\{(111, 5, 75, 106, 46, 52, 6, 22, 62, 32, 11)$,
$(111, 4, 73, 105, 47, 54, 7, 24, 63, 34, 12)$,
$(111, 3, 71, 104, 48, 56, 8, 26, 64, 36, 13)$,
$(111, 2, 69, 103, 49, 58, 9, 28, 65, 38, 14)$,
$(111, 1, 67, 102, 50, 60, 10, 30, 66, 40, 15)\}$.
(55 edges, 55 all lengths)
This stater comprises a balanced C_{11}-5-foil decomposition of K_{111}.

Example 2.6. Balanced C_{11}-6-foil design of K_{133}.
$\{(133, 6, 90, 127, 55, 62, 7, 26, 74, 38, 13)$,
$(133, 5, 88, 126, 56, 64, 8, 28, 75, 40, 14)$,
$(133, 4, 86, 125, 57, 66, 9, 30, 76, 42, 15)$,

$(133, 3, 84, 124, 58, 68, 10, 32, 77, 44, 16)$,
$(133, 2, 82, 123, 59, 70, 11, 34, 78, 46, 17)$,
$(133, 1, 80, 122, 60, 72, 12, 36, 79, 48, 18)\}$.
(66 edges, 66 all lengths)
This stater comprises a balanced C_{11}-6-foil decomposition of K_{133}.

3. Balanced (C_{10}, C_{12})-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_{10} and C_{12} be *the* 10-*cycle* and *the* 12-*cycle*, respectively. *The* (C_{10}, C_{12})-2t-*foil* is a graph of t edge-disjoint C_{10}'s and t edge-disjoint C_{12}'s with a common vertex and the common vertex is called *the center of the* (C_{10}, C_{12})-2t-*foil*. When K_n is decomposed into edge-disjoint sum of (C_{10}, C_{12})-2t-foils and every vertex of K_n appears in the same number of (C_{10}, C_{12})-2t-foils, we say that K_n *has a balanced* (C_{10}, C_{12})-2t-*foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced* (C_{10}, C_{12})-2t-*foil design*.

Theorem 3. K_n has a balanced (C_{10}, C_{12})-2t-foil design if and only if $n \equiv 1 \pmod{44t}$.

Proof. (Necessity) Suppose that K_n has a balanced (C_{10}, C_{12})-2t-foil decomposition. Let b be the number of (C_{10}, C_{12})-2t-foils and r be the replication number. Then $b = n(n-1)/44t$ and $r = (20t+1)(n-1)/44t$. Among r (C_{10}, C_{12})-2t-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_{10}, C_{12})-2t-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/44t$ and $r_2 = 20(n-1)/44$. Therefore, $n \equiv 1 \pmod{44t}$ is necessary.
(Sufficiency) Put $n = 44st+1$ and $T = st$. Then $n = 44T+1$. Construct a (C_{10}, C_{12})-2T-foil as follows:
$\{(44T+1, 2T, 30T, 42T+1, 18T+1, 36T+3, 18T+2, 42T, 30T-2, 2T-1)$,
$(44T+1, 2T+1, 8T+2, 24T+2, 12T+2, 4T+1, 8T+3, 4T+2, 12T+4, 24T+3, 8T+4, 2T+2)\} \cup$
$\{(44T+1, 2T-2, 30T-4, 42T-1, 18T+3, 36T+7, 18T+4, 42T-2, 30T-6, 2T-3)$,
$(44T+1, 2T+3, 8T+6, 24T+4, 12T+6, 4T+3, 8T+7, 4T+4, 12T+8, 24T+5, 8T+8, 2T+4)\} \cup$
$\{(44T+1, 2T-4, 30T-8, 42T-3, 18T+5, 36T+11, 18T+6, 42T-4, 30T-10, 2T-5)$,
$(44T+1, 2T+5, 8T+10, 24T+6, 12T+10, 4T+5, 8T+11, 4T+6, 12T+12, 24T+7, 8T+12, 2T+6)\} \cup$
... $\cup$
$\{(44T+1, 2, 26T+4, 40T+3, 20T-1, 40T-1, 20T, 40T+2, 26T+2, 1)$,
$(44T+1, 4T-1, 12T-2, 26T, 16T-2, 6T-1, 12T-1, 6T, 16T, 26T+1, 12T, 4T)\}$.
($22T$ edges, $22T$ all lengths)
Decompose the (C_{10}, C_{12})-2T-foil into s (C_{10}, C_{12})-2t-foils. Then these starters comprise a balanced (C_{10}, C_{12})-2t-foil decomposition of K_n.

Example 3.1. Balanced (C_{10}, C_{12})-2-foil design of K_{45}.
$\{(45, 2, 30, 43, 19, 39, 20, 42, 28, 1)$,
$(45, 3, 10, 26, 14, 5, 11, 6, 16, 27, 12, 4)\}$.
(22 edges, 22 all lengths)
This starter comprises a balanced (C_{10}, C_{12})-2-foil decomposition of K_{45}.

Example 3.2. Balanced (C_{10}, C_{12})-4-foil design of K_{89}.
$\{(89, 4, 60, 85, 37, 75, 38, 84, 58, 3)$,
$(89, 2, 56, 83, 39, 79, 40, 82, 54, 1)\}$
$\cup$
$\{(89, 5, 18, 50, 26, 9, 19, 10, 28, 51, 20, 6)$,
$(89, 7, 22, 52, 30, 11, 23, 12, 32, 53, 24, 8)\}$.
(44 edges, 44 all lengths)
This starter comprises a balanced (C_{10}, C_{12})-4-foil decomposition of K_{89}.

Example 3.3. Balanced (C_{10}, C_{12})-6-foil design of K_{133}.
$\{(133, 6, 90, 127, 55, 111, 56, 126, 88, 5)$,

$(133, 4, 86, 125, 57, 115, 58, 124, 84, 3)$,
$(133, 2, 82, 123, 59, 119, 60, 122, 80, 1)\}$
$\cup$
$\{(133, 7, 26, 74, 38, 13, 27, 14, 40, 75, 28, 8)$,
$(133, 9, 30, 76, 42, 15, 31, 16, 44, 77, 32, 10)$,
$(133, 11, 34, 78, 46, 17, 35, 18, 48, 79, 36, 12)\}$.
(66 edges, 66 all lengths)
This starter comprises a balanced (C_{10}, C_{12})-6-foil decomposition of K_{133}.

Example 3.4. Balanced (C_{10}, C_{12})-8-foil design of K_{177}.
$\{(177, 8, 120, 169, 73, 147, 74, 168, 118, 7)$,
$(177, 6, 116, 167, 75, 151, 76, 166, 114, 5)$,
$(177, 4, 112, 165, 77, 155, 78, 164, 110, 3)$,
$(177, 2, 108, 163, 79, 159, 80, 162, 106, 1)\}$
$\cup$
$\{(177, 9, 34, 98, 50, 17, 35, 18, 52, 99, 36, 10)$,
$(177, 11, 38, 100, 54, 19, 39, 20, 56, 101, 40, 12)$,
$(177, 13, 42, 102, 58, 21, 43, 22, 60, 103, 44, 14)$,
$(177, 15, 46, 104, 62, 23, 47, 24, 64, 105, 48, 16)\}$.
(88 edges, 88 all lengths)
This starter comprises a balanced (C_{10}, C_{12})-8-foil decomposition of K_{177}.

Example 3.5. Balanced (C_{10}, C_{12})-10-foil design of K_{221}.
$\{(221, 10, 150, 211, 91, 183, 92, 210, 148, 9)$,
$(221, 8, 146, 209, 93, 187, 94, 208, 144, 7)$,
$(221, 6, 142, 207, 95, 191, 96, 206, 140, 5)$,
$(221, 4, 138, 205, 97, 195, 98, 204, 136, 3)$,
$(221, 2, 134, 203, 99, 199, 100, 202, 132, 1)\}$
$\cup$
$\{(221, 11, 42, 122, 62, 21, 43, 22, 64, 123, 44, 12)$,
$(221, 13, 46, 124, 66, 23, 47, 24, 68, 125, 48, 14)$,
$(221, 15, 50, 126, 70, 25, 51, 26, 72, 127, 52, 16)$,
$(221, 17, 54, 128, 74, 27, 55, 28, 76, 129, 56, 18)$,
$(221, 19, 58, 130, 78, 29, 59, 30, 80, 131, 60, 20)\}$.
(110 edges, 110 all lengths)
This starter comprises a balanced (C_{10}, C_{12})-10-foil decomposition of K_{221}.

4. Balanced C_{22}-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_{22} be *the 22-cycle*. *The C_{22}-t-foil* is a graph of t edge-disjoint C_{22}'s with a common vertex and the common vertex is called *the center of the C_{22}-t-foil*. When K_n is decomposed into edge-disjoint sum of C_{22}-t-foils and every vertex of K_n appears in the same number of C_{22}-t-foils, it is called that K_n *has a balanced C_{22}-t-foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced C_{22}-t-foil design*.

Theorem 4. K_n has a balanced C_{22}-t-foil design if and only if $n \equiv 1 \pmod{44t}$.

Proof. (Necessity) Suppose that K_n has a balanced C_{22}-t-foil decomposition. Let b be the number of C_{22}-t-foils and r be the replication number. Then $b = n(n-1)/44t$ and $r = (21t+1)(n-1)/44t$. Among r C_{22}-t-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of C_{42}-t-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $2tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/44t$ and $r_2 = 21(n-1)/44$. Therefore, $n \equiv 1$ (mod $44t$) is necessary.
(Sufficiency) Put $n = 44st + 1, T = st$. Then $n = 44T + 1$. Construct a C_{22}-T-foil as follows:
$\{\ (44T+1, 2T, 30T, 42T+1, 18T+1, 20T+2, 2T+1, 8T+2, 24T+2, 12T+2, 4T+1, 8T+3, 4T+2, 12T+4, 24T+3, 8T+4, 2T+2, 20T+4, 18T+2, 42T, 30T-2, 2T-1)$,
$(44T+1, 2T-2, 30T-4, 42T-1, 18T+3, 20T+6, 2T+3, 8T+6, 24T+4, 12T+6, 4T+3, 8T+7, 4T+4, 12T+8, 24T+5, 8T+8, 2T+4, 20T+8, 18T+4, 42T-2, 30T-6, 2T-3)$,
$(44T+1, 2T-4, 30T-8, 42T-3, 18T+5, 20T+10, 2T+5, 8T+10, 24T+6, 12T+$

$10, 4T+5, 8T+11, 4T+6, 12T+12, 24T+7, 8T+12, 2T+6, 20T+12, 18T+6, 42T-4, 30T-10, 2T-5)$,
...,
$(44T+1, 2, 26T+4, 40T+3, 20T-1, 24T-2, 4T-1, 12T-2, 26T, 16T-2, 6T-1, 12T-1, 6T, 16T, 26T+1, 12T, 4T, 24T, 20T, 40T+2, 26T+2, 1)\ \}$.
($22T$ edges, $22T$ all lengths)
Decompose this C_{22}-T-foil into s C_{22}-t-foils. Then these starters comprise a balanced C_{22}-t-foil decomposition of K_n.

Example 4.1. Balanced C_{22} design of K_{45}.
$\{(45, 2, 30, 43, 19, 22, 3, 10, 26, 14, 5, 11, 6, 16, 27, 12, 4, 24, 20, 42, 28, 1)\}$.
(22 edges, 22 all lengths)
This starter comprises a balanced C_{22}-decomposition of K_{45}.

Example 4.2. Balanced C_{22}-2-foil design of K_{89}.
$\{(89, 4, 60, 85, 37, 42, 5, 18, 50, 26, 9, 19, 10, 28, 51, 20, 6, 44, 38, 84, 58, 3)$,
$(89, 2, 56, 83, 39, 46, 7, 22, 52, 30, 11, 23, 12, 32, 53, 24, 8, 48, 40, 82, 54, 1)\}$.
(44 edges, 44 all lengths)
This starter comprises a balanced C_{22}-2-foil decomposition of K_{89}.

Example 4.3. Balanced C_{22}-3-foil design of K_{133}.
$\{(133, 6, 90, 127, 55, 62, 7, 26, 74, 38, 13, 27, 14, 40, 75, 28, 8, 64, 56, 126, 88, 5)$,
$(133, 4, 86, 125, 57, 66, 9, 30, 76, 42, 15, 31, 16, 44, 77, 32, 10, 68, 58, 124, 84, 3)$,
$(133, 2, 82, 123, 59, 70, 11, 34, 78, 46, 17, 35, 18, 48, 79, 36, 12, 72, 60, 122, 80, 1)\}$.
(66 edges, 66 all lengths)
This starter comprises a balanced C_{22}-3-foil decomposition of K_{133}.

Example 4.4. Balanced C_{22}-4-foil design of K_{177}.
$\{(177, 8, 120, 169, 73, 82, 9, 34, 98, 50, 17, 35, 18, 52, 99, 36, 10, 84, 74, 168, 118, 7)$,
$(177, 6, 116, 167, 75, 86, 11, 38, 100, 54, 19, 39, 20, 56, 101, 40, 12, 88, 76, 166, 114, 5)$,
$(177, 4, 112, 165, 77, 90, 13, 42, 102, 58, 21, 43, 22, 60, 103, 44, 14, 92, 78, 164, 110, 3)$,
$(177, 2, 108, 163, 79, 94, 15, 46, 104, 62, 23, 47, 24, 64, 105, 48, 16, 96, 80, 162, 106, 1)\}$.
(88 edges, 88 all lengths)
This starter comprises a balanced C_{22}-4-foil decomposition of K_{177}.

Example 4.5. Balanced C_{22}-5-foil design of K_{221}.
$\{(221, 10, 150, 211, 91, 102, 11, 42, 122, 62, 21, 43, 22, 64, 123, 44, 12, 104, 92, 210, 148, 9)$,
$(221, 8, 146, 209, 93, 106, 13, 46, 124, 66, 23, 47, 24, 68, 125, 48, 14, 108, 94, 208, 144, 7)$,
$(221, 6, 142, 207, 95, 110, 15, 50, 126, 70, 25, 51, 26, 72, 127, 52, 16, 112, 96, 206, 140, 5)$,
$(221, 4, 138, 205, 97, 114, 17, 54, 128, 74, 27, 55, 28, 76, 129, 56, 18, 116, 98, 204, 136, 3)$,
$(221, 2, 134, 203, 99, 118, 19, 58, 130, 78, 29, 59, 30, 80, 131, 60, 20, 120, 100, 202, 132, 1)\}$.
(110 edges, 110 all lengths)
This starter comprises a balanced C_{22}-5-foil decomposition of K_{221}.

5. Balanced C_{11m}-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_{11m} be *the* $11m$*-cycle*. *The* C_{11m}*-*t*-foil* is a graph of t edge-disjoint C_{11m}'s with a common vertex and the common vertex is called *the center of the* C_{11m}*-*t*-foil*. When K_n is decomposed into edge-disjoint sum of C_{11m}-t-foils and every vertex of K_n appears in the same number of C_{11m}-t-foils, it is called that K_n *has a balanced* C_{11m}*-*t*-foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced* C_{11m}*-*t*-foil design.*

Theorem 5. K_n has a balanced C_{33}-t-foil design if and only if $n \equiv 1 \pmod{66t}$.

Example 5.1. Balanced C_{33} design of K_{67}.
Starter: $\{(67, 7, 20, 38, 14, 4, 32, 28, 64, 45, 42, 44,$
$2, 43, 63, 29, 34, 5, 16, 39, 22, 8, 17,$
$9, 24, 40, 18, 6, 36, 30, 62, 41, 1)\}$.

Example 5.2. Balanced C_{33}-2-foil design of K_{133}.

Starter: $\{(133, 13, 38, 74, 26, 7, 62, 55, 127, 90, 6, 89,$
$83, 88, 126, 56, 64, 8, 28, 75, 40, 14, 29,$
$15, 42, 76, 30, 9, 66, 57, 125, 86, 4),$
$(133, 16, 44, 77, 32, 10, 68, 58, 124, 84, 3, 5,$
$2, 82, 123, 59, 70, 11, 34, 78, 46, 17, 35,$
$18, 48, 79, 36, 12, 72, 60, 122, 80, 1).\}.$

Theorem 6. K_n has a balanced C_{44}-t-foil design if and only if $n \equiv 1 \pmod{88t}$.

Example 6.1. Balanced C_{44} design of K_{89}.
Starter: $\{(89, 4, 60, 85, 37, 42, 5, 18, 50, 26, 9, 19,$
$10, 28, 51, 20, 6, 44, 38, 84, 58, 55, 57,$
$2, 56, 83, 39, 46, 7, 22, 52, 30, 11, 23,$
$12, 32, 53, 24, 8, 48, 40, 82, 54, 1)\}.$

Example 6.2. Balanced C_{44}-2-foil design of K_{177}.
Starter: $\{(177, 8, 120, 169, 73, 82, 9, 34, 98, 50, 17, 35,$
$18, 52, 99, 36, 10, 84, 74, 168, 118, 111, 117,$
$6, 116, 167, 75, 86, 11, 38, 100, 54, 19, 39,$
$20, 56, 101, 40, 12, 88, 76, 166, 114, 5),$
$(177, 4, 112, 165, 77, 90, 13, 42, 102, 58, 21, 43,$
$22, 60, 103, 44, 14, 92, 78, 164, 110, 107, 109,$
$2, 108, 163, 79, 94, 15, 46, 104, 62, 23, 47,$
$24, 64, 105, 48, 16, 96, 80, 162, 106, 1).\}.$

Theorem 7. K_n has a balanced C_{55}-t-foil design if and only if $n \equiv 1 \pmod{110t}$.

Example 7.1. Balanced C_{55} design of K_{111}.
Starter: $\{(111, 11, 32, 62, 22, 6, 52, 46, 106, 75, 70, 74,$
$4, 73, 105, 47, 54, 7, 24, 63, 34, 12, 25,$
$13, 36, 64, 26, 8, 56, 48, 104, 71, 3, 5,$
$2, 69, 103, 49, 58, 9, 28, 65, 38, 14, 29,$
$15, 40, 66, 30, 10, 60, 50, 102, 67, 1)\}.$

Example 7.2. Balanced C_{55}-2-foil design of K_{221}.
Starter: $\{(221, 21, 62, 122, 42, 11, 102, 91, 211, 150, 10, 149,$
$139, 148, 210, 92, 104, 12, 44, 123, 64, 22, 45,$
$23, 66, 124, 46, 13, 106, 93, 209, 146, 8, 145,$
$137, 144, 208, 94, 108, 14, 48, 125, 68, 24, 49,$
$25, 70, 126, 50, 15, 110, 95, 207, 142, 6),$
$(221, 26, 72, 127, 52, 16, 112, 96, 206, 140, 5, 9,$
$4, 138, 205, 97, 114, 17, 54, 128, 74, 27, 55,$
$28, 76, 129, 56, 18, 116, 98, 204, 136, 133, 135,$
$2, 134, 203, 99, 118, 19, 58, 130, 78, 29, 59,$
$30, 80, 131, 60, 20, 120, 100, 202, 132, 1).\}.$

Theorem 8. K_n has a balanced C_{66}-t-foil design if and only if $n \equiv 1 \pmod{132t}$.

Example 8.1. Balanced C_{66} design of K_{133}.
Starter: $\{(133, 6, 90, 127, 55, 62, 7, 26, 74, 38, 13, 27,$
$14, 40, 75, 28, 8, 64, 56, 126, 88, 83, 87,$
$4, 86, 125, 57, 66, 9, 30, 76, 42, 15, 31,$
$16, 44, 77, 32, 10, 68, 58, 124, 84, 3, 5,$
$2, 82, 123, 59, 70, 11, 34, 78, 46, 17, 35,$
$18, 48, 79, 36, 12, 72, 60, 122, 80, 1)\}.$

Theorem 9. K_n has a balanced C_{77}-t-foil design if and only if $n \equiv 1 \pmod{154t}$.

Example 9.1. Balanced C_{77} design of K_{155}.
Starter: $\{(155, 15, 44, 86, 30, 8, 72, 64, 148, 105, 98, 104,$
$6, 103, 147, 65, 74, 9, 32, 87, 46, 16, 33,$
$17, 48, 88, 34, 10, 76, 66, 146, 101, 96, 100,$

$4, 99, 145, 67, 78, 11, 36, 89, 50, 18, 37,$
$19, 52, 90, 38, 12, 80, 68, 144, 97, 3, 5,$
$2, 95, 143, 69, 82, 13, 40, 91, 54, 20, 41,$
$21, 56, 92, 42, 14, 84, 70, 142, 93, 1)\}.$

Theorem 10. K_n has a balanced C_{88}-t-foil design if and only if $n \equiv 1 \pmod{176t}$.

Example 10.1. Balanced C_{88} design of K_{177}.
Starter: $\{(177, 8, 120, 169, 73, 82, 9, 34, 98, 50, 17, 35,$
$18, 52, 99, 36, 10, 84, 74, 168, 118, 111, 117,$
$6, 116, 167, 75, 86, 11, 38, 100, 54, 19, 39,$
$20, 56, 101, 40, 12, 88, 76, 166, 114, 109, 113,$
$4, 112, 165, 77, 90, 13, 42, 102, 58, 21, 43,$
$22, 60, 103, 44, 14, 92, 78, 164, 110, 3, 5,$
$2, 108, 163, 79, 94, 15, 46, 104, 62, 23, 47,$
$24, 64, 105, 48, 16, 96, 80, 162, 106, 1)\}.$

Theorem 11. K_n has a balanced C_{99}-t-foil design if and only if $n \equiv 1 \pmod{198t}$.

Example 11.1. Balanced C_{99} design of K_{199}.
Starter: $\{(199, 19, 56, 110, 38, 10, 92, 82, 190, 135, 126, 134,$
$8, 133, 189, 83, 94, 11, 40, 111, 58, 20, 41,$
$21, 60, 112, 42, 12, 96, 84, 188, 131, 124, 130,$
$6, 129, 187, 85, 98, 13, 44, 113, 62, 22, 45,$
$23, 64, 114, 46, 14, 100, 86, 186, 127, 5, 9,$
$4, 125, 185, 87, 102, 15, 48, 115, 66, 24, 49,$
$25, 68, 116, 50, 16, 104, 88, 184, 123, 120, 122,$
$2, 121, 183, 89, 106, 17, 52, 117, 70, 26, 53,$
$27, 72, 118, 54, 18, 108, 90, 182, 119, 1)\}.$

Theorem 12. K_n has a balanced C_{110}-t-foil design if and only if $n \equiv 1 \pmod{220t}$.

Example 12.1. Balanced C_{110} design of K_{221}.
Starter: $\{(221, 10, 150, 211, 91, 102, 11, 42, 122, 62, 21, 43,$
$22, 64, 123, 44, 12, 104, 92, 210, 148, 139, 147,$
$8, 146, 209, 93, 106, 13, 46, 124, 66, 23, 47,$
$24, 68, 125, 48, 14, 108, 94, 208, 144, 137, 143,$
$6, 142, 207, 95, 110, 15, 50, 126, 70, 25, 51,$
$26, 72, 127, 52, 16, 112, 96, 206, 140, 5, 9,$
$4, 138, 205, 97, 114, 17, 54, 128, 74, 27, 55,$
$28, 76, 129, 56, 18, 116, 98, 204, 136, 133, 135,$
$2, 134, 203, 99, 118, 19, 58, 130, 78, 29, 59,$
$30, 80, 131, 60, 20, 120, 100, 202, 132, 1)\}.$

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