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Simple Statistical Tests to Identify Model Limitation or Accuracy of Lattice Approximation for Deterministic Volatility Models

Momchil Marinov,[†1] Koichi Miyazaki[†1] and Junji Mawaribuchi[†1]

In this research we investigate both the model limitation of the fitting to the market price and the accuracy of the approximation in a Lattice Construction method over Deterministic Volatility Models (DVM) using simple statistical tests. Li (2000/2001) proposed the Lattice Construction method which can express the market price flexibly using appropriate DVMs. However, this method has the implicit influence of approximation caused by recombining which is visible for DVM Lattice and is not common for Black-Scholes Lattice model. As a novelty approach we propose a new verification methodology to identify the model limitation or accuracy of lattice approximation for DVMs. It is difficult to discuss about the approximation issue in DVMs because they don't have closed-form solutions like Black-Scholes model. The presented statistical tests use the option model price distribution generated by Monte Carlo simulation technique which doesn't include the approximation of recombining. This specific characteristic of Monte Carlo method is used for capturing the influence of approximation caused by recombining in the DVMs. In that way we can verify whether the estimated models have model limitation of fitting to the market price or (and) whether there is a problem regarding the accuracy of lattice approximation in reproducing the market price.

1. Introduction

An option is a contract that includes right to buy (call option) and right to sell (put option) the underlying asset at an initially agreed price, the strike price. This work is dedicated to European types of options which can be exercised only in the expiration date. In a concord with the development of option markets, various option valuation models have been proposed. The most essential and well known one is Black-Scholes model (BS model)[2]. This model adopts geometric Brownian motion and its volatility of equity return is constant. Applying the Black-Scholes' formula to actual option market prices, we obtain the implied volatility curve. In this curve the volatility values are low for At-The-Money (ATM) options and they are higher when the strike price is away from ATM. In other words the volatility smile is obtained. In actual option market, the assumption of constant volatility is not sufficient because it doesn't capture the market price flexibly (the fitting between the model price and market price is not good). That's why so many volatility models were suggested to provide the flexibility of the volatility.

To represent the volatility of the model more flexibly, Deterministic Volatility Models (Dupire[3], Derman and Kani[4], Rubinstein[5] and so on) have been proposed. Later on, different numerical approaches solving the DVMs are suggested. Li[1] proposed an interesting solution for DVMs using Lattice Construction method. His method is based on a new algorithm for constructing implied binomial trees. In the DVMs the Local Volatility function plays main role. Our work examines four kinds of such functions (including BS model). In an important early contribution to our paper, Mawaribuchi, Miyazaki and Okamoto[6] have estimated the parameters of Local Volatility functions in a way that the difference between the lattice model price and the market price is the smallest possible, using the objective function (4) in Section 2.3 of this paper. Thus, the smaller the difference, the better the fitting of the model. In that way it can be verified which DVM is closer to the market values. In Mawaribuchi, Miyazaki and Okamoto, however, it is not studied about the accuracy of the lattice approximation.

In our paper we provide simple statistical tests to identify the limitation or accuracy of lattice approximation for Deterministic Volatility Models. The idea of capturing the accuracy of lattice approximation is as follows. The parameters in the different Local Volatility functions are estimated so as to minimize the difference between the option market prices and their corresponding model prices estimated by the Lattice Construction method. These parameters are the best ones regarding the lattice method. At the same time they contain the influence of the approximation caused by recombining. In our work, the influence of the approximation in question is examined by a comparison (comparison 1) between

†1 University of Electro-Communications

the option prices estimated by the lattice method with the ones estimated by Monte Carlo simulation (using the same parameters). Specific and important issue of Monte Carlo method is that it doesn't contain the influence of recombining to represent DVMs. To grasp the model limitations, in addition to the previous comparison, we also compare (comparison 2) the option market prices with the model prices estimated by the lattice method. In the case of small differences in both of the comparisons, it can be mentioned that the model captures well the option market prices and we have high accuracy of lattice approximation. On the other hand, when the difference in comparison 2 is large even though the difference in comparison 1 is small, it is concluded that the model has limitation in capturing the option market prices.

This paper is organized as follows: In Section 2 is briefly summarized the preceding literature related to the Lattice Construction method over Deterministic Volatility Models. In Section 3, we describe the purpose of this paper and propose simple statistical tests to identify model limitation or accuracy of lattice approximation for Deterministic Volatility Models. Section 4 is dedicated to empirical analyses and demonstrates the main results. The last section contains the conclusion.

2. Deterministic Volatility Models and Their Lattices (Literature Review)

2.1 Deterministic Volatility Models

The stock price process of Deterministic Volatility Model follows the stochastic differential equation

$$\frac{dS_t}{S_t} = rdt + \sigma\left(S_t, t\right) d\hat{W}, \tag{1}$$

where S_t, r, $\sigma(\cdot)$ and $d\hat{W}$ are underlying asset, risk-free interest rate, local volatility (this function includes the underlying asset S_t and certain period of discretization t), and Brownian motion under risk-neutral measure, respectively. The DVM is specified by the functional form of the local volatility. Four kinds of such models are listed in **Table 1** (including 1 parameter model which is like BS model). Below we explain each of 2 parameter, 3 parameter and 5 parameter models. (Refer to Mawaribuchi, Miyazaki and Okamoto [6] regarding the features of functions $tanh(x)$ and $sech(x)$).

Table 1 Deterministic Volatility Models.

DVM	Local Volatility	
1P	$\sigma\left(S_t, t\right) = a$	BS Model
2P	$\sigma\left(S_t, t\right) = aS_t^b$	CEV Model
3P	$\sigma\left(S_t, t\right) = c + a\left\{1 - \tanh\left[b\left(\frac{S_t - S_0}{S_0}\right)\right]\right\}$	Li Model
5P	$\sigma\left(S_t, t\right) = c + a\left\{1 - \tanh\left[b\left(\frac{S_t - S_0}{S_0}\right)\right]\right\} + d\left\{1 - \operatorname{sech}\left[e\left(\frac{S_t - S_0}{S_0}\right)\right]\right\}$	MMO Model

2 parameter model has two parameters such as (a, b) and its local volatility is b -power of the underlying asset S_t, multiplied by a. The model is often called Constant Elasticity of Variance model (CEV model) and it is known that the model represents skewness of the risk-neutral distribution. Nevertheless its flexibility is not quite big. 3 parameter model has three parameters (a, b, c) and its local volatility contains function $tanh(x)$ [1]. It is able to represent skewness of the risk-neutral distribution more flexibly than 2 parameter model. 5 parameter model has five parameters (a, b, c, d, e) and its local volatility is extension of 3 parameter model by including function $sech(x)$ in addition to $tanh(x)$ [6]. Function $sech(x)$ is upward convex and useful to represent kurtosis of the risk-neutral distribution. Important point is that function $tanh(x)$ can express skewness of the risk-neutral distribution and function $sech(x)$ can express kurtosis flexibly.

2.2 Option Pricing and Lattice Construction Method in DVM (Li [1])

Having in mind that our paper is based on European call and put options, the valuation formulas for these cases are

$$\begin{aligned} \text{Call Price} &= \mathrm{e}^{\text{-rT}} \int_0^\infty \max\left(S_T - K, 0\right) f\left(S_T\right) dS_T \\ \text{Put Price} &= \mathrm{e}^{\text{-rT}} \int_0^\infty \max\left(K - S_T, 0\right) f\left(S_T\right) dS_T, \end{aligned} \tag{2}$$

where r, S_T, K and $f\left(S_T\right)$ are risk-free rate, equity price at the maturity, strike price, and probability density function at the maturity, respectively. In order to evaluate Eq. (2) it is useful to derive numerically (using Lattice Construction method) the density function at the maturity - $f\left(S_T\right)$.

To construct the binomial lattice for each DVM from Table 1, it is convenient

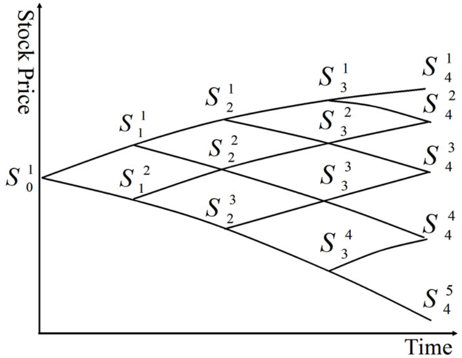

Fig. 1 Basic structure of Li Model Binomial Lattice.

to adopt Li algorithm that proposes setting both up and down transition probabilities at 50%. Hoshika and Miyazaki [7] noticed that the robustness of the Li algorithm is higher compared to that of Derman and Kani [4]. In another paper Mawaribuchi, Miyazaki and Okamoto [6] demonstrated that 5 parameter model can express flexibly various kinds of option market prices. The conceptual graphic of the lattice is shown in **Fig. 1**. S_t^i denotes the underlying asset price at time t, which falls on the i-th node (counting up, starting from the top of the t period). On the figure is shown the skewness of the DVM Lattice.

Li Algorithm

The asset price dynamics between two consecutive time periods (time interval is Δt) are given in Eq. (3). The stock prices in the current period (t) are expressed by using the stock prices from the previous period ($t-1$).

$$\begin{aligned} S_t^1 &= S_{t-1}^1 \left[1 + r\Delta t + \sigma\left(S_{t-1}^1, t\right)\sqrt{\Delta t}\right], \\ S_t^{t+1} &= S_{t-1}^t \left[1 + r\Delta t - \sigma\left(S_{t-1}^t, t\right)\sqrt{\Delta t}\right], \\ S_t^{i+1} &= \frac{1}{2}\left\{\begin{array}{l} S_{t-1}^i \left[1 + r\Delta t - \sigma\left(S_{t-1}^i, t\right)\sqrt{\Delta t}\right] \\ +S_{t-1}^{i+1} \left[1 + r\Delta t + \sigma\left(S_{t-1}^{i+1}, t\right)\sqrt{\Delta t}\right] \end{array}\right\}. \quad (i \neq 0, t) \end{aligned} \tag{3}$$

The first and the second equations generate the top and the bottom stock paths in the lattice and the third equation describes all the stock paths inside the lattice.

2.3 Fitting to the Market Price in the Lattice Construction Method

In option pricing theory, it is important to check whether the adopted model has enough flexibility to express the market price precisely. Below we discuss this issue considering optimization in the Lattice Construction method.

In aim to minimize the sum of square errors (the differences between the model prices and their corresponding market prices) it is convenient to calibrate the parameters of each model and identify which model well replicates the cross-sectional option market prices. The smaller the minimized sum of square errors the better the calibration. Totally 6 kinds of Out-of-The-Money (OTM) options are used in the calibration and these are OTM1 (the strike price is the closest to the current equity price), OTM2 (the strike price is the second closest to the current equity price), OTM3 (the strike price is the third closest to the current equity price) call and put options. In this study, the models are estimated in a way to minimize their objective functions (Eq. (4)). Once the model is identified with the minimum objective functional value, the estimated parameters are the best for the model to replicate the cross-sectional option market prices.

Objective function

$$\min \sum_{i=1}^{6} (P_i' - P_i)^2/6, \tag{4}$$

where P_i and P_i' are option market price and option model price, respectively. i indicates type of option and i = 1, 2, 3, 4, 5 and 6 represents Call OTM1, Call OTM2, Call OTM3, Put OTM1, Put OTM2 and Put OTM3, in order. The reason why we adopt this objective function is to discuss whether the DVM is able to express the cross-sectional option market price (of various strike prices) using same parameters.

In the following empirical analyses, the options are monthly contracts and their maturities are 15 business days. The result of the analyses is attained one for each month. The covered data period is from June 2003 until July 2007, or totally 50 months. The number of discretization periods in the lattice and Monte Carlo methods is 30.

Empirical Results

Figure 2 shows the average absolute difference between the lattice model prices and the corresponding market prices about the four observed models, when the minimum of Eq. (4) is attained. This result is suitable for comparison among DVMs (taking into account several options) but not for comparison among different kinds of options because their market price are different from each other.

The results are of course quite similar to those of Mawaribuchi, Miyazaki and Okamoto [6] although the data covering period is a little different. 1 parameter model (which is like BS model) is a simple one and it can't capture the market price flexibly. 2 parameter model includes skewness to a certain level and it behaves better than 1 parameter model. 3 parameter model represents skewness more flexibly than 2 parameter model and its fitting is better. From all of the observed DVMs it is obvious that the difference between 5 parameter model and the market prices is the lowest. This result justifies the existence of skewness and kurtosis in actual option market, and the fitting of 5 parameter model is the best among the observed four DVMs.

Having in mind that implied volatility skew and smile are observed in actual option market, and considering the results, 1 and 2 parameter models seem to be not good and the use of appropriate DVM like 5 parameter model is recommended.

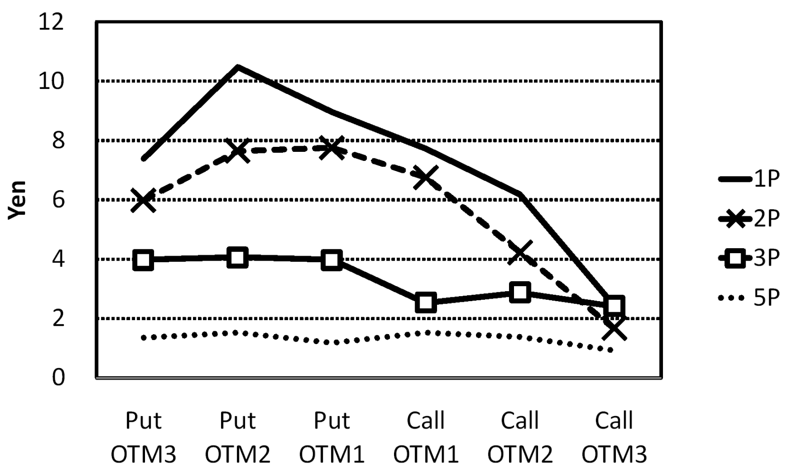

Fig. 2 Absolute difference of lattice and Market prices (average).

3. Purpose and Method of Our Analyses

3.1 Purpose of Our Analyses

In the Li-algorithm, the third equation in Eq. (3) contains a value of 1/2 which is the approximation of recombining. **Figure 3** shows geometrically the stock prices obtained by Li algorithm regarding BS Lattice and DVM Lattice. Looking at the stock price S, one may notice that under suffix is the time, upper suffix are up-movements of the previous stock price $(+)$ and down-movements of the previous stock price $(-)$. For example, S_{t+2}^{+-} is stock price at time $t+2$ and the previous movements of it are up and down. In BS Lattice, up and down range is constant in all stock price periods because the volatility is constant. Stock prices S_{t+2}^{+-} and S_{t+2}^{-+} are the same values in BS Lattice but in DVM Lattice they are not equal because the volatility changes. So, as from Eq. (3), Li algorithm uses approximation of recombining of $\left(S_{t+2}^{+-}+S_{t+2}^{-+}\right)/2$ and the influence of approximation caused by recombining is included when we fit the option model prices to the market prices in the lattice method. Thus, the purpose of our analyses is to identify model limitation or accuracy of lattice approximation for Deterministic Volatility Models.

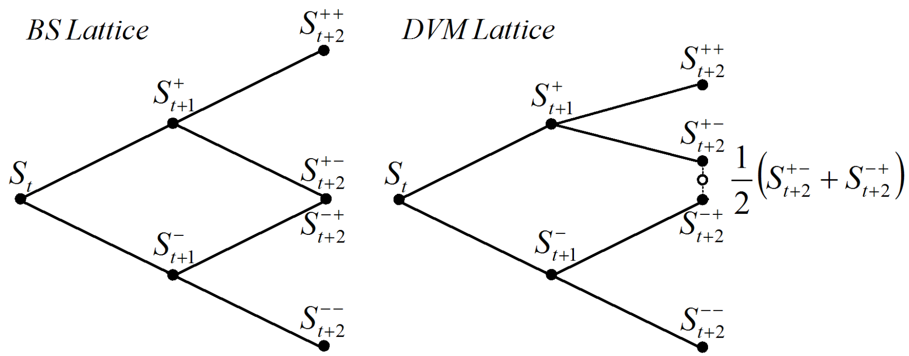

Fig. 3 BS Lattice and DVM Lattice.

3.2 Simple Statistical Tests Based on Monte Carlo Simulation

We propose simple statistical tests to identify model limitation or accuracy of lattice approximation for Deterministic Volatility Models.

As mentioned more up, we use Monte Carlo simulation in this paper for the proposed statistical tests. This numerical method is often used in problems that cannot be solved analytically. Compared to other numerical methods Monte Carlo is easy to be applied for valuing financial products. There are typically three steps: generating sample paths, evaluating the payoff along each path, and taking the average to obtain the option prices. Keeping these steps we simulate and obtain the option prices for all of the estimated DVMs.

Equation (1) takes part in applying the Monte Carlo method to the problem. Equation (1) is transformed to Eq. (5) using the Ito's lemma.

$$S_t = S_0 e^{\int_0^t \left(r - \frac{\sigma^2(S_u,u)}{2}\right)du + \int_0^t \sigma(S_u,u)dW_u}, \qquad (5)$$

where S_t, S_0, r, $\sigma(S_u, u)$ and W are the equity price at t, the initial stock price, the risk-free rate, the Local Volatility function with the parameters estimated by the lattice method and the Brownian motion, respectively. Using this Local Volatility function we can carve out the influence of approximation. The discrete version of Eq. (5) is Eq. (6).

$$S_t = S_0 e^{\sum_{u=0}^{n-1} \left(r - \frac{\sigma^2(S_{u\Delta t}, u\Delta t)}{2}\right)\Delta t + \sum_{u=0}^{n-1} \sigma(S_{u\Delta t}, u\Delta t) z_u \sqrt{\Delta t}} \qquad (6)$$

Here $z \in N(0,1)$, and z is independent and identically distributed (i.i.d.) variable. Δt is the time interval. $n = t/\Delta t$ is the discretization period (n is integer).

In that way the local volatilities in each of the periods of the discretization (Δt) in the discrete model are applied. And the time interval Δt of the Monte Carlo simulation is set to be the same one like in the lattice method.

In this work we use 1000 Monte Carlo paths and the discretization periods are 30. To derive call (put) option price, we compute the value of the discounted payoff function $e^{-rT}\max(S_T - K, 0)$ ($e^{-rT}\max(K - S_T, 0)$) for each one of 1000 Monte Carlo paths and take average of the 1000 values. We define the average as Monte Carlo model price. We iterate the simulation 5000 times and attain 5000 Monte Carlo model prices. Having these 5000 values, later in the paper we present the Monte Carlo model price distribution for all of the estimated DVMs.

Below, the ideas behind the conducted tests are presented. Based on the combination of the results for the two statistical tests with 99% confidence interval (test 1 and test 2), we identify model limitation or accuracy of lattice approximation for Deterministic Volatility Models.

Test 1: Examination of the Accuracy of Lattice Approximation

Test 1 is a comparison between the lattice model price and the Monte Carlo model price. We make a verification concerning the accuracy of approximation caused by recombining for DVM. Thus, the hypothesis of test 1 is that the lattice model price is equal to the Monte Carlo model price. If the lattice model price is rejected by 99% confidence interval of the Monte Carlo model price distribution, it means that the lattice model price is significantly different from the average of Monte Carlo model prices. In other words the approximation caused by recombining for DVM is not accurate. In contrary, if the lattice model price is not rejected the accuracy of the approximation is acceptable.

Test 2: Examination of the Model Limitation without Influence of the Lattice Approximation

Test 2 is a comparison between the market price and the lattice model price. We make a verification concerning the model limitation without influence of the lattice approximation caused by recombining for DVM. Thus, the hypothesis of test 2 is that the market price is equal to the lattice model price. Regarding test 2 we need the lattice model price distribution. For the test, we adopt adjusted Monte Carlo model price distribution which mean is replaced from Monte Carlo model price to lattice model price in aim to remove the influence of the lattice approximation. Therefore we test the difference between the market price and lattice model price using Monte Carlo distribution, which mean is shifted to zero.

Implication of the results: The way to identify model limitation or accuracy of lattice approximation for Deterministic Volatility Models based on the combination of the two statistical tests

The results of using test 1 and test 2 can be divided into the following four cases: (case 1), (case 2), (case 3) and (case 4) show that (test 1, test 2) are (reject, reject), (accept, reject), (reject, accept) and (accept, accept), respectively.

Table 2 Implication of the statistical tests.

	test1	test2	implication
case 1	reject	reject	undecidable
case 2	accept	reject	model limitation
case 3	reject	accept	This model is good for lattice, but we cannot use the parameters of this model for continuous model.
case 4	accept	accept	ideal

First, we consider the cases in which test 2 is rejected. Just because test 2 is rejected, we cannot conclude DVM to have limitation of expressing the market price. In the cases that test 1 is also rejected, there is possibility that test 2 is rejected due to inaccuracy of the approximation caused by recombining. In this case, there is a chance to obtain closer fitting to the market price, if we revamp the approximation method. Taking that into account we can consider (case 1) as undecidable. On the other hand, in the case that test 1 is accepted (and test 2 is still rejected - (case 2)) it means that the DVM cannot reproduce market price even though the approximation caused by recombining is generally accurate. (case 2) means that this model has a limitation.

Next, we consider the cases in which test 2 is accepted. In (case 3) we have that test 1 is rejected and test 2 accepted. It means that the DVM can reproduce the market price using lattice method. However, the parameters of this DVM (originally given as a continuous model) estimated by lattice method contain the influence of approximation caused by recombining and we cannot use them as parameters of the continuous model. In (case 4) we have that test 1 and test 2 are accepted. It means that the estimated models can reproduce the market price and the parameters in the DVM (originally given as a continuous model) estimated by lattice model are suitable for other valuation method like Monte Carlo simulation method. We summarized the above interpretation in **Table 2**.

4. Empirical Analyses

4.1 99% Confidence Interval of the Monte Carlo Model Price Distribution

The two tests described in Section 3 highly depend on Monte Carlo model price distribution. In the case when test 1 and test 2 are accepted, it means that

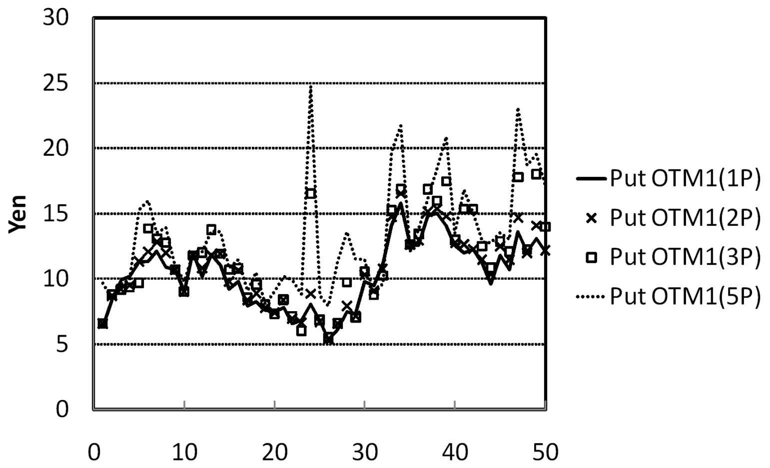

Fig. 4 99% confidence interval of the Monte Carlo model price distribution.

Monte Carlo model prices and lattice model prices are close each other. It also shows us that the lattice model prices and the market prices are close each other. However, we cannot say confidently that Monte Carlo model prices, lattice model prices and market prices are close if the 99% confidence interval of Monte Carlo price distribution is large. Thus, we need to verify the 99% confidence interval of Monte Carlo price distribution before we conduct the tests.

In **Fig. 4**, we show the 99% confidence interval of Monte Carlo model price distribution in Put OTM1. In the initial 30 contract months, the 99% confidence interval is around 10 yen. In the last 20 contract months, the stock market becomes volatile and the 99% confidence interval is around 15 yen. It is difficult to determine whether this value is large or small in absolute sense. For a good rule of thumb, we suggest the use of Bid-Offer Spread (which gives the difference between offer and bid prices) of the options market prices.

When options are traded on the market, the contracts are realized for either offer or bid prices. If the confidence interval of our hypothesis is nearly the same as Bid-Offer Spread, we can say that the lattice model price is close to the Monte Carlo model price (and the lattice model price is close to the market price). Kobayashi, Miyazaki and Tanaka [8] insist that it is reasonable assumption that the Bid-Offer Spread seems to be about 10 (5×2) yen in the usual market condition and a little bit larger in the volatile market condition due to the fact that option prices for Nikkei 225 option are set by 5 yen increments.

4.2 Results and Implications of the Two Tests

In our analyses we examine 6 different options (Call OTM1, Call OTM2, Call OTM3, Put OTM1, Put OTM2 and Put OTM3) which are used in the optimization of the model parameters. For each of the adopted DVMs, in **Table 3**, we summarize how many contract months (out of 50 months) are allocated to the four cases. In aim to make easy to determine the quality of each model, in Table 3, we also provide the weighted average of scores for 50 contract months assuming that scores for (case 4), (case 3), (case 2) and (case 1) are point 3, point 2, point 1 and point 0, respectively. We determine that the model with highest score is superior model.

From the analyses in Table 3 we can see the following four main results. First, in put options which strike price is further from the current stock price (far-Out-of-The Money, Put OTM2 and Put OTM3), 1 parameter model and 2 parameter model are categorized to (case 2) in more than half of total 50 contract months. This fact tells us that these models have limitation for reproducing option market prices.

Second, even for the options whose strike prices are further out from the current stock price, in the case of call options (Call OTM2, Call OTM3), 1 parameter model and 2 parameter model are categorized to (case 2) in less than half of total 50 contract months. Therefore, we can decide that these models have the limitation for reproducing especially the market prices for put options.

Third, regarding the options whose strike prices are close to the current stock price (Call OTM1, Put OTM1), the number of contract months that 5 parameter model is categorized to (case 3) increases. Due to the large influence of approximation caused by recombining, in the use of the parameters estimated by lattice method to other valuation method like Monte Carlo method, the model prices deviate from the corresponding market prices. To remove the deficiency, much better lattice approximation method is expected.

Lastly, we summarize the general evaluation for DVMs based on the score in Table 3. The models that have fewer parameters have lower score. The larger the number of parameters in the Local Volatility function the higher the score. Especially, due to the highest score for all kinds of the options except Call OTM1 for which 3 parameter model has the highest score, 5 parameter model could be considered as the reasonable and reliable model even taking the influence of the approximation caused by recombining in the lattice method into account.

Table 3 Result of the two tests (Two-tailed test (1%) for 50 months).

	case 1	case 2	case 3	case 4	score
Put OTM3(1P)	0	45	0	5	1.20
Put OTM3(2P)	0	37	0	13	1.52
Put OTM3(3P)	0	30	0	20	1.80
Put OTM3(5P)	0	4	0	46	2.84
Put OTM2(1P)	0	40	0	10	1.40
Put OTM2(2P)	0	35	0	15	1.60
Put OTM2(3P)	0	18	0	32	2.28
Put OTM2(5P)	0	3	3	44	2.82
Put OTM1(1P)	0	33	0	17	1.68
Put OTM1(2P)	0	28	0	22	1.88
Put OTM1(3P)	1	9	6	34	2.46
Put OTM1(5P)	0	0	16	34	2.68
Call OTM1(1P)	0	20	0	30	2.20
Call OTM1(2P)	0	21	0	29	2.16
Call OTM1(3P)	2	4	8	36	2.56
Call OTM1(5P)	1	1	23	25	2.44
Call OTM2(1P)	0	23	0	27	2.08
Call OTM2(2P)	0	19	0	31	2.24
Call OTM2(3P)	0	12	0	38	2.52
Call OTM2(5P)	0	0	2	48	2.96
Call OTM3(1P)	1	12	0	37	2.46
Call OTM3(2P)	1	12	0	37	2.46
Call OTM3(3P)	3	19	0	28	2.06
Call OTM3(5P)	0	1	0	49	2.96

5. Conclusions

When we adopt Deterministic Volatility Models with the use of the Lattice Construction method for the valuation of options, the influence of the lattice approximation caused by the recombining is implicitly in the model prices. In this study, we use Lattice Construction method suggested by Li [1] which approximation accuracy is better than other famous preceding methods. We propose simple statistical tests to identify whether the estimated models have limitation of fitting to the market price or (and) whether there is a problem regarding the accuracy of the lattice approximation in reproducing market prices using several DVMs. From the results of the empirical analyses based on the proposed statistical method we observed that in the models having less parameters in their Local Volatility functions, like 1 parameter model and 2 parameter model, the accuracy of the approximation for the lattice construction method is sufficient. However, these models have the limitation in the fitting to the market price especially for far-Out-of-The-Money Put options. On the other hand, 5 parameter model is the closest fitting model to the option market prices in general. However, for the options near At-The-Money, the approximation error caused by recombining in the lattice sometimes could not be dismissed, and we are not able to use the parameters estimated by lattice method for the Monte Carlo simulation method. Over all, we think 5 parameter model would be the best one among the four models taking both measures such as the model limitation and the accuracy of the approximation into account.

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Momchil Marinov was born in 1984. He received his B.E. and M.E. degrees from Sofia University, Sofia, Bulgaria in 2007 and 2009. He is a visiting researcher in University of Electro-Communications (UEC) since 2009.

Koichi Miyazaki was born in 1967. He received his Ph.D. degree from Tsukuba University in 2000. He is an associate professor in University of Electro-Communications (UEC) since 2007. He is a member of the Operations Research Society of Japan, JAFEE, the Japan Society for Industrial and Applied Mathematics and Japanese Society of Applied Statistics.

Junji Mawaribuchi was born in 1986. He received his B.E. degree from University of Electro-Communications (UEC) in 2009. He is a M.E. student at UEC since 2009.