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均衡型 (C_5, C_{12})-Foil デザインと関連デザイン

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グラフ理論において、グラフの分解問題は主要な研究テーマである。C_5 を５点を通るサイクル、C_{12} を１２点を通るサイクルとする。１点を共有する辺素な t 個の C_5 と t 個の C_{12} からなるグラフを (C_5, C_{12})-$2t$-foil という。本研究では、完全グラフ K_n を 均衡的に (C_5, C_{12})-$2t$-foil 部分グラフに分解する均衡型 (C_5, C_{12})-$2t$-foil デザインについて述べる。さらに、均衡型 C_{17}-t-foil デザイン、均衡型 (C_{10}, C_{24})-$2t$-foil デザイン、均衡型 C_{34}-t-foil デザインについて述べる。

Balanced (C_5, C_{12})-Foil Designs and Related Designs

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In graph theory, the decomposition problem of graphs is a very important topic. Various type of decompositions of many graphs can be seen in the literature of graph theory. This paper gives balanced (C_5, C_{12})-$2t$-foil designs, balanced C_{17}-t-foil designs, balanced (C_{10}, C_{24})-$2t$-foil designs, and balanced C_{34}-t-foil designs.

1. Balanced (C_5, C_{12})-$2t$-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_5 and C_{12} be *the* 5-*cycle* and *the* 12-*cycle*, respectively. *The* (C_5, C_{12})-$2t$-*foil* is a graph of t edge-disjoint C_5's and t edge-disjoint C_{12}'s with a common vertex and the common vertex is called *the center of the* (C_5, C_{12})-$2t$-*foil*. When K_n is decomposed into edge-disjoint sum of (C_5, C_{12})-$2t$-foils, we say that K_n *has a* (C_5, C_{12})-$2t$-*foil decomposition*. Moreover, when every vertex of K_n appears in the same number of (C_5, C_{12})-$2t$-foils, we say that K_n *has a balanced* (C_5, C_{12})-$2t$-*foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced* (C_5, C_{12})-$2t$-*foil design*.

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Theorem 1. K_n has a balanced (C_5, C_{12})-$2t$-foil decomposition if and only if $n \equiv 1 \pmod{34t}$.

Proof. **(Necessity)** Suppose that K_n has a balanced (C_5, C_{12})-$2t$-foil decomposition. Let b be the number of (C_5, C_{12})-$2t$-foils and r be the replication number. Then $b = n(n-1)/34t$ and $r = (15t+1)(n-1)/34t$. Among r (C_5, C_{12})-$2t$-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_5, C_{12})-$2t$-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/34t$ and $r_2 = 15(n-1)/34$. Therefore, $n \equiv 1 \pmod{34t}$ is necessary.
(Sufficiency) Put $n = 34st + 1$ and $T = st$. Then $n = 34T + 1$.
Construct a (C_5, C_{12})-$2T$-foil as follows:
$\{(34T+1, 1, 14T+2, 30T+2, 14T), (34T+1, 4T+1, 6T+2, 16T+2, 23T+3, 11T+2, 17T+3, 29T+3, 20T+3, 19T+2, 8T+2, 3T+1)\} \cup$
$\{(34T+1, 2, 14T+4, 30T+3, 14T-1), (34T+1, 4T+2, 6T+4, 16T+3, 23T+5, 11T+3, 17T+5, 29T+4, 20T+5, 19T+3, 8T+4, 3T+2)\} \cup$
$\{(34T+1, 3, 14T+6, 30T+4, 14T-2), (34T+1, 4T+3, 6T+6, 16T+4, 23T+7, 11T+4, 17T+7, 29T+5, 20T+7, 19T+4, 8T+6, 3T+3)\} \cup$
... $\cup$
$\{(34T+1, T-1, 16T-2, 31T, 13T+2), (34T+1, 5T-1, 8T-2, 17T, 25T-1, 12T, 19T-1, 30T+1, 22T-1, 20T, 10T-2, 4T-1)\} \cup$
$\{(34T+1, T, 16T, 31T+1, 13T+1), (34T+1, 5T, 8T, 17T+1, 25T+1, 12T+1, 19T+1, 11T, 22T+1, 20T+1, 10T, 4T)\}$.
($17T$ edges, $17T$ all lengths)
Decompose the (C_5, C_{12})-$2T$-foil into s (C_5, C_{12})-$2t$-foils. Then these starters comprise a balanced (C_5, C_{12})-$2t$-foil decomposition of K_n.

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Example 1.1. Balanced (C_5, C_{12})-2-foil decomposition of K_{35}.
$\{(35, 1, 16, 32, 14), (35, 5, 8, 18, 26, 13, 20, 11, 23, 21, 10, 4)\}$.
(17 edges, 17 all lengths)
This starter comprises a balanced (C_5, C_{12})-2-foil decomposition of K_{35}.

Example 1.2. Balanced (C_5, C_{12})-4-foil decomposition of K_{69}.
$\{(69, 1, 30, 62, 28), (69, 9, 14, 34, 49, 24, 37, 61, 43, 40, 18, 7)\} \cup$
$\{(69, 2, 32, 63, 27), (69, 10, 16, 35, 51, 25, 39, 22, 45, 41, 20, 8)\}$.
(34 edges, 34 all lengths)
This starter comprises a balanced (C_5, C_{12})-4-foil decomposition of K_{69}.

Example 1.3. Balanced (C_5, C_{12})-6-foil decomposition of K_{103}.
$\{(103, 1, 44, 92, 42), (103, 13, 20, 50, 72, 35, 54, 90, 63, 59, 26, 10)\} \cup$
$\{(103, 2, 46, 93, 41), (103, 14, 22, 51, 74, 36, 56, 91, 65, 60, 28, 11)\} \cup$
$\{(103, 3, 48, 94, 40), (103, 15, 24, 52, 76, 37, 58, 33, 67, 61, 30, 12)\}$.
(51 edges, 51 all lengths)
This starter comprises a balanced (C_5, C_{12})-6-foil decomposition of K_{103}.

Example 1.4. Balanced (C_5, C_{12})-8-foil decomposition of K_{137}.
$\{(137, 1, 58, 122, 56), (137, 17, 26, 66, 95, 46, 71, 119, 83, 78, 34, 13)\} \cup$
$\{(137, 2, 60, 123, 55), (137, 18, 28, 67, 97, 47, 73, 120, 85, 79, 36, 14)\} \cup$
$\{(137, 3, 62, 124, 54), (137, 19, 30, 68, 99, 48, 75, 121, 87, 80, 38, 15)\} \cup$
$\{(137, 4, 64, 125, 53), (137, 20, 32, 69, 101, 49, 77, 44, 89, 81, 40, 16)\}$.
(68 edges, 68 all lengths)
This starter comprises a balanced (C_5, C_{12})-8-foil decomposition of K_{137}.

Example 1.5. Balanced (C_5, C_{12})-10-foil decomposition of K_{171}.
$\{(171, 1, 72, 152, 70), (171, 21, 32, 82, 118, 57, 88, 148, 103, 97, 42, 16)\} \cup$
$\{(171, 2, 74, 153, 69), (171, 22, 34, 83, 120, 58, 90, 149, 105, 98, 44, 17)\} \cup$
$\{(171, 3, 76, 154, 68), (171, 23, 36, 84, 122, 59, 92, 150, 107, 99, 46, 18)\} \cup$
$\{(171, 4, 78, 155, 67), (171, 24, 38, 85, 124, 60, 94, 151, 109, 100, 48, 19)\} \cup$
$\{(171, 5, 80, 156, 66), (171, 25, 40, 86, 126, 61, 96, 55, 111, 101, 50, 20)\}$.
(85 edges, 85 all lengths)
This starter comprises a balanced (C_5, C_{12})-10-foil decomposition of K_{171}.

Example 1.6. Balanced (C_5, C_{12})-12-foil decomposition of K_{205}.
$\{(205, 1, 86, 182, 84), (205, 25, 38, 98, 141, 68, 105, 177, 123, 116, 50, 19)\} \cup$
$\{(205, 2, 88, 183, 83), (205, 26, 40, 99, 143, 69, 107, 178, 125, 117, 52, 20)\} \cup$
$\{(205, 3, 90, 184, 82), (205, 27, 42, 100, 145, 70, 109, 179, 127, 118, 54, 21)\} \cup$
$\{(205, 4, 92, 185, 81), (205, 28, 44, 101, 147, 71, 111, 180, 129, 119, 56, 22)\} \cup$
$\{(205, 5, 94, 186, 80), (205, 29, 46, 102, 149, 72, 113, 181, 131, 120, 58, 23)\} \cup$
$\{(205, 6, 96, 187, 79), (205, 30, 48, 103, 151, 73, 115, 66, 133, 121, 60, 24)\}$.
(102 edges, 102 all lengths)
This starter comprises a balanced (C_5, C_{12})-12-foil decomposition of K_{205}.

2. Balanced C_{17}-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_{17} be *the* 17*-cycle*. *The* C_{17}*-*t*-foil* is a graph of t edge-disjoint C_{17}'s with a common vertex and the common vertex is called *the center of the* C_{17}*-*t*-foil*. When K_n is decomposed into edge-disjoint sum of C_{17}-t-foils, it is called that K_n *has a* C_{17}*-*t*-foil decomposition*. Moreover, when every vertex of K_n appears in the same number of C_{17}-t-foils, it is called that K_n *has a balanced* C_{17}*-*t*-foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced* C_{17}*-*t*-foil design*.

Theorem 2. K_n has a balanced C_{17}-t-foil decomposition if and only if $n \equiv 1 \pmod{34t}$.

Proof. (Necessity) Suppose that K_n has a balanced C_{17}-t-foil decomposition. Let b be the number of C_{17}-t-foils and r be the replication number. Then $b = n(n-1)/34t$ and $r = (16t+1)(n-1)/34t$. Among r C_{17}-t-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of C_{17}-t-foils in which v is the center and v is not the center, respectively.

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Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $2tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/34t$ and $r_2 = 16(n-1)/34$. Therefore, $n \equiv 1 \pmod{34t}$ is necessary.

(Sufficiency) Put $n = 34st + 1, T = st$. Then $n = 34T + 1$. Construct a C_{17}-T-foil as follows:

$\{$ $(34T+1, T, 16T, 31T+1, 13T+1, 17T+2, 4T+1, 6T+2, 16T+2, 23T+3, 11T+2, 17T+3, 29T+3, 20T+3, 19T+2, 8T+2, 3T+1)$,
$(34T+1, T-1, 16T-2, 31T, 13T+2, 17T+4, 4T+2, 6T+4, 16T+3, 23T+5, 11T+3, 17T+5, 29T+4, 20T+5, 19T+3, 8T+4, 3T+2)$,
$(34T+1, T-2, 16T-4, 31T-1, 13T+3, 17T+6, 4T+3, 6T+6, 16T+4, 23T+7, 11T+4, 17T+7, 29T+5, 20T+7, 19T+4, 8T+6, 3T+3)$,
...,
$(34T+1, 2, 14T+4, 30T+3, 14T-1, 19T-2, 5T-1, 8T-2, 17T, 25T-1, 12T, 19T-1, 30T+1, 22T-1, 20T, 10T-2, 4T-1)$,
$(34T+1, 1, 14T+2, 30T+2, 14T, 19T, 5T, 8T, 17T+1, 25T+1, 12T+1, 19T+1, 11T, 22T+1, 20T+1, 10T, 4T)$ $\}$.
($17T$ edges, $17T$ all lengths)

Decompose this C_{17}-T-foil into s C_{17}-t-foils. Then these starters comprise a balanced C_{17}-t-foil decomposition of K_n.

Example 2.1. Balanced C_{17}-decomposition of K_{35}.
$\{(35, 1, 16, 32, 14, 19, 5, 8, 18, 26, 13, 20, 11, 23, 21, 10, 4)\}$.
(17 edges, 17 all lengths)
This stater comprises a balanced C_{17}-decomposition of K_{35}.

Example 2.2. Balanced C_{17}-2-foil decomposition of K_{69}.
$\{(69, 2, 32, 63, 27, 36, 9, 14, 34, 49, 24, 37, 61, 43, 40, 18, 7)$,
$(69, 1, 30, 62, 28, 38, 10, 16, 35, 51, 25, 39, 22, 45, 41, 20, 8)\}$.
(34 edges, 34 all lengths)
This stater comprises a balanced C_{17}-2-foil decomposition of K_{69}.

Example 2.3. Balanced C_{17}-3-foil decomposition of K_{103}.
$\{(103, 3, 48, 94, 40, 53, 13, 20, 50, 72, 35, 54, 90, 63, 59, 26, 10)$,
$(103, 2, 46, 93, 41, 55, 14, 22, 51, 74, 36, 56, 91, 65, 60, 28, 11)$,
$(103, 1, 44, 92, 42, 57, 15, 24, 52, 76, 37, 58, 33, 67, 61, 30, 12)\}$.
(51 edges, 51 all lengths)
This stater comprises a balanced C_{17}-3-foil decomposition of K_{103}.

Example 2.4. Balanced C_{17}-4-foil decomposition of K_{137}.
$\{(137, 4, 64, 125, 53, 70, 17, 26, 66, 95, 46, 71, 119, 83, 78, 34, 13)$,
$(137, 3, 62, 124, 54, 72, 18, 28, 67, 97, 47, 73, 120, 85, 79, 36, 14)$,
$(137, 2, 60, 123, 55, 74, 19, 30, 68, 99, 48, 75, 121, 87, 80, 38, 15)$,
$(137, 1, 58, 122, 56, 76, 20, 32, 69, 101, 49, 77, 44, 89, 81, 40, 16)\}$.
(68 edges, 68 all lengths)
This stater comprises a balanced C_{17}-4-foil decomposition of K_{137}.

Example 2.5. Balanced C_{17}-5-foil decomposition of K_{171}.
$\{(171, 5, 80, 156, 66, 87, 21, 32, 82, 118, 57, 88, 148, 103, 97, 42, 16)$,
$(171, 4, 78, 155, 67, 89, 22, 34, 83, 120, 58, 90, 149, 105, 98, 44, 17)$,
$(171, 3, 76, 154, 68, 91, 23, 36, 84, 122, 59, 92, 150, 107, 99, 46, 18)$,
$(171, 2, 74, 153, 69, 93, 24, 38, 85, 124, 60, 94, 151, 109, 100, 48, 19)$,
$(171, 1, 72, 152, 70, 95, 25, 40, 86, 126, 61, 96, 55, 111, 101, 50, 20)\}$.
(85 edges, 85 all lengths)
This stater comprises a balanced C_{17}-5-foil decomposition of K_{171}.

Example 2.6. Balanced C_{17}-6-foil decomposition of K_{205}.
$\{(205, 6, 96, 187, 79, 104, 25, 38, 98, 141, 68, 105, 177, 123, 116, 50, 19)$,
$(205, 5, 94, 186, 80, 106, 26, 40, 99, 143, 69, 107, 178, 125, 117, 52, 20)$,
$(205, 4, 92, 185, 81, 108, 27, 42, 100, 145, 70, 109, 179, 127, 118, 54, 21)$,
$(205, 3, 90, 184, 82, 110, 28, 44, 101, 147, 71, 111, 180, 129, 119, 56, 22)$,
$(205, 2, 88, 183, 83, 112, 29, 46, 102, 149, 72, 113, 181, 131, 120, 58, 23)$,
$(205, 1, 86, 182, 84, 114, 30, 48, 103, 151, 73, 115, 66, 133, 121, 60, 24)\}$.

(102 edges, 102 all lengths)
This stater comprises a balanced C_{17}-6-foil decomposition of K_{205}.

3. Balanced (C_{10}, C_{24})-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_{10} and C_{24} be *the* 10-*cycle* and *the* 24-*cycle*, respectively. *The* (C_{10}, C_{24})-2t-*foil* is a graph of t edge-disjoint C_{10}'s and t edge-disjoint C_{24}'s with a common vertex and the common vertex is called *the center of the* (C_{10}, C_{24})-2t-*foil*. When K_n is decomposed into edge-disjoint sum of (C_{10}, C_{24})-2t-foils, we say that K_n *has a* (C_{10}, C_{24})-2t-*foil decomposition*. Moreover, when every vertex of K_n appears in the same number of (C_{10}, C_{24})-2t-foils, we say that K_n *has a balanced* (C_{10}, C_{24})-2t-*foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced* (C_{10}, C_{24})-2t-*foil design*.

Theorem 3. K_n has a balanced (C_{10}, C_{24})-2t-foil decomposition if and only if $n \equiv 1 \pmod{68t}$.

Proof. **(Necessity)** Suppose that K_n has a balanced (C_{10}, C_{24})-2t-foil decomposition. Let b be the number of (C_{10}, C_{24})-2t-foils and r be the replication number. Then $b = n(n-1)/68t$ and $r = (32t+1)(n-1)/68t$. Among r (C_{10}, C_{24})-2t-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_{10}, C_{24})-2t-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/68t$ and $r_2 = 32(n-1)/68$. Therefore, $n \equiv 1 \pmod{68t}$ is necessary.
(Sufficiency) Put $n = 68st + 1$ and $T = st$. Then $n = 68T + 1$.
Construct a (C_{10}, C_{24})-2T-foil as follows:
$\{(68T+1, 1, 28T+2, 60T+2, 28T, 56T-1, 28T-1, 60T+3, 28T+4, 2), (68T+1, 8T+1, 12T+2, 32T+2, 46T+3, 22T+2, 34T+3, 58T+3, 40T+3, 38T+2, 16T+2, 6T+1, 12T+3, 6T+2, 16T+4, 38T+3, 40T+5, 58T+4, 34T+5, 22T+3, 46T+5, 32T+3, 12T+4, 8T+2)\} \cup$
$\{(68T+1, 3, 28T+6, 60T+4, 28T-2, 56T-5, 28T-3, 60T+5, 28T+8, 4), (68T+1, 8T+3, 12T+6, 32T+4, 46T+7, 22T+4, 34T+7, 58T+5, 40T+7, 38T+4, 16T+6, 6T+3, 12T+7, 6T+4, 16T+8, 38T+5, 40T+9, 58T+6, 34T+9, 22T+5, 46T+9, 32T+5, 12T+8, 8T+4)\} \cup$
$\{(68T+1, 5, 28T+10, 60T+6, 28T-4, 56T-9, 28T-5, 60T+7, 28T+12, 6), (68T+1, 8T+5, 12T+10, 32T+6, 46T+11, 22T+6, 34T+11, 58T+7, 40T+11, 38T+6, 16T+10, 6T+5, 12T+11, 6T+6, 16T+12, 38T+7, 40T+13, 58T+8, 34T+13, 22T+7, 46T+13, 32T+7, 12T+12, 8T+6)\} \cup$
... $\cup$
$\{(68T+1, 2T-1, 32T-2, 62T, 26T+2, 52T+3, 26T+1, 62T+1, 32T, 2T), (68T+1, 10T-1, 16T-2, 34T, 50T-1, 24T, 38T-1, 60T+1, 44T-1, 40T, 20T-2, 8T-1, 16T-1, 8T, 20T, 40T+1, 44T+1, 22T, 38T+1, 24T+1, 50T+1, 34T+1, 16T, 10T)\}$.
($34T$ edges, $34T$ all lengths)
Decompose the (C_{10}, C_{24})-2T-foil into s (C_{10}, C_{24})-2t-foils. Then these starters comprise a balanced (C_{10}, C_{24})-2t-foil decomposition of K_n.

Example 3.1. Balanced (C_{10}, C_{24})-2-foil decomposition of K_{69}.
$\{(69, 1, 30, 62, 28, 55, 27, 63, 32, 2),$
$(69, 9, 14, 34, 49, 24, 37, 61, 43, 40, 18, 7, 15, 8, 20, 41, 45, 22, 39, 25, 51, 35, 16, 10)\}$.
(34 edges, 34 all lengths)
This starter comprises a balanced (C_{10}, C_{24})-2-foil decomposition of K_{69}.

Example 3.2. Balanced (C_{10}, C_{24})-4-foil decomposition of K_{137}.
$\{(137, 1, 58, 122, 56, 111, 55, 123, 60, 2),$
$(137, 3, 62, 124, 54, 107, 53, 125, 64, 4)\} \cup$
$\{(137, 17, 26, 66, 95, 46, 71, 119, 83, 78, 34, 13, 27, 14, 36, 79, 85, 120, 73, 47, 97, 67, 28, 18),$
$(137, 19, 30, 68, 99, 48, 75, 121, 87, 80, 38, 15, 31, 16, 40, 81, 89, 44, 77, 49, 101, 69, 32, 20)\}$.
(68 edges, 68 all lengths)
This starter comprises a balanced (C_{10}, C_{24})-4-foil decomposition of K_{137}.

Example 3.3. Balanced (C_{10}, C_{24})-6-foil decomposition of K_{205}.
$\{(205, 1, 86, 182, 84, 167, 83, 183, 88, 2),$

$(205, 3, 90, 184, 82, 163, 81, 185, 92, 4)$,
$(205, 5, 94, 186, 80, 159, 79, 187, 96, 6)\} \quad \cup$
$\{(205, 25, 38, 98, 141, 68, 105, 177, 123, 116, 50, 19, 39, 20, 52, 117, 125, 178, 107, 69, 143, 99, 40, 26)$,
$(205, 27, 42, 100, 145, 70, 109, 179, 127, 118, 54, 21, 43, 22, 56, 119, 129, 180, 111, 71, 147, 101, 44, 28)$,
$(205, 29, 46, 102, 149, 72, 113, 181, 131, 120, 58, 23, 47, 24, 60, 121, 133, 66, 115, 73, 151, 103, 48, 30)\}$.
(102 edges, 102 all lengths)

This starter comprises a balanced (C_{10}, C_{24})-6-foil decomposition of K_{205}.

Example 3.4. Balanced (C_{10}, C_{24})-8-foil decomposition of K_{273}.
$\{(273, 1, 114, 242, 112, 223, 111, 243, 116, 2)$,
$(273, 3, 118, 244, 110, 219, 109, 245, 120, 4)$,
$(273, 5, 122, 246, 108, 215, 107, 247, 124, 6)$,
$(273, 7, 126, 248, 106, 211, 105, 249, 128, 8)\} \quad \cup$
$\{(273, 33, 50, 130, 187, 90, 139, 235, 163, 154, 66, 25, 51, 26, 68, 155, 165, 236, 141, 91, 189, 131, 52, 34)$,
$(273, 35, 54, 132, 191, 92, 143, 237, 167, 156, 70, 27, 55, 28, 72, 157, 169, 238, 145, 93, 193, 133, 56, 36)$,
$(273, 37, 58, 134, 195, 94, 147, 239, 171, 158, 74, 29, 59, 30, 76, 159, 173, 240, 149, 95, 197, 135, 60, 38)$,
$(273, 39, 62, 136, 199, 96, 151, 241, 175, 160, 78, 31, 63, 32, 80, 161, 177, 88, 153, 97, 201, 137, 64, 40)\}$.
(136 edges, 136 all lengths)

This starter comprises a balanced (C_{10}, C_{24})-8-foil decomposition of K_{273}.

4. Balanced C_{34}-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_{34} be *the 34-cycle*. *The C_{34}-t-foil* is a graph of t edge-disjoint C_{34}'s with a common vertex and the common vertex is called *the center of the C_{34}-t-foil*. When K_n is decomposed into edge-disjoint sum of C_{34}-t-foils, it is called that K_n *has a C_{34}-t-foil decomposition*. Moreover, when every vertex of K_n appears in the same number of C_{34}-t-foils, it is called that K_n *has a balanced C_{34}-t-foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced C_{34}-t-foil design*.

Theorem 4. K_n has a balanced C_{34}-t-foil decomposition if and only if $n \equiv 1 \pmod{68t}$.

Proof. (Necessity) Suppose that K_n has a balanced C_{34}-t-foil decomposition. Let b be the number of C_{34}-t-foils and r be the replication number. Then $b = n(n-1)/68t$ and $r = (33t+1)(n-1)/68t$. Among r C_{34}-t-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of C_{34}-t-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $2tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/68t$ and $r_2 = 33(n-1)/68$. Therefore, $n \equiv 1 \pmod{68t}$ is necessary.
(Sufficiency) Put $n = 68st + 1, T = st$. Then $n = 68T + 1$. Construct a C_{34}-T-foil as follows:
$\{ (68T+1, 2T, 32T, 62T+1, 26T+1, 34T+2, 8T+1, 12T+2, 32T+2, 46T+3, 22T+2, 34T+3, 58T+3, 40T+3, 38T+2, 16T+2, 6T+1, 12T+3, 6T+2, 16T+4, 38T+3, 40T+5, 58T+4, 34T+5, 22T+3, 46T+5, 32T+3, 12T+4, 8T+2, 34T+4, 26T+2, 62T, 32T-2, 2T-1)$,
$(68T+1, 2T-2, 32T-4, 62T-1, 26T+3, 34T+6, 8T+3, 12T+6, 32T+4, 46T+7, 22T+4, 34T+7, 58T+5, 40T+7, 38T+4, 16T+6, 6T+3, 12T+7, 6T+4, 16T+8, 38T+5, 40T+9, 58T+6, 34T+9, 22T+5, 46T+9, 32T+5, 12T+8, 8T+4, 34T+8, 26T+4, 62T-2, 32T-6, 2T-3)$,
$(68T+1, 2T-4, 32T-8, 62T-3, 26T+5, 34T+10, 8T+5, 12T+10, 32T+6, 46T+11, 22T+6, 34T+11, 58T+7, 40T+11, 38T+6, 16T+10, 6T+5, 12T+11, 6T+6, 16T+12, 38T+7, 40T+13, 58T+8, 34T+13, 22T+7, 46T+13, 32T+7, 12T+12, 8T+6, 34T+12, 26T+6, 62T-4, 32T-10, 2T-5)$,
...,
$(68T+1, 2, 28T+4, 60T+3, 28T-1, 38T-2, 10T-1, 16T-2, 34T, 50T-1, 24T, 38T-1, 60T+1, 44T-1, 40T, 20T-2, 8T-1, 16T-1, 8T, 20T, 40T+1, 44T+1, 22T, 38T+1, 24T+1, 50T+1, 34T+1, 16T, 10T, 38T, 28T, 60T+2, 28T+2, 1) \}$.
($34T$ edges, $34T$ all lengths)
Decompose this C_{34}-T-foil into s C_{34}-t-foils. Then these starters comprise a balanced C_{34}-t-foil decomposition of K_n.

Example 4.1. Balanced C_{34}-decomposition of K_{69}.

$\{(69, 2, 32, 63, 27, 36, 9, 14, 34, 49, 24, 37, 61, 43, 40, 18, 7, 15, 8, 20, 41, 45, 22, 39, 25, 51, 35, 16, 10, 38, 28, 62, 30, 1)\}$.

(34 edges, 34 all lengths)

This stater comprises a balanced C_{34}-decomposition of K_{69}.

Example 4.2. Balanced C_{34}-2-foil decomposition of K_{137}.

$\{(137, 4, 64, 125, 53, 70, 17, 26, 66, 95, 46, 71, 119, 83, 78, 34, 13, 27, 14, 36, 79, 85, 120, 73, 47, 97, 67, 28, 18, 72, 54, 124, 62, 3)$,

$(137, 2, 60, 123, 55, 74, 19, 30, 68, 99, 48, 75, 121, 87, 80, 38, 15, 31, 16, 40, 81, 89, 44, 77, 49, 101, 69, 32, 20, 76, 56, 122, 58, 1)\}$.

(68 edges, 68 all lengths)

This stater comprises a balanced C_{34}-2-foil decomposition of K_{137}.

Example 4.3. Balanced C_{34}-3-foil decomposition of K_{205}.

$\{(205, 6, 96, 187, 79, 104, 25, 38, 98, 141, 68, 105, 177, 123, 116, 50, 19, 39, 20, 52, 117, 125, 178, 107, 69, 143, 99, 40, 26, 106, 80, 186, 94, 5)$,

$(205, 4, 92, 185, 81, 108, 27, 42, 100, 145, 70, 109, 179, 127, 118, 54, 21, 43, 22, 56, 119, 129, 180, 111, 71, 147, 101, 44, 28, 110, 82, 184, 90, 3)$,

$(205, 2, 88, 183, 83, 112, 29, 46, 102, 149, 72, 113, 181, 131, 120, 58, 23, 47, 24, 60, 121, 133, 66, 115, 73, 151, 103, 48, 30, 114, 84, 182, 86, 1)\}$.

(102 edges, 102 all lengths)

This stater comprises a balanced C_{34}-3-foil decomposition of K_{205}.

Example 4.4. Balanced C_{34}-4-foil decomposition of K_{273}.

$\{(273, 8, 128, 249, 105, 138, 33, 50, 130, 187, 90, 139, 235, 163, 154, 66, 25, 51, 26, 68, 155, 165, 236, 141, 91, 189, 131, 52, 34, 140, 106, 248, 126, 7)$,

$(273, 6, 124, 247, 107, 142, 35, 54, 132, 191, 92, 143, 237, 167, 156, 70, 27, 55, 28, 72, 157, 169, 238, 145, 93, 193, 133, 56, 36, 144, 108, 246, 122, 5)$,

$(273, 4, 120, 245, 109, 146, 37, 58, 134, 195, 94, 147, 239, 171, 158, 74, 29, 59, 30, 76, 159, 173, 240, 149, 95, 197, 135, 60, 38, 148, 110, 244, 118, 3)$,

$(273, 2, 116, 243, 111, 150, 39, 62, 136, 199, 96, 151, 241, 175, 160, 78, 31, 63, 32, 80, 161, 177, 88, 153, 97, 201, 137, 64, 40, 152, 112, 242, 114, 1)\}$.

(136 edges, 136 all lengths)

This stater comprises a balanced C_{34}-4-foil decomposition of K_{273}.

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