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均衡型 (C_5, C_8)-Foil デザインと関連デザイン

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グラフ理論において、グラフの分解問題は主要な研究テーマである。C_5 を5点を通るサイクル、C_8 を8点を通るサイクルとする。1点を共有する辺素な t 個の C_5 と t 個の C_8 からなるグラフを (C_5, C_8)-$2t$-foil という。本研究では、完全グラフ K_n を 均衡的に (C_5, C_8)-$2t$-foil 部分グラフに分解する均衡型 (C_5, C_8)-$2t$-foil デザインについて述べる。さらに、均衡型 C_{13}-t-foil デザイン、均衡型 (C_{10}, C_{16})-$2t$-foil デザイン、均衡型 C_{26}-t-foil デザインについて述べる。

Balanced (C_5, C_8)-Foil Designs and Related Designs

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In graph theory, the decomposition problem of graphs is a very important topic. Various type of decompositions of many graphs can be seen in the literature of graph theory. This paper gives balanced (C_5, C_8)-$2t$-foil designs, balanced C_{13}-t-foil designs, balanced (C_{10}, C_{16})-$2t$-foil designs, and balanced C_{26}-t-foil designs.

1. Balanced (C_5, C_8)-$2t$-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_5 and C_8 be *the* 5-*cycle* and *the* 8-*cycle*, respectively. *The* (C_5, C_8)-$2t$-*foil* is a graph of t edge-disjoint C_5's and t edge-disjoint C_8's with a common vertex and the common vertex is called *the center of the* (C_5, C_8)-$2t$-*foil*. In particular, the (C_5, C_8)-2-foil is called *the* (C_5, C_8)-*bowtie*. When K_n is decomposed into edge-disjoint sum of (C_5, C_8)-$2t$-foils, we say that K_n *has a* (C_5, C_8)-$2t$-*foil decomposition*. Moreover, when every vertex of K_n appears in the same number of (C_5, C_8)-$2t$-foils, we say that K_n *has a balanced* (C_5, C_8)-$2t$-*foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced* (C_5, C_8)-$2t$-*foil design*.

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Theorem 1. K_n has a balanced (C_5, C_8)-$2t$-foil decomposition if and only if $n \equiv 1 \pmod{26t}$.

Proof. **(Necessity)** Suppose that K_n has a balanced (C_5, C_8)-$2t$-foil decomposition. Let b be the number of (C_5, C_8)-$2t$-foils and r be the replication number. Then $b = n(n-1)/26t$ and $r = (11t+1)(n-1)/26t$. Among r (C_5, C_8)-$2t$-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_5, C_8)-$2t$-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n-1$. From these relations, $r_1 = (n-1)/26t$ and $r_2 = 11(n-1)/26$. Therefore, $n \equiv 1 \pmod{26t}$ is necessary.
(Sufficiency) Put $n = 26st+1$ and $T = st$. Then $n = 26T+1$. Construct a (C_5, C_8)-$2T$-foil as follows:
$\{(26T+1, 1, 10T+2, 22T+2, 10T), (26T+1, T+1, 19T+2, 24T+2, 3T+2, 23T+2, 5T+2, 2T+1)\} \cup$
$\{(26T+1, 2, 10T+4, 22T+3, 10T-1), (26T+1, T+2, 19T+4, 24T+3, 3T+4, 23T+3, 5T+4, 2T+2)\} \cup$
$\{(26T+1, 3, 10T+6, 22T+4, 10T-2), (26T+1, T+3, 19T+6, 24T+4, 3T+6, 23T+4, 5T+6, 2T+3)\} \cup$
... $\cup$
$\{(26T+1, T, 12T, 23T+1, 9T+1), (26T+1, 2T, 21T, 25T+1, 5T, 24T+1, 7T, 3T)\}$.
Decompose the (C_5, C_8)-$2T$-foil into s (C_5, C_8)-$2t$-foils. Then these starters comprise a balanced (C_5, C_8)-$2t$-foil decomposition of K_n.

Corollary 1. K_n has a balanced (C_5, C_8)-bowtie decomposition if and only if $n \equiv 1 \pmod{26}$.

Example 1.1. Balanced (C_5, C_8)-2-foil decomposition of K_{27}.
$\{(27, 1, 12, 24, 10), (27, 2, 21, 26, 5, 25, 7, 3)\}$.
This starter comprises a balanced (C_5, C_8)-2-foil decomposition of K_{27}.

Example 1.2. Balanced (C_5, C_8)-4-foil decomposition of K_{53}.
$\{(53, 1, 22, 46, 20), (53, 3, 40, 50, 8, 48, 12, 5)\} \cup$
$\{(53, 2, 24, 47, 19), (53, 4, 42, 51, 10, 49, 14, 6)\}$.
This starter comprises a balanced (C_5, C_8)-4-foil decomposition of K_{53}.

Example 1.3. Balanced (C_5, C_8)-6-foil decomposition of K_{79}.
$\{(79, 1, 32, 68, 30), (79, 4, 59, 74, 11, 71, 17, 7)\} \cup$
$\{(79, 2, 34, 69, 29), (79, 5, 61, 75, 13, 72, 19, 8)\} \cup$
$\{(79, 3, 36, 70, 28), (79, 6, 63, 76, 15, 73, 21, 9)\}$.
This starter comprises a balanced (C_5, C_8)-6-foil decomposition of K_{79}.

Example 1.4. Balanced (C_5, C_8)-8-foil decomposition of K_{105}.
$\{(105, 1, 42, 90, 40), (105, 5, 78, 98, 14, 94, 22, 9)\} \cup$
$\{(105, 2, 44, 91, 39), (105, 6, 80, 99, 16, 95, 24, 10)\} \cup$
$\{(105, 3, 46, 92, 38), (105, 7, 82, 100, 18, 96, 26, 11)\} \cup$
$\{(105, 4, 48, 93, 37), (105, 8, 84, 101, 20, 97, 28, 12)\}$.
This starter comprises a balanced (C_5, C_8)-8-foil decomposition of K_{105}.

Example 1.5. Balanced (C_5, C_8)-10-foil decomposition of K_{131}.
$\{(131, 1, 52, 112, 50), (131, 6, 97, 122, 17, 117, 27, 11)\} \cup$
$\{(131, 2, 54, 113, 49), (131, 7, 99, 123, 19, 118, 29, 12)\} \cup$
$\{(131, 3, 56, 114, 48), (131, 8, 101, 124, 21, 119, 31, 13)\} \cup$
$\{(131, 4, 58, 115, 47), (131, 9, 103, 125, 23, 120, 33, 14)\} \cup$
$\{(131, 5, 60, 116, 46), (131, 10, 105, 126, 25, 121, 35, 15)\}$.
This starter comprises a balanced (C_5, C_8)-10-foil decomposition of K_{131}.

Example 1.6. Balanced (C_5, C_8)-12-foil decomposition of K_{157}.
$\{(157, 1, 62, 134, 60), (157, 7, 116, 146, 20, 140, 32, 13)\} \cup$
$\{(157, 2, 64, 135, 59), (157, 8, 118, 147, 22, 141, 34, 14)\} \cup$
$\{(157, 3, 66, 136, 58), (157, 9, 120, 148, 24, 142, 36, 15)\} \cup$
$\{(157, 4, 68, 137, 57), (157, 10, 122, 149, 26, 143, 38, 16)\} \cup$
$\{(157, 5, 70, 138, 56), (157, 11, 124, 150, 28, 144, 40, 17)\} \cup$
$\{(157, 6, 72, 139, 55), (157, 12, 126, 151, 30, 145, 42, 18)\}$.
This starter comprises a balanced (C_5, C_8)-12-foil decomposition of K_{157}.

2. Balanced C_{13}-t-Foil Designs

Let C_{13} be *the cycle* on 13 vertices. *The C_{13}-t-foil* is a graph of t edge-disjoint C_{13}'s with a common vertex and the common vertex is called *the center of the C_{13}-t-foil*. In particular, the C_{13}-2-foil and the C_{13}-3-foil are called *the C_{13}-bowtie* and *the C_{13}-trefoil*, respectively. When K_n is decomposed into edge-disjoint sum of C_{13}-t-foils, it is called that K_n *has a C_{13}-t-foil decomposition*. Moreover, when every vertex of K_n appears in the same number of C_{13}-t-foils, it is called that K_n *has a balanced C_{13}-t-foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced C_{13}-t-foil design*.

Theorem 2. K_n has a balanced C_{13}-t-foil decomposition if and only if $n \equiv 1 \pmod{26t}$.

Proof. (Necessity) Suppose that K_n has a balanced C_{13}-t-foil decomposition. Let b be the number of C_{13}-t-foils and r be the replication number. Then $b = n(n-1)/26t$ and $r = (12t+1)(n-1)/26t$. Among r C_{13}-t-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of C_{13}-t-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $2tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/26t$ and $r_2 = 12(n-1)/26$. Therefore, $n \equiv 1 \pmod{26t}$ is necessary.
(Sufficiency) Put $n = 26st + 1, T = st$. Then $n = 26T + 1$. Construct a C_{13}-T-foil as follows:

$\{ (26T+1, T, 12T, 23T+1, 14T, 15T+1, T+1, 19T+2, 24T+2, 3T+2, 23T+2, 5T+2, 2T+1),$
$(26T+1, T-1, 12T-2, 23T, 14T-2, 15T, T+2, 19T+4, 24T+3, 3T+4, 23T+3, 5T+4, 2T+2),$
$(26T+1, T-2, 12T-4, 23T-1, 14T-4, 15T-1, T+3, 19T+6, 24T+4, 3T+6, 23T+4, 5T+6, 2T+3),$
$...,$
$(26T+1, 1, 10T+2, 22T+2, 12T+2, 14T+2, 2T, 21T, 25T+1, 5T, 24T+1, 7T, 3T) \}.$
Decompose this C_{13}-T-foil into s C_{13}-t-foils. Then these starters comprise a balanced C_{13}-t-foil decomposition of K_n.

Corollary 2.1. K_n has a balanced C_{13}-bowtie decomposition if and only if $n \equiv 1 \pmod{52}$.

Corollary 2.2. K_n has a balanced C_{13}-trefoil decomposition if and only if $n \equiv 1 \pmod{78}$.

Example 2.1. Balanced C_{13}-decomposition of K_{27}.
$\{(27, 1, 12, 24, 14, 16, 2, 21, 26, 5, 25, 7, 3)\}$.
This stater comprises a balanced C_{13}-decomposition of K_{27}.

Example 2.2. Balanced C_{13}-2-foil decomposition of K_{53}.
$\{(53, 2, 24, 47, 28, 31, 3, 40, 50, 8, 48, 12, 5),$
$(53, 1, 22, 46, 26, 30, 4, 42, 51, 10, 49, 14, 6)\}$.
This stater comprises a balanced C_{13}-2-foil decomposition of K_{53}.

Example 2.3. Balanced C_{13}-3-foil decomposition of K_{79}.
$\{(79, 3, 36, 70, 42, 46, 4, 59, 74, 11, 71, 17, 7),$
$(79, 2, 34, 69, 40, 45, 5, 61, 75, 13, 72, 19, 8),$
$(79, 1, 32, 68, 38, 44, 6, 63, 76, 15, 73, 21, 9)\}$.
This stater comprises a balanced C_{13}-3-foil decomposition of K_{79}.

Example 2.4. Balanced C_{13}-4-foil decomposition of K_{105}.
$\{(105, 4, 48, 93, 56, 61, 5, 78, 98, 14, 94, 22, 9),$
$(105, 3, 46, 92, 54, 60, 6, 80, 99, 16, 95, 24, 10),$
$(105, 2, 44, 91, 52, 59, 7, 82, 100, 18, 96, 26, 11),$
$(105, 1, 42, 90, 50, 58, 8, 84, 101, 20, 97, 28, 12)\}$.
This stater comprises a balanced C_{13}-4-foil decomposition of K_{105}.

Example 2.5. Balanced C_{13}-5-foil decomposition of K_{131}.
$\{(131, 5, 60, 116, 70, 76, 6, 97, 122, 17, 117, 27, 11),$
$(131, 4, 58, 115, 68, 75, 7, 99, 123, 19, 118, 29, 12),$
$(131, 3, 56, 114, 66, 74, 8, 101, 124, 21, 119, 31, 13),$
$(131, 2, 54, 113, 64, 73, 9, 103, 125, 23, 120, 33, 14),$
$(131, 1, 52, 112, 62, 72, 10, 105, 126, 25, 121, 35, 15)\}$.
This stater comprises a balanced C_{13}-5-foil decomposition of K_{131}.

Example 2.6. Balanced C_{13}-6-foil decomposition of K_{157}.
$\{(157, 6, 72, 139, 84, 91, 7, 116, 146, 20, 140, 32, 13),$
$(157, 5, 70, 138, 82, 90, 8, 118, 147, 22, 141, 34, 14),$
$(157, 4, 68, 137, 80, 89, 9, 120, 148, 24, 142, 36, 15),$
$(157, 3, 66, 136, 78, 88, 10, 122, 149, 26, 143, 38, 16),$
$(157, 2, 64, 135, 76, 87, 11, 124, 150, 28, 144, 40, 17),$
$(157, 1, 62, 134, 74, 86, 12, 126, 151, 30, 145, 42, 18)\}$.
This stater comprises a balanced C_{13}-6-foil decomposition of K_{157}.

3. Balanced (C_{10}, C_{16})-$2t$-Foil Designs

Let C_{10} and C_{16} be *the* 10-*cycle* and *the* 16-*cycle*, respectively. *The* (C_{10}, C_{16})-$2t$-*foil* is a graph of t edge-disjoint C_{10}'s and t edge-disjoint C_{16}'s with a common vertex and the common vertex is called *the center of the* (C_{10}, C_{16})-$2t$-*foil*. In particular, the (C_{10}, C_{16})-2-foil is called *the* (C_{10}, C_{16})-*bowtie*. When K_n is decomposed into edge-disjoint sum of (C_{10}, C_{16})-$2t$-foils, we say that K_n *has a* (C_{10}, C_{16})*-2t-foil decomposition*. Moreover, when every vertex of K_n appears in the same number of (C_{10}, C_{16})-$2t$-foils, we say that K_n *has a balanced* (C_{10}, C_{16})*-2t-foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced* (C_{10}, C_{16})*-2t-foil design*.

Theorem 3. K_n has a balanced (C_{10}, C_{16})-$2t$-foil decomposition if and only if $n \equiv 1 \pmod{52t}$.

Proof. (Necessity) Suppose that K_n has a balanced (C_{10}, C_{16})-$2t$-foil decomposition. Let b be the number of (C_{10}, C_{16})-$2t$-foils and r be the replication number. Then $b = n(n-1)/52t$ and $r = (24t+1)(n-1)/52t$. Among r (C_{10}, C_{16})-$2t$-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_{10}, C_{16})-$2t$-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/52t$ and $r_2 = 24(n-1)/52$. Therefore, $n \equiv 1 \pmod{52t}$ is necessary.
(Sufficiency) Put $n = 52st+1$ and $T = st$. Then $n = 52T+1$. Construct a (C_{10}, C_{16})-$2T$-foil as follows:
$\{(52T+1, 1, 20T+2, 44T+2, 20T, 40T-1, 20T-1, 44T+3, 20T+4, 2), (52T+1, 2T+1, 38T+2, 48T+2, 6T+2, 46T+2, 10T+2, 4T+1, 8T+3, 4T+2, 10T+4, 46T+3, 6T+4, 48T+3, 38T+4, 2T+2)\} \cup$
$\{(52T+1, 3, 20T+6, 44T+4, 20T-2, 40T-5, 20T-3, 44T+5, 20T+8, 4), (52T+1, 2T+3, 38T+6, 48T+4, 6T+6, 46T+4, 10T+6, 4T+3, 8T+7, 4T+4, 10T+8, 46T+5, 6T+8, 48T+5, 38T+8, 2T+4)\} \cup$
$\{(52T+1, 5, 20T+10, 44T+6, 20T-4, 40T-9, 20T-5, 44T+7, 20T+12, 6), (52T+1, 2T+5, 38T+10, 48T+6, 6T+10, 46T+6, 10T+10, 4T+5, 8T+11, 4T+6, 10T+12, 46T+7, 6T+12, 48T+7, 38T+12, 2T+6)\} \cup$
... $\cup$
$\{(52T+1, 2T-1, 24T-2, 46T, 18T+2, 36T+3, 18T+1, 46T+1, 24T, 2T), (52T+1, 4T-1, 42T-2, 50T, 10T-2, 48T, 14T-2, 6T-1, 12T-1, 6T, 14T, 48T+1, 10T, 50T+1, 42T, 4T)\}$.
Decompose the (C_{10}, C_{16})-$2T$-foil into s (C_{10}, C_{16})-$2t$-foils. Then these starters comprise a balanced (C_{10}, C_{16})-$2t$-foil decomposition of K_n.

Corollary 3. K_n has a balanced (C_{10}, C_{16})-bowtie decomposition if and only if $n \equiv 1 \pmod{52}$.

Example 3.1. Balanced (C_{10}, C_{16})-2-foil decomposition of K_{53}.
$\{(53, 1, 22, 46, 20, 39, 19, 47, 24, 2), (53, 3, 40, 50, 8, 48, 12, 5, 11, 6, 14, 49, 10, 51, 42, 4)\}$.
This starter comprises a balanced (C_{10}, C_{16})-2-foil decomposition of K_{53}.

Example 3.2. Balanced (C_{10}, C_{16})-4-foil decomposition of K_{105}.
$\{(105, 1, 42, 90, 40, 79, 39, 91, 44, 2),$
$(105, 3, 46, 92, 38, 75, 37, 93, 48, 4)\}$
$\cup$
$\{(105, 5, 78, 98, 14, 94, 22, 9, 19, 10, 24, 95, 16, 99, 80, 6),$
$(105, 7, 82, 100, 18, 96, 26, 11, 23, 12, 28, 97, 20, 101, 84, 8)\}$.
This starter comprises a balanced (C_{10}, C_{16})-4-foil decomposition of K_{105}.

Example 3.3. Balanced (C_{10}, C_{16})-6-foil decomposition of K_{157}.
$\{(157, 1, 62, 134, 60, 119, 59, 135, 64, 2),$
$(157, 3, 66, 136, 58, 115, 57, 137, 68, 4),$
$(157, 5, 70, 138, 56, 111, 55, 139, 72, 6)\}$
$\cup$
$\{(157, 7, 116, 146, 20, 140, 32, 13, 27, 14, 34, 141, 22, 147, 118, 8),$
$(157, 9, 120, 148, 24, 142, 36, 15, 31, 16, 38, 143, 26, 149, 122, 10),$
$(157, 11, 124, 150, 28, 144, 40, 17, 35, 18, 42, 145, 30, 151, 126, 12)\}$.
This starter comprises a balanced (C_{10}, C_{16})-6-foil decomposition of K_{157}.

Example 3.4. Balanced (C_{10}, C_{16})-8-foil decomposition of K_{209}.
$\{(209, 1, 82, 178, 80, 159, 79, 179, 84, 2),$
$(209, 3, 86, 180, 78, 155, 77, 181, 88, 4),$
$(209, 5, 90, 182, 76, 151, 75, 183, 92, 6),$

$(209, 7, 94, 184, 74, 147, 73, 185, 96, 8)\}$
$\cup$
$\{(209, 9, 154, 194, 26, 186, 42, 17, 35, 18, 44, 187, 28, 195, 156, 10),$
$(209, 11, 158, 196, 30, 188, 46, 19, 39, 20, 48, 189, 32, 197, 160, 12),$
$(209, 13, 162, 198, 34, 190, 50, 21, 43, 22, 52, 191, 36, 199, 164, 14),$
$(209, 15, 166, 200, 38, 192, 54, 23, 47, 24, 56, 193, 40, 201, 168, 16)\}.$
This starter comprises a balanced (C_{10}, C_{16})-8-foil decomposition of K_{209}.

Example 3.5. Balanced (C_{10}, C_{16})-10-foil decomposition of K_{261}.
$\{(261, 1, 102, 222, 100, 199, 99, 223, 104, 2),$
$(261, 3, 106, 224, 98, 195, 97, 225, 108, 4),$
$(261, 5, 110, 226, 96, 191, 95, 227, 112, 6),$
$(261, 7, 114, 228, 94, 187, 93, 229, 116, 8),$
$(261, 9, 118, 230, 92, 183, 91, 231, 120, 10)\}$
$\cup$
$\{(261, 11, 192, 242, 32, 232, 52, 21, 43, 22, 54, 233, 34, 243, 194, 12),$
$(261, 13, 196, 244, 36, 234, 56, 23, 47, 24, 58, 235, 38, 245, 198, 14),$
$(261, 15, 200, 246, 40, 236, 60, 25, 51, 26, 62, 237, 42, 247, 202, 16),$
$(261, 17, 204, 248, 44, 238, 64, 27, 55, 28, 66, 239, 46, 249, 206, 18),$
$(261, 19, 208, 250, 48, 240, 68, 29, 59, 30, 70, 241, 50, 251, 210, 20)\}.$
This starter comprises a balanced (C_{10}, C_{16})-10-foil decomposition of K_{261}.

4. Balanced C_{26}-t-Foil Designs

Let C_{26} be *the cycle* on 26 vertices. *The C_{26}-t-foil* is a graph of t edge-disjoint C_{26}'s with a common vertex and the common vertex is called *the center of the C_{26}-t-foil*. In particular, the C_{26}-2-foil and the C_{26}-3-foil are called *the C_{26}-bowtie* and *the C_{26}-trefoil*, respectively. When K_n is decomposed into edge-disjoint sum of C_{26}-t-foils, it is called that K_n *has a C_{26}-t-foil decomposition*. Moreover, when every vertex of K_n appears in the same number of C_{26}-t-foils, it is called that K_n *has a balanced C_{26}-t-foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced C_{26}-t-foil design*.

Theorem 4. K_n has a balanced C_{26}-t-foil decomposition if and only if $n \equiv 1 \pmod{52t}$.

Proof. (Necessity) Suppose that K_n has a balanced C_{26}-t-foil decomposition. Let b be the number of C_{26}-t-foils and r be the replication number. Then $b = n(n-1)/52t$ and $r = (25t+1)(n-1)/52t$. Among r C_{26}-t-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of C_{26}-t-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $2tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/52t$ and $r_2 = 25(n-1)/52$. Therefore, $n \equiv 1 \pmod{52t}$ is necessary.
(Sufficiency) Put $n = 52st + 1, T = st$. Then $n = 52T + 1$. Construct a C_{26}-T-foil as follows:
$\{\ (52T+1, 2T, 24T, 46T+1, 28T, 30T+1, 2T+1, 38T+2, 48T+2, 6T+2, 46T+2, 10T+2, 4T+1, 8T+3, 4T+2, 10T+4, 46T+3, 6T+4, 48T+3, 38T+4, 2T+2, 30T, 28T-2, 46T, 24T-2, 2T-1),$
$(52T+1, 2T-2, 24T-4, 46T-1, 28T-4, 30T-4, 2T+3, 38T+6, 48T+4, 6T+6, 46T+4, 10T+6, 4T+3, 8T+7, 4T+4, 10T+8, 46T+5, 6T+8, 48T+5, 38T+8, 2T+4, 30T-2, 28T-6, 46T-2, 24T-6, 2T-3),$
$(52T+1, 2T-4, 24T-8, 46T-3, 28T-8, 30T-3, 2T+5, 38T+10, 48T+6, 6T+10, 46T+6, 10T+10, 4T+5, 8T+11, 4T+6, 10T+12, 46T+7, 6T+12, 48T+7, 38T+12, 2T+6, 30T-4, 28T-10, 46T-4, 24T-10, 2T-5),$
...,
$(52T+1, 2, 20T+4, 44T+3, 24T+4, 28T+3, 4T-1, 42T-2, 50T, 10T-2, 48T, 14T-2, 6T-1, 12T-1, 6T, 14T, 48T+1, 10T, 50T+1, 42T, 4T, 28T+2, 24T+2, 44T+2, 20T+2, 1)\ \}.$
Decompose this C_{26}-T-foil into s C_{26}-t-foils. Then these starters comprise a balanced C_{26}-t-foil decomposition of K_n.

Corollary 4.1. K_n has a balanced C_{26}-bowtie decomposition if and only if $n \equiv 1$ (mod

104).

Corollary 4.2. K_n has a balanced C_{26}-trefoil decomposition if and only if $n \equiv 1 \pmod{156}$.

Example 4.1. Balanced C_{26}-decomposition of K_{53}.
$\{(53, 2, 24, 47, 28, 31, 3, 40, 50, 8, 48, 12, 5, 11, 6, 14, 49, 10, 51, 42, 4, 30, 26, 46, 22, 1)\}$.
This stater comprises a balanced C_{26}-decomposition of K_{53}.

Example 4.2. Balanced C_{26}-2-foil decomposition of K_{105}.
$\{(105, 4, 48, 93, 56, 61, 5, 78, 98, 14, 94, 22, 9, 19, 10, 24, 95, 16, 99, 80, 6, 60, 54, 92, 46, 3)$,
$(105, 2, 44, 91, 52, 59, 7, 82, 100, 18, 96, 26, 11, 23, 12, 28, 97, 20, 101, 84, 8, 58, 50, 90, 42, 1)\}$.
This stater comprises a balanced C_{26}-2-foil decomposition of K_{105}.

Example 4.3. Balanced C_{26}-3-foil decomposition of K_{157}.
$\{(157, 6, 72, 139, 84, 91, 7, 116, 146, 20, 140, 32, 13, 27, 14, 34, 141, 22, 147, 118, 8, 90, 82, 138, 70, 5)$,
$(157, 4, 68, 137, 80, 89, 9, 120, 148, 24, 142, 36, 15, 31, 16, 38, 143, 26, 149, 122, 10, 88, 78, 136, 66, 3)$,
$(157, 2, 64, 135, 76, 87, 11, 124, 150, 28, 144, 40, 17, 35, 18, 42, 145, 30, 151, 126, 12, 86, 74, 134, 62, 1)\}$.
This stater comprises a balanced C_{26}-3-foil decomposition of K_{157}.

Example 4.4. Balanced C_{26}-4-foil decomposition of K_{209}.
$\{(209, 8, 96, 185, 112, 121, 9, 154, 194, 26, 186, 42, 17, 35, 18, 44, 187, 28, 195, 156, 10, 120, 110, 184, 94, 7)$,
$(209, 6, 92, 183, 108, 119, 11, 158, 196, 30, 188, 46, 19, 39, 20, 48, 189, 32, 197, 160, 12, 118, 106, 182, 90, 5)$,
$(209, 4, 88, 181, 104, 117, 13, 162, 198, 34, 190, 50, 21, 43, 22, 52, 191, 36, 199, 164, 14, 116, 102, 180, 86, 3)$,
$(209, 2, 84, 179, 100, 115, 15, 166, 200, 38, 192, 54, 23, 47, 24, 56, 193, 40, 201, 168, 16, 114, 98, 178, 82, 1)\}$.
This stater comprises a balanced C_{26}-4-foil decomposition of K_{209}.

Example 4.5. Balanced C_{26}-5-foil decomposition of K_{261}.
$\{(261, 10, 120, 231, 140, 151, 11, 192, 242, 32, 232, 52, 21, 43, 22, 54, 233, 34, 243, 194, 12, 150, 138, 230, 118, 9)$,
$(261, 8, 116, 229, 136, 149, 13, 196, 244, 36, 234, 56, 23, 47, 24, 58, 235, 38, 245, 198, 14, 148, 134, 228, 114, 7)$,
$(261, 6, 112, 227, 132, 147, 15, 200, 246, 40, 236, 60, 25, 51, 26, 62, 237, 42, 247, 202, 16, 146, 130, 226, 110, 5)$,
$(261, 4, 108, 225, 128, 145, 17, 204, 248, 44, 238, 64, 27, 55, 28, 66, 239, 46, 249, 206, 18, 144, 126, 224, 106, 3)$,
$(261, 2, 104, 223, 124, 143, 19, 208, 250, 48, 240, 68, 29, 59, 30, 70, 241, 50, 251, 210, 20, 142, 122, 222, 102, 1)\}$.
This stater comprises a balanced C_{26}-5-foil decomposition of K_{261}.

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