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Effectiveness of Genetic Multi-Step Search on Unsupervised Design of Morphological Filters for Noise Removal

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This paper shows the effectiveness of deterministic Multi-step Crossover Fusion (dMSXF) on an unsupervised design problem of suitable structuring elements (SEs) of a morphological filter. In our previous work, it was shown that dMSXF worked very well for solving combinatorial optimization problems, especially on problems for which the landscape is an AR(1) landscape observed in the NK model. In addition, the effectiveness on reproduction mechanisms of offspring of dMSXF was shown to be kept through increases in the level of epistasis. In this paper, we show that a characteristic of the AR(1) landscape is observed in an objective function for the unsupervised design of SEs, and the superior search performance of dMSXF to a conventional crossover is shown. The processing results of the obtained SEs are also compared to that of a conventional filter for impulse noise removal.

1. Introduction

Mathematical morphology is a fundamental framework for image manipulation, and a wide range of nonlinear image processing filters can be unified within this framework[1]. *Opening* and *closing*, which is the dual of opening, are typical morphological operations and fundamental morphological filters that have idempotence. They are used for various methods of noise reduction, object extraction, etc. Mathematical morphological operations manipulate an image with a small object called a *structuring element* (SE), which is equivalent to the window of image processing filters. The opening operator composes the resultant image object by arranging the SE inside a target object and removes residual regions that are too small to locate the SE inside. The significance of opening is its quantitativeness with respect to the sizes of image objects. The impulse noise removal by opening achieves a quantitative operation in the sense that the noise objects smaller than the SE are precisely removed. Since this operator composes an image by repetitively locating an SE, its shape and gray scale distribution appear directly in the resultant image. In the case that the SE is inappropriate to the image, it causes the appearance of undesired microstructures which are not related to the original image. These problems can be avoided by the use of an appropriate gray scale SE that resembles the objects in the target image. Thus determination of the shape and gray scale distribution of the SEs is an important problem. Here, we use the opening operator for noise removal in textures corrupted by impulse noise. For practical use, we design suitable SEs for the textures, which remove the noise and reconstruct the image with high accuracy, directly from the corrupted image only. A genetic algorithm (GA) is adopted as the optimization algorithm for the unsupervised design.

When we apply a GA to a particular problem, especially for combinatorial problems, it is important to design a crossover method with emphasis on the heredity of favorable characteristics of parents. Deterministic Multi-step Crossover Fusion (dMSXF)[2], which is a kind of genetic multi-step search based on neighborhood search mechanisms, is a promising crossover operator for combinatorial problems. DMSXF can be constructed by introducing both a problem-specific neighborhood structure and a distance measure. By the mechanism of multi-step search, dMSXF can generate a wide variety of offspring between parents and it performs especially well on problems for which the landscape is an *AR(1) landscape*[2,3]. In addition, we have shown that the high search performance of dMSXF was achieved by setting the neighborhood size to the near value to the correlation length which reflects the level of epistasis[4].

In this paper, dMSXF is adopted as the crossover method to optimize SEs of morphological filters for the texture image. First we introduce our proposed unsupervised design. The landscape of objective function for the design of SEs is experimentally shown to be similar to AR(1) landscape observed in an NK model[5]. In numerical experiments, we apply dMSXF to typical textures, and show the effectiveness of dMSXF as a main search operator of GA. The processing results of obtained SEs are compared to that of a conventional filter for impulse noise removal.

2. Deterministic Multi-step Crossover Fusion

Genetic algorithms (GAs) are among the most effective approximation algorithms for optimization problems. GAs are applicable to a wide range of problems and have been

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applied to numerous combinatorial problems, such as Traveling Salesman Problem and various scheduling problems. To apply GAs to these problems, it is important to design a crossover method to consider problem-specific structures and characteristics. Various crossovers focusing on the inheritance of parents' characteristics have been discussed. Among them, deterministic Multi-step Crossover Fusion (dMSXF)[2)], which is a type of genetic multi-step searches based on neighborhood search mechanisms, has been proposed, since the incorporation of neighborhood searches into GAs is essential in order to adjust the structural details of solutions in combinatorial problems.

DMSXF advances the neighborhood search from a parent p_1 in the direction approaching the other parent p_2. The procedure of dMSXF is as follows. A set of offspring generated by parents p_1, p_2 is indicated by $C(p_1, p_2)$.

Procedure of dMSXF

0. Let p_1, p_2 be parents and set their offspring $C(p_1, p_2) = \phi$.
1. k=1. Set the initial search point $x_1 = p_1$ and add x_1 into $C(p_1, p_2)$.
2. /Step k/ Prepare $N(x_k)$ composed of μ neighbors generated from the current solution x_k. $\forall y_i \in N(x_k)$ must satisfy $d(y_i, p_2) < d(x_k, p_2)$.
3. Select the best solution y from $N(x_k)$. Let the next search point x_{k+1} be y and add x_{k+1} into $C(p_1, p_2)$.
4. Set $k = k + 1$ and go to 2. until $k = k_{max}$ or x_k equals p_2.

At step 2 of the procedure of dMSXF, every neighborhood candidate y_i ($1 \leq i \leq \mu$) generated from x_k must be closer to p_2 than x_k. In addition, dMSXF necessarily moves its transition toward p_2 even if all solutions in $N(x_k)$ are inferior to the current solution x_k. Table 1 shows an example of application of dMSFX to a 1-max problem. In this problem, the Hamming distance is adopted as the distance measure. The bits copied from p_2 to x_k are chosen randomly at each step.

3. Unsupervised Design of Structuring Elements for Noise Removal

3.1 Mathematical Morphology and Opening

In the context of mathematical morphology, an image object is defined by a set. In the case of binary images, this set contains the pixel positions included in the object, i.e., those of white pixels. In the case of gray scale images, an image object is defined by an *umbra* set. If the pixel value distribution of an image object is denoted as $f(\boldsymbol{x})$, where $\boldsymbol{x} \in R^2$ is a pixel position, its umbra $U[f(\boldsymbol{x})]$ is defined as follows:

$$U[f(\boldsymbol{x})] = \{(\boldsymbol{x}, t) \in R^3 | f(\boldsymbol{x}) \geq t > -\infty\} \quad (1)$$

Consequently, when we assume a solid whose support is the same as a gray scale image object and whose height at each pixel position is the same as the pixel value at this position, the umbra is equivalent to this solid and the whole volume below this solid within the support.

Another object, called a *structuring element* (SE), is defined in the same manner as above. The SE is equivalent to the window of image processing filters, and is considered to be much smaller than the image object. *Opening* and *closing* are typical morphological operations, and fundamental morphological filters that have idempotence. They are used for various methods of noise reduction, object extraction, etc. In the case of a binary image and an SE, the opening of an image object X with respect to an SE B, denoted X_B, has the following properties:

$$X_B = \{B_z | B_z \subset X, z \in R^2\} \quad (2)$$

where B_z indicates the translation of B by z.

Here, we concentrate on the opening, since the operations on closing are regarded as the dual of the opening. In the case of the gray scale image and an SE, the opening is similarly defined by replacing the sets X and B with their umbrae, respectively, and supposing that $z \in R^3$. This property indicates that opening is the regeneration of an image produced by arranging the SE, and removes smaller white regions in the binary case or smaller regions composed of brighter pixels than its neighborhood in the gray scale case than the SE. The fact that the opening operator eliminates smaller structures and smaller bright peaks than the SE indicates that it works as a filter to distinguish object structures by their sizes. In this paper, we adopt the opening with SEs as an impulse noise removal filter in texture images.

Table 1 Application of dMSXF to the 1-max Problem: $L_{bit} = 10$, $k_{max} = 3$, $\mu = 3$

p_1	$step1$	$step2$	p_2
(base solution x_k)	$\bar{0}\bar{1}\bar{1}\bar{1}11\bar{0}\bar{0}\bar{0}0$ (5)	$\bar{0}\bar{1}\bar{1}\bar{1}11\bar{0}110$ (7)	$1\bar{1}\bar{0}1111110$ (8)
0111110000 (5)	$01\hat{0}\hat{0}11\hat{1}000$ (4)	$\hat{1}1\hat{0}111\hat{1}110$ (8) →	1000111110 (6)
	$01\hat{0}11100\hat{1}0$ (5)	$0\hat{0}1\hat{0}110110$ (5)	
	$0111110\hat{1}\hat{1}0$ (7) →	$\hat{1}1\hat{0}\hat{0}110110$ (6)	

$\bar{*}$ of x_k means the difference between x_k and p_2, and $\hat{*}$ is introduced from p_2.
$d(p_1, p_2)/k_{max}$(= 7/3 = 2 or 3) bits are introduced from p_2 at each transition.

3.2 Unsupervised Design of SEs

For textures corrupted by the impulse noise, we design suitable SEs which can remove the noise and reconstruct the image with accuracy. In our approach, for practical use, optimal SEs are designed directly from the corrupted image and no training images are required. In this section, we identify the problem and design the objective function.

3.2.1 Noise Model

Images are often corrupted by impulse noises due to a noisy circuit or channel transmission errors. Here, we consider gray scale texture images whose gray level is 256. There are several impulse noise models for images. We adopt the model below. In this model, $x_o(i,j)$ indicates the pixel values of the original image, and l represents a non-negative integer with uniform distribution. $x(i,j)$ is rounded to 255 if $x_o(i,j)+l$ exceeds 255.

$$x(i,j) = \begin{cases} x_o(i,j)+l & \text{prob. } p \\ x_o(i,j) & \text{prob. } 1-p \end{cases} \tag{3}$$

3.2.2 Design of Objective Function

The opening of image X with respect to an SE B means that the residue of X is obtained by removing smaller structures than B. Let rB be a result of r-times of homothetic magnification of B, which is defined as (r-1)-times of $Minkowski$ set additions between B and another small element. We perform the opening of X with respect to the homothetic SEs rB and obtain the image sequence $\{X, X_B, X_{2B}, \cdots, X_{rB}, \cdots\}$. In this sequence, X_{rB} is obtained by removing the regions smaller than rB. In the case of gray scale images, at each r, we calculate a ratio of value at each pixel position of X_{rB} to that of the original X, and then calculate a sum of the ratio. The function from size r to the corresponding ratio decreases monotonically, and becomes unity when the size is 0. This function is called the *size distribution function*[6]. The size distribution function of size r, $F(r)$, indicates the area ratio of the regions whose sizes are greater than r or equal to r. Here, the integral of $F(r)$ is used as the objective function and SEs are obtained by minimizing it.

3.2.3 Application of GA to the Design of SEs

For simplicity, we fix the shape of each SE to a full square 3x3 pixels and only optimize the pixel values for each element of this square. Each SE is an individual, a candidate solution, of GA and each element of the SE takes a value of the range [0, 255] for gray scale textures. At the initial population, a random value of this range is assigned to each element. During the search using the GA, these values are coded to a binary string of length 8; consequently, each individual of GA is expressed as a binary string of length 72. This design problem is a binary problem and dMSXF is implemented as the bitwise operator shown in Table 1.

The generation alternation model we used for dMSXF in this paper is outlined below. This model focuses on a local search performance and it showed effectiveness in combinatorial optimization problems[2),3)].

Generation Alternation Model

0. Generate the initial population composed of N_{pop} random solutions, individuals, $\{x_1, x_2, \cdots, x_{N_{pop}}\}$.
1. Reset indexes $\{1, 2, \cdots, N_{pop}\}$ to each individual randomly.
2. Select N_{pop} pairs of parents (x_i, x_{i+1}) $(1 \le i \le N_{pop})$ where $x_{N_{pop}+1} = x_1$.
3. Apply dMSXF for each pair (x_i, x_{i+1}).
4. For each pair (x_i, x_{i+1}), select the best individual c from offspring $C(x_i, x_{i+1})$ generated by parents (x_i, x_{i+1}) and replace the parent x_i with c.
5. Go to 1 until some terminal criterion is satisfied, e.g., generations and/or the number of evaluations.

4. Problem Difficulties Analysis

4.1 Properties of Fitness Landscape

It is essential to investigate problem difficulties before applying a GA to an individual problem. There are several measurements for comprehending complexity in the fitness landscapes of objective function. Epistatic interactions among design variables affect the features of local landscapes. An intensity of fitness correlation among neighborhood solutions, which appears as a local ruggedness in a fitness landscape, reflects the level of epistasis. The *correlation length* ℓ_c is an indicator of the level of epistasis[7)], and it is derived from the *random walk correlation function* $r(s)$ as equation (4), where a time series $\{f(x_t)\}$ defines the correlation of two points s steps away along a random walk of length m through the fitness landscape. $\bar{f}$ and σ_f^2 respectively denote the average and the variance of the fitness values.

$$r(s) = \lim_{m\to\infty} \frac{1}{\sigma_f^2(m-s)} \sum_{t=1}^{m-s} (f(x_t)-\bar{f})(f(x_{t+s})-\bar{f})$$
$$\ell_c = -1/\ln(|r(1)|) \tag{4}$$

If a time series $\{f(x_t)\}$ is *isotropic, Gaussian, and Markovian*, then the function $r(s)$ is of the form $r(s) = r(1)^s = e^{-s/\ell_c}$. In this case, the landscape is called an *AR(1) landscape*. AR(1) landscapes have been found in various combinatorial optimization problems and can be created by the NK model. This landscape feature is also observed in the objective function for optimizing structuring elements, which is demonstrated later in the section 4.2. In an AR(1) landscape, the lower the value for ℓ_c, the more rugged the landscape. The fitness landscape of the NK model shows the AR(1) function and the value of ℓ_c has been proved to be $N/(K+1)$[8]. The indicator ℓ_c has an intensified impact on the performance of any neighborhood search method, and we can obtain beneficial information from the current solution to generate neighborhood solutions if the scope of the neighborhood is smaller than ℓ_c.

4.2 Fitness Landscape Feature of the Objective Function of SEs

Here, we show that the objective function to design SEs for noise removal, the integral of the size distribution, has a feature of an AR(1) landscape. Random walk correlation functions of this design problem were examined to analyze aspects of the fitness landscape and problem difficulties. At each step of the random walk, a neighborhood solution was generated by the bit flip operator. Two kinds of images, D57 and D74 of Brodatz textures[9], were used for the examination. These instances are described in detail in section 5. The probability of the impulse noise was set to 0.25 or 0.5 for both.

The computed correlation functions are plotted in Fig. 1. These functions have been estimated experimentally by performing random walks of length 1x10^5. Random walk correlation functions of instances of the NK model are also plotted in the figure for comparison. All functions here are 72 bit problems, which is the same bit size used to represent the individuals for designing SEs.

From the figure, we can see that the random walk correlation functions for the design problem of SEs have an exponentially decaying form as expected for the landscape of the NK model. The aspect of local ruggedness of objective functions of these textures in the optimization of SEs is supposed to be extremely similar to the NK model. It is observed that the ruggedness of function is different among textures. In these instances, D74 is more rugged than D57. In addition, the level of epistasis of the objective function decreases against increasing in the noise rate.

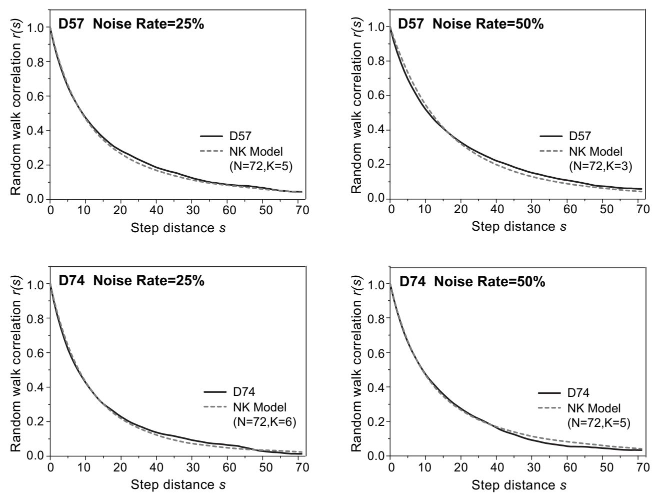

Fig. 1 Random Walk Correlation Functions for Texture Images

It has been shown that dMSXF performs very well on an AR(1) landscape[4]. Therefore, it is expected that dMSXF will work well for optimizing SEs for noise removal. In addition, we can preliminarily examine the appropriate setting parameters of dMSXF with NK models, before applying it to design SEs.

5. Numerical Experiments

We next discuss the suitability of dMSXF for application to the design problem of SEs. To show their effectiveness, we compare it with the conventional crossover operator, the uniform crossover (UX) that generates mostly intermediate offspring between parents in any rugged landscape. The processing performance of designed SEs for impulse noise removal is also examined by comparing it to another typical filter. Eight kinds of images, D3, D20, D57, D74, D87, D98, D101 and D112 of Brodatz textures[9] which have wide variety in shape characteristics and the brightness, were used for the examination. The size of each image is 128x128 and the gray level of each image is 256. These textures are illustrated in Fig. 2.

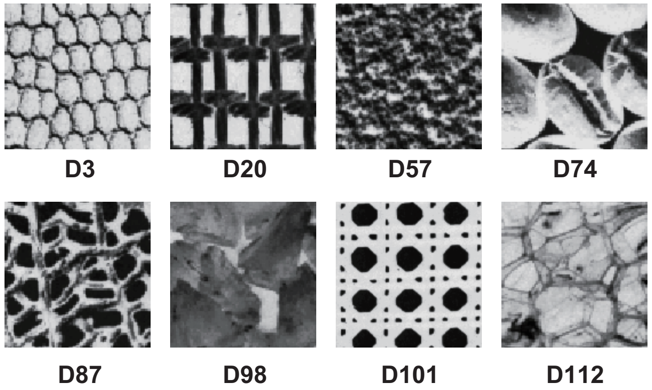

Fig. 2 Brodatz Textures

Before applying dMSXF to the design problem, the search performance on NK models is examined to determine the parameter settings where it works effectively.

5.1 Performance of dMSXF against the Level of Epistasis

We confirm appropriate setting of parameters of dMSXF using instances of the NK model of N=72. In the experiments, the population size was set to 10. The generation alternation model based on elitist recombination model (ER)[10] was adopted for UX.

Each trial was terminated after 25 generations in dMSXF, and 50 generations in UX in all instances of the NK Model. The total number of evaluations was the same between dMSXF and UX. For dMSXF, k_{max} was set to 2, 4, 8 and 16. The number of offspring generated by each pair of parents, N_C, was set to 32 for both methods. μ is calculated by N_C/k_{max}.

Table 2 Search Performance of dMSXF and UX (N=72)

Instance		k_{max}=2		k_{max}=4		k_{max}=8		k_{max}=16		UX=32	
K	ℓ_c	avg.	std.	avg.	std.	avg.	std.	avg.	std.	avg.	std.
3	18.0	0.908	0.028	0.909	0.032	0.905	0.030	0.888	0.032	0.884	0.035
6	10.3	0.903	0.036	0.908	0.038	0.895	0.037	0.869	0.041	0.879	0.035
9	7.2	0.736	0.022	0.738	0.023	0.739	0.026	0.723	0.024	0.728	0.025
12	5.5	0.694	0.022	0.702	0.016	0.700	0.018	0.687	0.017	0.690	0.019

Table 2 shows search performances on NK models of N=72 tuning K in the range of 3 to 12. These results show the average and the standard deviation of fitness out of 100 trials. Bigger values are considered better. At each trial, the same initial population was used among GAs.

From Table 2, dMSXF shows a completely superior performance to UX at all levels of epistasis. Among the settings of dMSXF, k_{max}=4 performs satisfactorily in these instances.

5.2 Effectiveness of dMSXF in the Design of SEs

Here we apply dMSXF to optimization of SEs which work effectively on impulse noise removal for textures. Eight kinds of images were used for the examination, and the probability of impulse noise was set to 0.25 and 0.5 at each image. The parameters of GAs were the same as in the previous section. In dMSXF, k_{max} was set to 4 which is considered the most productive.

Table 3 shows the best value of MSE between the processing results and the original image, the worst MSE (wst.) the averaged MSE (avg.) and the standard deviation of MSE (std.) out of 20 trials. Smaller values are considered better. The correlation length ℓ_c at each image in the table was estimated experimentally by performing random walks of length 1x10^5. The processing performance of designed SEs was also compared to the processing results of PSWA[11] which is a promising filter for impulse noise removal. A full 3x3 square window was adopted in this filter. The examples of estimation result with the best solutions of dMSXF are shown in Fig. 3.

From the comparison between GAs and PSWA, it is shown that our unsupervised approach can design effective SEs which remove the noise and reconstruct the image with high accuracy. Especially in the results of noise rate 0.5, even the worst MSE of GAs is smaller than that of PSWA in most textures. DMSXF performs better than the conventional crossover and obtains good solutions. The remarkable improvement in processing performance is found in the worst MSE, which indicates that dMSXF can design suitable SEs stably.

6. Conclusions

Deterministic Multi-step Crossover Fusion (dMSXF) is a superior genetic multi-step operator for inheritance of characteristics and works especially well on problems associated with AR(1) landscapes. In this paper, dMSXF was adopted as the crossover method to optimize structuring elements (SEs) of morphological filters for texture images. A feature of an AR(1) landscape was observed in the objective function for the

Table 3 Processing results (MSE): dMSXF, UX and PSWA

Noise Rate = 25%											
Instance			**GA (dMSXF)**				**GA (UX)**				**PSWA**
#	MSE	ℓ_c	best	wst.	avg.	std.	best	wst.	avg.	std.	best
D3	1506.4	11.2	260.6	356.6	292.0	21.2	266.7	438.0	294.8	35.7	490.0
D20	3052.3	9.7	105.9	245.0	121.5	30.1	106.4	239.6	133.9	37.3	245.2
D57	3996.3	10.2	274.5	362.8	287.9	24.1	275.4	439.6	312.1	53.8	299.1
D74	2102.7	9.4	110.3	132.8	118.0	6.4	111.9	136.2	121.3	6.5	210.0
D87	3298.5	10.6	280.7	311.1	291.0	6.5	287.0	336.9	299.4	12.5	402.8
D98	3712.0	9.1	27.7	36.7	30.4	2.4	27.7	52.1	32.9	6.8	43.2
D101	1819.3	9.3	261.3	580.6	354.9	95.6	282.9	688.3	397.9	114.3	344.7
D112	1682.6	9.6	87.7	105.7	93.2	5.2	88.5	130.9	99.4	10.6	145.2
Noise Rate = 50%											
Instance			**GA (dMSXF)**				**GA (UX)**				**PSWA**
#	MSE	ℓ_c	best	wst.	avg.	std.	best	wst.	avg.	std.	best
D3	2918.7	11.6	441.3	505.3	462.7	18.9	443.1	543.2	471.5	31.3	773.7
D20	6331.9	11.4	265.3	319.4	273.1	13.2	266.4	331.7	277.2	14.5	562.2
D57	7979.4	11.7	383.7	442.4	392.3	12.7	384.9	463.7	404.9	23.7	537.0
D74	4076.7	10.9	226.2	286.2	237.5	13.4	226.4	289.3	252.4	22.0	382.5
D87	6580.7	11.6	472.4	516.4	486.3	12.2	473.0	529.8	491.3	15.0	701.8
D98	7409.7	9.6	69.5	76.3	71.7	1.8	70.1	123.6	84.7	13.6	87.9
D101	3667.0	11.1	438.9	513.8	462.4	20.1	440.2	533.7	470.8	24.8	458.2
D112	3427.6	10.9	180.7	230.3	188.6	10.5	181.9	256.4	200.9	20.7	257.4

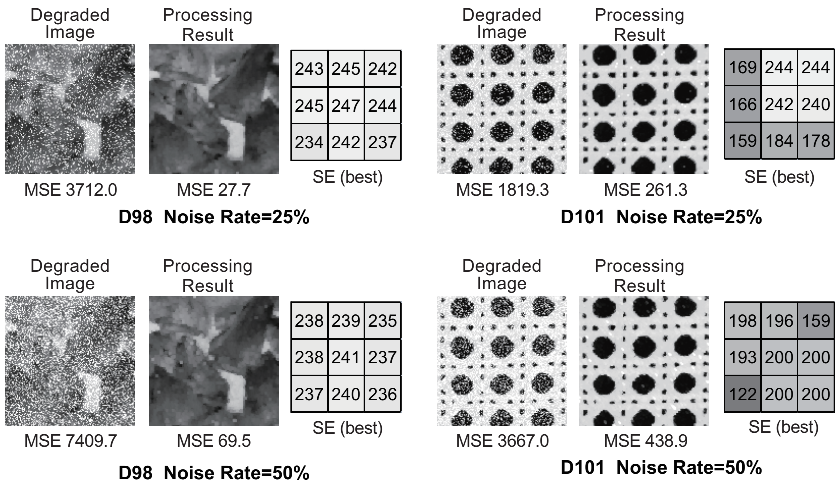

Fig. 3 The processing results obtained by dMSXF

design of SEs, and it was shown that dMSXF can estimate more suitable SEs stably. SEs obtained by dMSXF also outperformed a promising conventional filter. Here, we applied this unsupervised design method to the case of non-negative integer impulse noises; however, it is extendable to the combination of opening and closing to suppress more practical salt-and-pepper noise. This goal is left for future works.

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References

1) Soille, P.: Morphological Image Analysis, 2nd Ed., Springer, 2003.
2) Ikeda, K., and Kobayashi, S.: deterministic Multi-step Crossover Fusion: A Handy Crossover for GAs, PPSN VII, pp.162–171, 2002.
3) Hanada, Y., Hiroyasu, T. and Miki, M.: Genetic Multi-step Search in Interpolation and Extrapolation domain, Proc. of GECCO 2007, pp. 1242-1249, 2007.
4) Hanada, Y., Hiroyasu, T. and Muneyasu, M.: Analysis of the Performance of Genetic Multi-Step Search in Interpolation and Extrapolation Domain, Transactions of the Japanese Society for Artificial Intelligence, Vol. 24, No. 1, pp.136-146, 2009 (in Japanese) / GECCO 2008, pp.1107-1108, 2008
5) Kauffman, S. A.: Adaptation on rugged fitness landscapes, Lectures in the Sciences of Complexity, Vol. 1, Addison Wesley, 1989.
6) Fujiki A., Asano A., and Muneyasu, M.: Unsupervised optimization of morphological filters for noise removal in texture images. Proc of SCIS & ISIS2006, pp. 1794–1799, 2006.
7) Weinberger, E. D.: Correlated and uncorrelated fitness landscapes and how to tell the difference, Biological Cybernetics, Vol. 63, pp. 325-336, 1990.
8) Fontana, W., Stadler, P. F., Tarazona, P., Weinberger, E. D. and Schuster, P.: RNA folding and combinatory landscapes, Phys. Rev. E, Vol. 47, No. 3, pp. 2083–2099, 1993.
9) Brodatz, P.: Textures: A Photographic Album for Artists and Designers, Dover Pubns, 1999. http://www.ux.uis.no/tranden/brodatz.html
10) Thierens, D. and Goldberg, D. E.: Elitist Recombination: an integrated selection recombination GA. Proceedings of the 1st IEEE Conference on Evolutionary Computation, pp. 508-512, 1994.
11) Matsumoto, T. and Taguchi, A.: Removal of impulse noise from highly corrupted images by using noise position information and directional information of image, in Proc. Conference on Nonlinear Image Processing and Pattern Analysis XII. SPIE, pp. 188–196, 2001.