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完全グラフの均衡型 (C_5, C_{18})-$2t$-Foil 分解

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グラフ理論において、グラフの分解問題は主要な研究テーマである。C_5 を５点を通るサイクル、C_{18} を１８点を通るサイクルとする。１点を共有する辺素な t 個の C_5 と t 個の C_{18} からなるグラフを (C_5, C_{18})-$2t$-foil という。本研究では、完全グラフ K_n を 均衡的に (C_5, C_{18})-$2t$-foil 部分グラフに分解する組合せデザインについて述べる。

Balanced (C_5, C_{18})-$2t$-Foil Decomposition of Complete Graphs

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In graph theory, the decomposition problem of graphs is a very important topic. Various type of decompositions of many graphs can be seen in the literature of graph theory. This paper gives a balanced (C_5, C_{18})-$2t$-foil decomposition of complete graph K_n.

1. Introduction

Let K_n denote *the complete graph* of n vertices. Let C_5 and C_{18} be *the* 5-*cycle* and *the* 18-*cycle*, respectively. *The* (C_5, C_{18})-$2t$-*foil* is a graph of t edge-disjoint C_5's and t edge-disjoint C_{18}'s with a common vertex and the common vertex is called *the center of the* (C_5, C_{18})-$2t$-*foil*. In particular, the (C_5, C_{18})-2-foil is called *the* (C_5, C_{18})-*bowtie*. When K_n is decomposed into edge-disjoint sum of (C_5, C_{18})-$2t$-foils, we say that K_n *has a* (C_5, C_{18})-$2t$-*foil decomposition*. Moreover, when every vertex of K_n appears in the same number of (C_5, C_{18})-$2t$-foils, we say that K_n *has a balanced* (C_5, C_{18})-$2t$-*foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced* (C_5, C_{18})-$2t$-*foil system*.

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2. Balanced (C_5, C_{18})-$2t$-foil decomposition of K_n

Theorem. K_n has a balanced (C_5, C_{18})-$2t$-foil decomposition if and only if $n \equiv 1 \pmod{46t}$.

Proof. (Necessity) Suppose that K_n has a balanced (C_5, C_{18})-$2t$-foil decomposition. Let b be the number of (C_5, C_{18})-$2t$-foils and r be the replication number. Then $b = n(n-1)/46t$ and $r = (21t+1)(n-1)/46t$. Among r (C_5, C_{18})-$2t$-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_5, C_{18})-$2t$-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n-1$. From these relations, $r_1 = (n-1)/46t$ and $r_2 = 21(n-1)/46$. Therefore, $n \equiv 1 \pmod{46t}$ is necessary.
(Sufficiency) Put $n = 46st + 1$ and $T = st$. Then $n = 46T + 1$.
Case 1. $\mathbf{n = 47}$. (Example 1. Balanced (C_5, C_{18})-2-foil decomposition of K_{47}.)
Case 2. $\mathbf{n = 46T + 1, T \geq 2}$. Construct a (C_5, C_{18})-$2T$-foil as follows:
$\{(46T+1, 1, 18T+2, 43T+2, 21T), (46T+1, 4T+1, 16T+2, 26T+2, 32T+2, 34T+3, 14T+3, 28T+3, 36T+4, 21T+3, 38T+4, 9T+3, 40T+4, 29T+3, 18T+3, 11T+2, 7T+2, 6T+1)\}$
$\cup$
$\{(46T+1, 2, 18T+4, 43T+3, 21T-1), (46T+1, 4T+2, 16T+4, 22T+3, 24T+5, 34T+4, 14T+5, 28T+4, 36T+6, 21T+4, 38T+6, 9T+4, 40T+6, 29T+4, 18T+5, 11T+3, 7T+4, 6T+2)\} \cup$
$\{(46T+1, 3, 18T+6, 43T+4, 21T-2), (46T+1, 4T+3, 16T+6, 22T+4, 24T+7, 34T+5, 14T+7, 28T+5, 36T+8, 21T+5, 38T+8, 9T+5, 40T+8, 29T+5, 18T+7, 11T+4, 7T+6, 6T+3)\} \cup$
$\{(46T+1, 4, 18T+8, 43T+5, 21T-3), (46T+1, 4T+4, 16T+8, 22T+5, 24T+9, 34T+6, 14T+9, 28T+6, 36T+10, 21T+6, 38T+10, 9T+6, 40T+10, 29T+6, 18T+9, 11T+5, 7T+8, 6T+4)\} \cup$

... $\cup$
$\{(46T+1, T-1, 20T-2, 44T, 20T+2), (46T+1, 5T-1, 18T-2, 23T, 26T-1, 35T+1, 16T-1, 29T+1, 38T, 22T+1, 40T, 10T+1, 42T, 30T+1, 20T-1, 12T, 9T-2, 7T-1)\}$
$\cup$
$\{(46T+1, T, 20T, 44T+1, 20T+1), (46T+1, 5T, 18T, 23T+1, 26T+1, 35T+2, 16T+1, 29T+2, 38T+2, 22T+2, 40T+2, 10T+2, 42T+2, 32T+1, 24T+1, 12T+1, 9T, 7T)\}$.
Decompose the (C_5, C_{18})-$2T$-foil into s (C_5, C_{18})-$2t$-foils. Then these starters comprise a balanced (C_5, C_{18})-$2t$-foil decomposition of K_n.

Corollary. K_n has a balanced (C_5, C_{18})-bowtie decomposition if and only if $n \equiv 1 \pmod{46}$.

Example 1. Balanced (C_5, C_{18})-2-foil decomposition of K_{47}.
$\{(47, 1, 20, 45, 21), (47, 5, 18, 28, 34, 37, 17, 31, 40, 24, 42, 12, 44, 33, 25, 13, 9, 7)\}$.
This starter comprises a balanced (C_5, C_{18})-2-foil decomposition of K_{47}.

Example 2. Balanced (C_5, C_{18})-4-foil decomposition of K_{93}.
$\{(93, 1, 38, 88, 42),$
$(93, 2, 40, 89, 41)\}$
$\cup$
$\{(93, 9, 34, 54, 66, 71, 31, 59, 76, 45, 80, 21, 84, 61, 39, 24, 16, 13),$
$(93, 10, 36, 47, 53, 72, 33, 60, 78, 46, 82, 22, 86, 65, 49, 25, 18, 14)\}$.
This starter comprises a balanced (C_5, C_{18})-4-foil decomposition of K_{93}.

Example 3. Balanced (C_5, C_{18})-6-foil decomposition of K_{139}.
$\{(139, 1, 56, 131, 63),$
$(139, 2, 58, 132, 62),$
$(139, 3, 60, 133, 61)\}$
$\cup$
$\{(139, 13, 50, 80, 98, 105, 45, 87, 112, 66, 118, 30, 124, 90, 57, 35, 23, 19),$
$(139, 14, 52, 69, 77, 106, 47, 88, 114, 67, 120, 31, 126, 91, 59, 36, 25, 20),$
$(139, 15, 54, 70, 79, 107, 49, 89, 116, 68, 122, 32, 128, 97, 73, 37, 27, 21)\}$.
This starter comprises a balanced (C_5, C_{18})-6-foil decomposition of K_{139}.

Example 4. Balanced (C_5, C_{18})-8-foil decomposition of K_{185}.
$\{(185, 1, 74, 174, 84),$
$(185, 2, 76, 175, 83),$
$(185, 3, 78, 176, 82),$
$(185, 4, 80, 177, 81)\}$
$\cup$
$\{(185, 17, 66, 106, 130, 139, 59, 115, 148, 87, 156, 39, 164, 119, 75, 46, 30, 25),$
$(185, 18, 68, 91, 101, 140, 61, 116, 150, 88, 158, 40, 166, 120, 77, 47, 32, 26),$
$(185, 19, 70, 92, 103, 141, 63, 117, 152, 89, 160, 41, 168, 121, 79, 48, 34, 27),$
$(185, 20, 72, 93, 105, 142, 65, 118, 154, 90, 162, 42, 170, 129, 97, 49, 36, 28)\}$.
This starter comprises a balanced (C_5, C_{18})-8-foil decomposition of K_{185}.

Example 5. Balanced (C_5, C_{18})-10-foil decomposition of K_{231}.
$\{(231, 1, 92, 217, 105),$
$(231, 2, 94, 218, 104),$
$(231, 3, 96, 219, 103),$
$(231, 4, 98, 220, 102),$
$(231, 5, 100, 221, 101)\}$
$\cup$
$\{(231, 21, 82, 132, 162, 173, 73, 143, 184, 108, 194, 48, 204, 148, 93, 57, 37, 31),$
$(231, 22, 84, 113, 125, 174, 75, 144, 186, 109, 196, 49, 206, 149, 95, 58, 39, 32),$
$(231, 23, 86, 114, 127, 175, 77, 145, 188, 110, 198, 50, 208, 150, 97, 59, 41, 33),$
$(231, 24, 88, 115, 129, 176, 79, 146, 190, 111, 200, 51, 210, 151, 99, 60, 43, 34),$
$(231, 25, 90, 116, 131, 177, 81, 147, 192, 112, 202, 52, 212, 161, 121, 61, 45, 35)\}$.
This starter comprises a balanced (C_5, C_{18})-10-foil decomposition of K_{231}.

Example 6. Balanced (C_5, C_{18})-12-foil decomposition of K_{277}.
$\{(277, 1, 110, 260, 126),$

$(277, 2, 112, 261, 125)$,
$(277, 3, 114, 262, 124)$,
$(277, 4, 116, 263, 123)$,
$(277, 5, 118, 264, 122)$,
$(277, 6, 120, 265, 121)\}$
$\cup$
$\{(277, 25, 98, 158, 194, 207, 87, 171, 220, 129, 232, 57, 244, 177, 111, 68, 44, 37)$,
$(277, 26, 100, 135, 149, 208, 89, 172, 222, 130, 234, 58, 246, 178, 113, 69, 46, 38)$,
$(277, 27, 102, 136, 151, 209, 91, 173, 224, 131, 236, 59, 248, 179, 115, 70, 48, 39)$,
$(277, 28, 104, 137, 153, 210, 93, 174, 226, 132, 238, 60, 250, 180, 117, 71, 50, 40)$,
$(277, 29, 106, 138, 155, 211, 95, 175, 228, 133, 240, 61, 252, 181, 119, 72, 52, 41)$,
$(277, 30, 108, 139, 157, 212, 97, 176, 230, 134, 242, 62, 254, 193, 145, 73, 54, 42)\}$.
This starter comprises a balanced (C_5, C_{18})-12-foil decomposition of K_{277}.

Example 7. Balanced (C_5, C_{18})-14-foil decomposition of K_{323}.
$\{(323, 1, 128, 303, 147)$,
$(323, 2, 130, 304, 146)$,
$(323, 3, 132, 305, 145)$,
$(323, 4, 134, 306, 144)$,
$(323, 5, 136, 307, 143)$,
$(323, 6, 138, 308, 142)$,
$(323, 7, 140, 309, 141)\}$
$\cup$
$\{(323, 29, 114, 184, 226, 241, 101, 199, 256, 150, 270, 66, 284, 206, 129, 79, 51, 43)$,
$(323, 30, 116, 157, 173, 242, 103, 200, 258, 151, 272, 67, 286, 207, 131, 80, 53, 44)$,
$(323, 31, 118, 158, 175, 243, 105, 201, 260, 152, 274, 68, 288, 208, 133, 81, 55, 45)$,
$(323, 32, 120, 159, 177, 244, 107, 202, 262, 153, 276, 69, 290, 209, 135, 82, 57, 46)$,
$(323, 33, 122, 160, 179, 245, 109, 203, 264, 154, 278, 70, 292, 210, 137, 83, 59, 47)$,
$(323, 34, 124, 161, 181, 246, 111, 204, 266, 155, 280, 71, 294, 211, 139, 84, 61, 48)$,
$(323, 35, 126, 162, 183, 247, 113, 205, 268, 156, 282, 72, 296, 225, 169, 85, 63, 49)\}$.
This starter comprises a balanced (C_5, C_{18})-14-foil decomposition of K_{323}.

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