

On the Number of Rooms in a Rectangular Solid Dissection

HIDENORI OHTA,^{†1} TOSHINORI YAMADA^{†2}
and KUNIHIRO FUJIYOSHI^{†1}

In these years, 3D-LSIs which consist of several silicon layers have been developed and attracted attention. For floorplaning of 3D-LSIs, a rectangular solid dissection, which is a dissection of a rectangular solid into smaller rectangular solids by planes, also has attracted attention and been studied much. However, not so many properties have been clarified about a rectangular solid dissection. This paper presents the relation between the number of rooms and that of walls in a rectangular solid dissection.

1. Introduction

A rectangular dissection (also called floorplan) is a dissection of a rectangle into smaller rectangles (called rooms) by horizontal and vertical line segments (called segs), and is used for a 2D-LSI layout design by placing at most one module in each room and by wiring between modules along segs. In order that the given modules may be embedded into the rooms, the floorplan is adjusted without topological changing. It is well known that $\mathcal{N} = seg + 1$ in a rectangular dissection without crossing any two segs¹⁾, where \mathcal{N} and seg are the number of rooms and segs (not containing four edges of the bounding rectangle), respectively. The Q-sequence proposed by Sakanushi, et al.²⁾ is based on the property of $\mathcal{N} = seg + 1$, and can represent any rectangular dissection without crossing any two segs. It is known that a near optimal floorplan (e.g., a floorplan with even smaller area) can be obtained in a practical time by searching the corresponding Q-sequences by simulated annealing.

^{†1} Department of Electrical and Information Engineering, Tokyo University of Agriculture and Technology

^{†2} Division of Mathematics, Electronics and Informations, Saitama University

Recently, the development of 3D-MCMs (Multi Chip Modules) which a number of semiconductor chips pack into and 3D-LSIs consisting of a number of layers integrating many active devices draws attention industrially. The following problem, so called “3D block packing problem,” has been studied for their layout designs^{3)–6)}: Given rectangular solid blocks, place these blocks in the minimum bounding rectangular solid without overlap. However, the problem of wiring between modules is not considered in the 3D block packing problem at all, and therefore routing in a 3D-LSI design may be carried out in disorder, which leads to the increase of the wire length in the resulting 3D-LSI and a degradation in performance.

In order to solve this problem, we can use a “rectangular solid dissection,” which is a dissection of the rectangular solid into smaller rectangular solids (called rooms) by dissection faces (called walls). Each of the modules is located in a room and we can wire between modules along walls, which is similar to wiring along segs in the 2D-LSI layout design. As a consequence, we can avoid disorderly routing in a 3D-LSI and the degradation in performance.

As representations of a rectangular solid dissection, Slicing-tree⁷⁾ and O-Sequence⁸⁾ were proposed. Slicing-tree can represent any slicing structure and the O-sequence can represent any rectangular solid dissection if the rectangular solid dissection is under the following two restrictions. The first restriction is that the shapes of all the walls are rectangles. The second one is that any two walls do not cross each other. Any slicing structure is also under the two restrictions.

By using Slicing-tree or O-Sequence with simulated annealing, we can search for good rectangular solid dissection (3D-Floorplan). However, there are so many rectangular solid dissections which the representations can not express. Those rectangular solid dissections may include optimal solutions and we may miss good solutions. By removing these restrictions and considering a general structure of rectangular solid dissections, we will be able to search for a better solution. However, no representation to express the general structure of rectangular solid dissections has been proposed.

One of the reasons why such a good method has not been invented is that the properties of a rectangular solid dissection are not clear. Basic properties of a

rectangular dissection are well known while not so many properties have been clarified about a rectangular solid dissection.

In this paper, we give one of the important basic properties, that is, the relation between the number of rooms and that of walls in a rectangular solid dissection.

The rest of the paper is organized as follows: In Section 2, we present the relation between the number of rooms and that of segs in an orthogonal dissection of a rectangle, which is a generalization of a rectangle dissection. Using the relation, Section 3 gives the relation between the number of rooms and that of walls in a rectangular solid dissection of a rectangular solid. Finally, Section 4 gives our conclusion.

2. Orthogonal Dissection of Rectangle

Consider a rectangle, which is called an *entire rectangle*. The four edges of the entire rectangle are called *external segs*. An *orthogonal dissection* of the entire rectangle is a dissection of the rectangle into smaller regions, called rooms, by horizontal and vertical line segments, called *internal segs*. **Figure 1** shows an orthogonal dissection of the rectangle into 13 rooms a, b, \dots , and m . Throughout the section, a “seg” means an internal or external seg. Let $\mathfrak{S}(\Delta)$ denote the number of internal segs in an orthogonal dissection Δ .

2.1 Segs

In the paper, we assume that each endpoint of a seg is on exactly one other segment perpendicularly. By the assumption, the corner among four rooms b, c, e , and f in Fig. 1 must be viewed as either of the following two cases (C1) and (C2), but not as the case in **Fig. 2** (a):

- (C1) The point of intersection of two segs (Fig. 2 (b));
- (C2) The common endpoint of two vertical [horizontal] segs on a horizontal

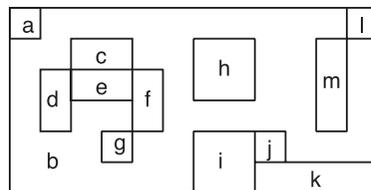


Fig. 1 Orthogonal dissection of rectangle which is dissected into 13 rooms a, b, \dots , and m .

[vertical] seg (Fig. 2 (c) [Fig. 2 (d)]).

The points in (C1) and (C2) are called a *cross point* and *cross-shaped corner*, respectively. Let $\mathfrak{C}(\Delta)$ denote the number of cross points in an orthogonal dissection Δ .

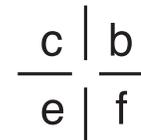
Similarly, the corner among three rooms b, i , and j must be viewed as an endpoint of one seg on the other seg. An endpoint of a seg is called a *T-shaped corner* if the endpoint is not an endpoint of the other seg.

We call the common endpoint of a vertical internal seg and horizontal internal seg an *L-shaped corner*. Let $\mathfrak{L}(\Delta)$ denote the number of L-shaped corners in an orthogonal dissection Δ .

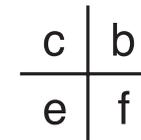
2.2 Rooms and Holes

A room in an orthogonal dissection of the entire rectangle is each of maximally connected regions obtained from the entire rectangle by deleting all segments. Notice that a room contains no segment in its boundary, that is a room is open. Let $\mathfrak{R}(\Delta)$ denote the number of rooms in an orthogonal dissection Δ .

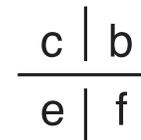
A hole of room R is a maximally connected region surrounded by R . Notice that a hole contains a boundary to R , that is a hole is closed. Therefore, room b in Fig. 1 has two holes (**Fig. 3**), and any region containing room m is not a hole of b . Let $\mathfrak{H}(\Delta)$ denote the summation of the number of holes over all the rooms in an orthogonal dissection Δ .



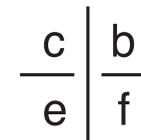
(a) Two vertical segs and two horizontal segs



(b) Intersection of two segs



(c) Two vertical segs and one horizontal seg



(d) One vertical seg and two horizontal segs

Fig. 2 The corner among 4 rooms b, c, e , and f in Fig. 1.

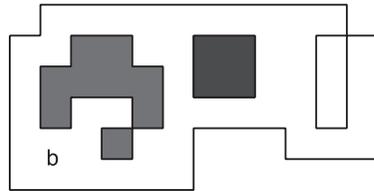


Fig. 3 Two holes of room *b* in Fig. 1.

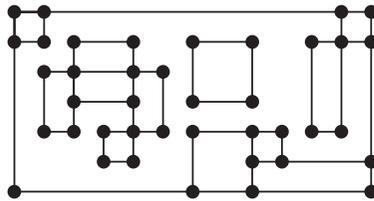


Fig. 4 Graph *G* obtained from Fig. 1.

2.3 Number of Rooms in an Orthogonal Dissection of a Rectangle

In the paper, we use the following famous theorem:

Theorem 1 (Euler’s polyhedron theorem) Let V , E , F , and C be the numbers of vertices, edges, faces, and connected components in a plane graph. Then, $V - E + F = C + 1$. \square

And we have the following theorem:

Theorem 2 For any orthogonal dissection Δ of an entire rectangle,
 $\mathfrak{R}(\Delta) = \mathfrak{S}(\Delta) + \mathfrak{C}(\Delta) - \mathfrak{L}(\Delta) + \mathfrak{H}(\Delta) + 1$.

Proof Consider any orthogonal dissection Δ of an entire rectangle. Assume without loss of generality that there exists no cross-shaped corner in Δ ; Otherwise, we can move a seg to remove a cross-shaped corner without changing the number of rooms, segs, cross points, L-shaped corners, and holes in Δ . Let G be a graph obtained from Δ in a natural way, each vertex of which represents a T-shaped corner, L-shaped corner, cross point, or a vertex of the entire rectangle. **Figure 4** shows a graph obtained from the orthogonal dissection in Fig. 1. Notice that the number of T-shaped corners in the dissection Δ is $(\mathfrak{S}(\Delta) - \mathfrak{L}(\Delta)) \times 2$. The number of vertices in G is

$$V = (\mathfrak{S}(\Delta) - \mathfrak{L}(\Delta)) \times 2 + \mathfrak{L}(\Delta) + \mathfrak{C}(\Delta) + 4,$$

and the number of connected components in G is $C = \mathfrak{H}(\Delta) + 1$. Each seg l in Δ is divided into

(# of T-shaped corners on l but endpoints of l) + (# of segs across l) + 1 edges in G , and so the number of edges in G is

$$E = (\mathfrak{S}(\Delta) - \mathfrak{L}(\Delta)) \times 2 + \mathfrak{C}(\Delta) \times 2 + (\mathfrak{S}(\Delta) + 4).$$

It is easy to see that G is a plane graph and that the number of faces in G is $F = \mathfrak{R}(\Delta) + 1$. Hence we conclude by Theorem 1 that

$$\mathfrak{L}(\Delta) - \mathfrak{C}(\Delta) - \mathfrak{S}(\Delta) + \mathfrak{R}(\Delta) + 1 = \mathfrak{H}(\Delta) + 2,$$

that is

$$\mathfrak{R}(\Delta) = \mathfrak{S}(\Delta) + \mathfrak{C}(\Delta) - \mathfrak{L}(\Delta) + \mathfrak{H}(\Delta) + 1. \quad \square$$

An orthogonal dissection Δ of the entire rectangle is called a rectangular dissection if every room in Δ is a rectangle. Since a rectangular dissection has no L-shaped corner and no hole, we have the following corollary from Theorem 2:

Corollary 1 $\mathfrak{R}(\Delta) = \mathfrak{S}(\Delta) + \mathfrak{C}(\Delta) + 1$ for any rectangular dissection Δ . \square

3. Rectangular Solid Dissection of Rectangular Solid

Consider a rectangular solid, called an *entire rectangular solid*, along the x -, y -, and z -axes in the space. The six faces consisting of the entire rectangular solid are called the *external walls*. A rectangular solid dissection of the entire rectangular solid is a dissection of the rectangular solid into smaller rectangular solids, called *rooms*, by planes, called *internal walls*, perpendicular to the x -, y -, or z -axis. Throughout the section, a “wall” means an internal or external wall. Each wall does not contain its boundary. A room contains no wall and does not intersect any other room. **Figure 5** shows a rectangular solid dissection into nine rooms a, b, \dots , and i . **Figure 6** shows a rectangular solid dissection into 6 rooms a, b, \dots , and f .

In a rectangular solid dissection, each room touches six faces of walls. A boundary of an internal wall perpendicular to the x -axis consists of line segments along the y - or z -axis. In the paper, we assume that for each line segment l along the y -axis [z -axis] in a boundary of an internal wall perpendicular to the x -axis, there is exactly one wall containing l and perpendicular to the z -axis [y -axis] wall. We have a similar assumption to the internal wall perpendicular to the

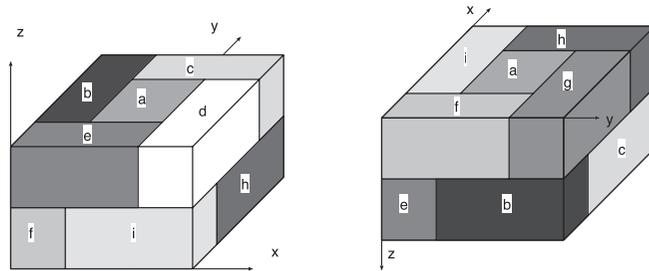


Fig. 5 Rectangular solid dissection.

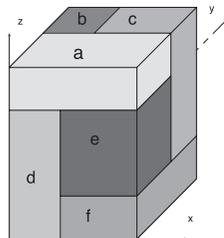


Fig. 6 Rectangular solid dissection 2.

y - or z -axis. These assumptions were also applied in the literature⁸⁾. With the assumptions, a boundary and a shape of each of the walls are specified. According to these assumptions, the rectangular solid dissection in Fig. 5 must be dissected by five non-rectangle internal walls. **Figure 7** shows five internal walls of rectangular solid dissection in Fig. 5. Figure 6 must be dissected by two rectangles and two non-rectangle internal walls. Note that two internal walls cross each other. **Figure 8** shows four internal walls of rectangular solid dissection in Fig. 6. Let $\mathcal{W}(\Phi)$ be the number of internal walls in a rectangular solid dissection Φ .

A *hole* of a wall is defined by a similar way of that in a room in Section 2.2. That is a hole of a wall W is a maximally connected region surrounded by W . Notice that a hole contains a boundary to W , that is a hole is closed. Let $\mathcal{H}(\Phi)$ be the total number of holes over all the internal walls in a rectangular solid dissection Φ .

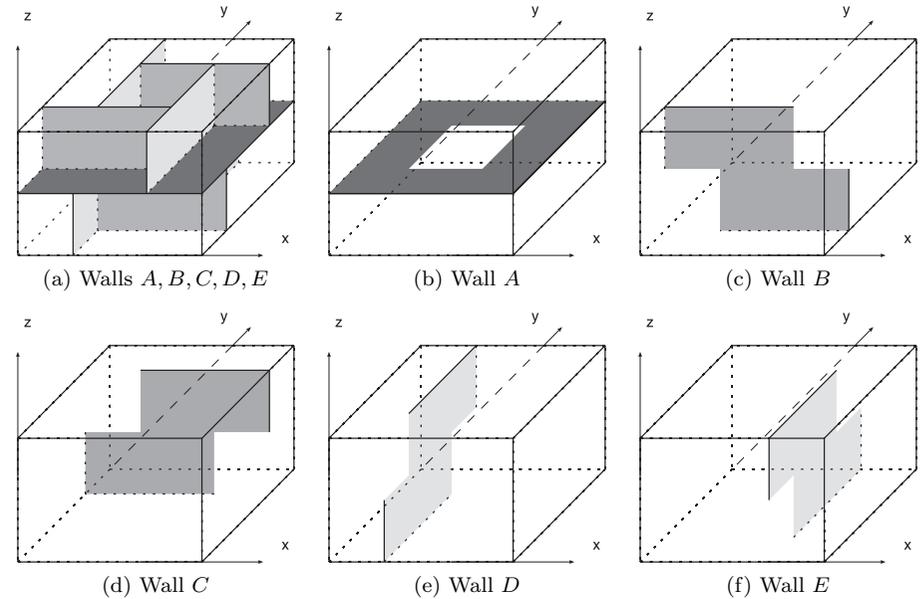


Fig. 7 Five walls in rectangular solid dissection in Fig. 5.

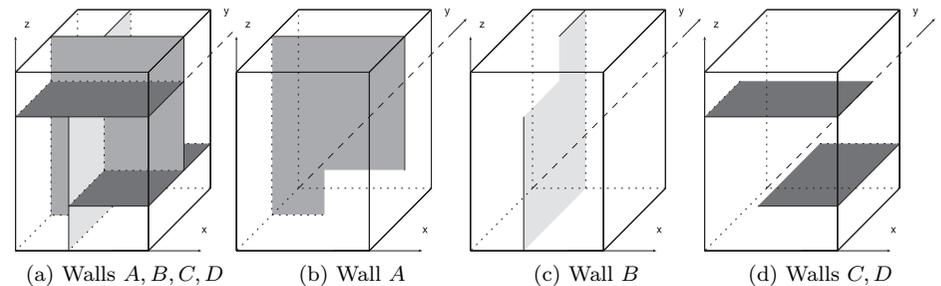


Fig. 8 Four walls in rectangular solid dissection in Fig. 6.

3.1 Crosses—Intersections of Two Walls—

A *cross* of two walls is a line segment of their intersection in a rectangular solid dissection. Notice that there exist multiple crosses of two walls, e.g., two crosses ab and cd of walls A and B in **Fig. 9**. Let $\mathcal{I}(\Phi)$ be the number of crosses in a

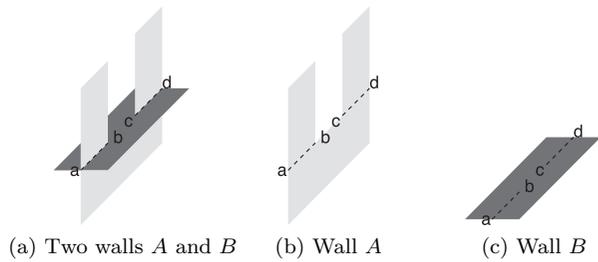


Fig. 9 Two crosses ab and cd of two walls A and B .

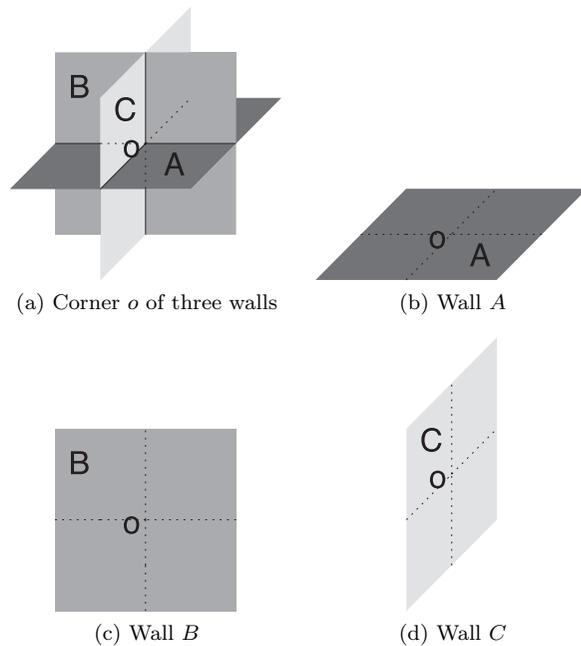


Fig. 10 Corner o of three walls A , B , and C .

rectangular solid dissection Φ .

3.2 Corners—Intersections of Three Walls—

The *corner* of three walls is the point of their intersection in a rectangular solid dissection. Then, the corner is on the cross of any two of these walls. **Figure 10**

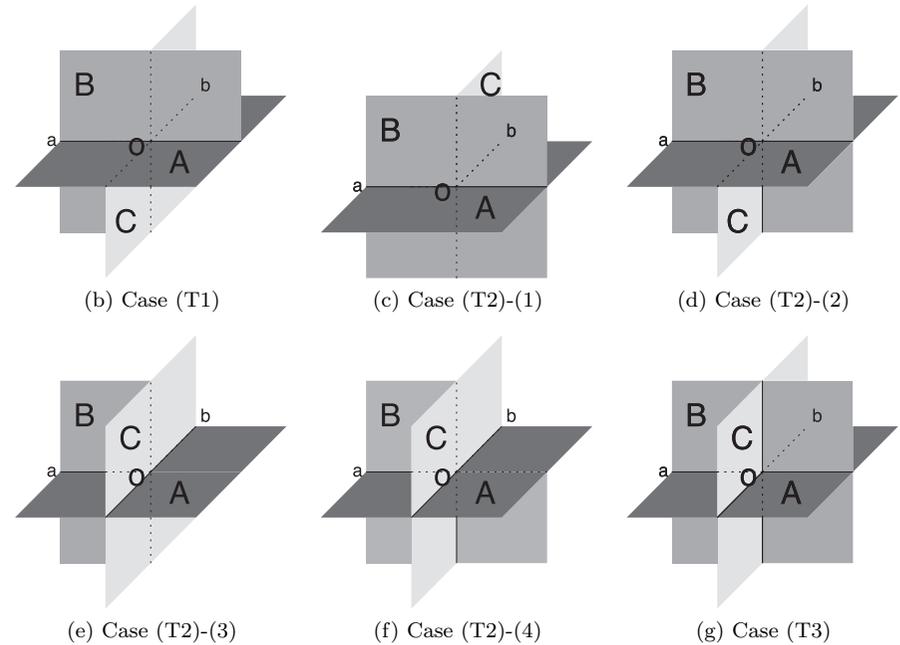
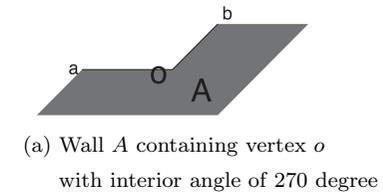


Fig. 11 Wall A containing vertex o with interior angle of 270 degree (Note that vertex o in (b) is a three-cornered point).

shows the corner o of three walls A , B , and C . Let $\mathcal{C}(\Phi)$ be the number of corners in a rectangular solid dissection Φ .

3.3 Three-cornered Points

Consider a wall A whose boundary containing a vertex o with an interior angle of 270 degree (**Fig. 11** (a)). By the definition of a wall, we have a wall B [C] containing a line segment ao [bo] and perpendicular to wall A . There are three

cases for the dissection to be a rectangular solid dissection:

- (T1) The dissection has no cross of A and B with endpoint o , and no cross of A and C with endpoint o as in Fig. 11 (b);
- (T2) The dissection has a cross with endpoint o and on wall A , either that of A and B as in Fig. 11 (c) or (d), or that of A and C as in Fig. 11 (e) or (f);
- (T3) The dissection has two crosses with endpoint o and on wall A , that of A and B , and that of A and C (Fig. 11 (g)).

The vertex o in (T1) is called a *three-cornered point*. Then, three walls have interior angle of 270 degree at vertex o . Let $\mathcal{T}(\Phi)$ be the number of three-cornered points in a rectangular solid dissection Φ .

3.4 Number of Rooms in a Rectangular Solid Dissection

Theorem 3 For any rectangular solid dissection Φ ,

$$\mathcal{R}(\Phi) = \mathcal{W}(\Phi) - \mathcal{H}(\Phi) + \mathcal{I}(\Phi) + \mathcal{C}(\Phi) + \mathcal{T}(\Phi) + 1.$$

Proof The theorem is proved by induction on $\mathcal{R}(\Phi)$.

If $\mathcal{R}(\Phi) = 1$ then the theorem holds since $\mathcal{W}(\Phi) = 0$, which implies that $\mathcal{H}(\Phi) = \mathcal{I}(\Phi) = \mathcal{C}(\Phi) = \mathcal{T}(\Phi) = 0$.

Assume that $\mathcal{R}(\Phi) \geq 2$. Assume for the inductive step that the theorem holds for any rectangular solid dissection Ψ with $\mathcal{R}(\Psi) < \mathcal{R}(\Phi)$. Since $\mathcal{R}(\Phi) \geq 2$, we have $\mathcal{W}(\Phi) \geq 1$. Fix any internal wall α (Fig. 12 (a)). Assume without loss of generality that α is perpendicular to the z -axis, and that the infinity plane containing α does not contain any other wall; Otherwise, we can move α in the z -direction. Consider a rectangular solid dissection Φ' obtained from Φ by replacing α with α' that is the wall of the cross section of the entire rectangular solid containing α (Fig. 12 (b)). Let

$$\begin{aligned} \mathcal{R}(\Phi') &= \mathcal{R}(\Phi) + r, & \mathcal{H}(\Phi') &= \mathcal{H}(\Phi) - h, \\ \mathcal{I}(\Phi') &= \mathcal{I}(\Phi) + s, & \mathcal{C}(\Phi') &= \mathcal{C}(\Phi) + c, \\ \mathcal{T}(\Phi') &= \mathcal{T}(\Phi) - t. \end{aligned}$$

Then, we have the following lemma:

Lemma 1 $r = s + c - t + h$.

Proof of Lemma 1 Consider the orthogonal dissection Δ such that the entire rectangle in Δ corresponds to the rectangle containing four edges around α' , and each internal seg in Δ corresponds to a line segment of intersection of

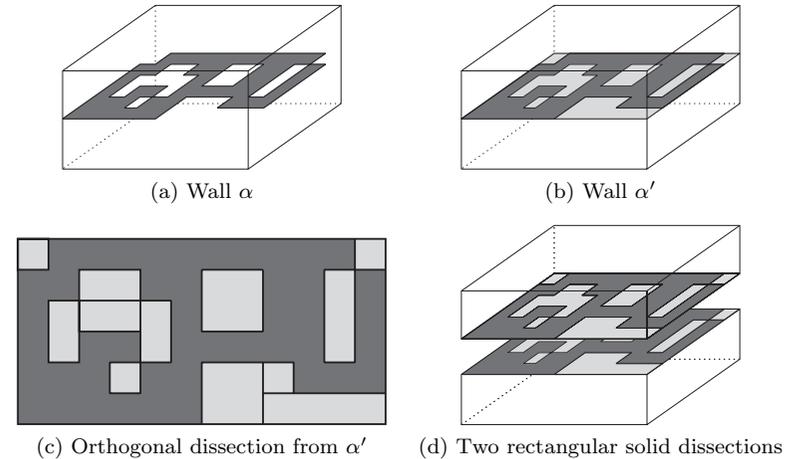


Fig. 12 Proof of Theorem 3.

$\alpha' - \alpha$ and an internal wall in Φ' , where $\alpha' - \alpha$ is the region obtained from α' by deleting a region in α (Fig. 12 (c)). In the following, we will estimate each term in equation

$$\mathfrak{R}(\Delta) = \mathfrak{S}(\Delta) + \mathfrak{C}(\Delta) - \mathfrak{L}(\Delta) + \mathfrak{H}(\Delta) + 1.$$

Each of all the rooms, except α , in Δ corresponds to a distinct room in Φ which is divided into two rooms by replacing α in Φ with α' in Φ' . Thus, $\mathfrak{R}(\Delta) = r + 1$.

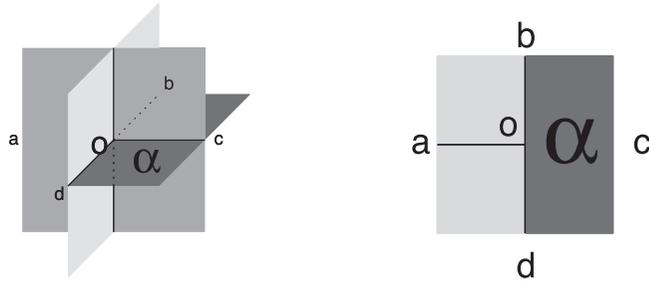
Consider any endpoint of an internal seg in Δ . Notice that each internal seg in Δ is contained in a cross in Φ' . So, we have the following two cases, (E1) and (E2):

- (E1) The endpoint corresponds to the endpoint of the cross;
- (E2) The endpoint corresponds to neither of the endpoints of the cross.

Therefore, each internal seg l in Δ can be classified into the following three groups, (S0), (S1), and (S2):

- (S0) Both endpoints of l are in (E1);
- (S1) One endpoint of l is in (E1) and the other endpoint is in (E2);
- (S2) Both endpoints of l are in (E2).

Let s_0, s_1 and s_3 denote the numbers of segs in (S0), (S1), and (S2) respectively. Then, $\mathfrak{S}(\Delta) = s_0 + s_1 + s_2$.



(a) Point o in Φ corresponding to a corner in Φ' (b) Δ corresponding to (a)

Fig. 13 Corner o in Φ' corresponding to T-shaped corner in Ψ .

Let q denote the number of cross points in Δ . That is, $q = \mathfrak{C}(\Delta)$.

Each L-shaped corner in Δ corresponds to a point in Φ corresponding to vertex o in either of (T1)–(T3) of Fig. 11, where A corresponds to α . Let t_1, t_2 , and t_3 denote the numbers of points in Φ corresponding to o in (T1), (T2), and (T3) of Fig. 11, respectively. Then, $\mathfrak{L}(\Delta) = t_1 + t_2 + t_3$.

The holes of room α in Δ are the same as those of wall α in Φ , which disappear in Φ' by replacing α with α' . Therefore, $\mathfrak{H}(\Delta) = h$.

From the above argument, we conclude by Theorem 2 that

$$r + 1 = (s_0 + s_1 + s_2) + q - (t_1 + t_2 + t_3) + h + 1. \tag{1}$$

Any seg in (S0) corresponds to a new cross generated in replacing α with α' , and one in (S2) corresponds to a cross obtained from two crosses by merging them. Thus,

$$s = s_0 - s_2. \tag{2}$$

Let p denote the number of corners in Φ' corresponding to T-shaped corners in Δ (**Fig. 13**). Since $p + q + t_3$ corners are generated and t_1 three-cornered points disappear by replacing α with α' , we obtain that

$$c = p + q + t_3 \tag{3}$$

and

$$t = t_1. \tag{4}$$

Δ has $s_1 + 2s_2$ endpoints of case (E2). On the other hand, each endpoint of case (E2) corresponds to a point o in either of (a) of Fig. 13, (T2) of Fig. 11 and (T3) of Fig. 11. Note that we count two endpoints for o in (T3) of Fig. 11, one for seg

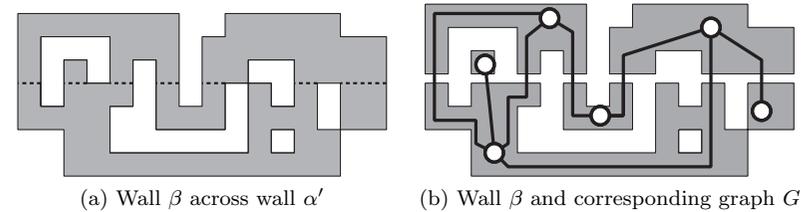


Fig. 14 Conversion of Wall β across Wall α' .

ao and the other for seg bo . Therefore, we obtain that

$$p + t_2 + 2t_3 = s_1 + 2s_2. \tag{5}$$

By Eqs. (1)–(5), we conclude that

$$\begin{aligned} r + 1 &= (s_0 + s_1 + s_2) + q - (t_1 + t_2 + t_3) + h + 1, \\ r &= (s_0 - s_2) + (p + q + t_3) - t_1 + h \\ &\quad + (s_1 + 2s_2) - (p + t_2 + 2t_3) \\ &= s + c - t + h. \end{aligned}$$

□

Let Φ'' be the rectangular solid dissection obtained from Φ' by cutting each wall across α' into multiple walls by α' .

Lemma 2

$$\begin{aligned} \mathcal{W}(\Phi') - \mathcal{H}(\Phi') + \mathcal{I}(\Phi') + \mathcal{C}(\Phi') \\ = \mathcal{W}(\Phi'') - \mathcal{H}(\Phi'') + \mathcal{I}(\Phi'') + \mathcal{C}(\Phi''). \end{aligned}$$

Proof of Lemma 2 Fix any internal wall β across α' in Φ' (**Fig. 14** (a)), and consider a graph G obtained from β by replacing each wall in Φ'' corresponding to β with a vertex and replacing each cross of α' and β with an edge (**Fig. 14** (b)). It is easy to see that G is a connected plane graph, and that each hole of β corresponds to an inner face in G . Let F be the number of walls in Φ'' corresponding to β , H be the number of holes of β , and C be the number of cross segments by α' and β . Since the numbers of vertices, edges, and faces are F, C , and $H + 1$, respectively, we have by Theorem 1

$$F + (H + 1) - C = 2.$$

That is,

$$1 - H + C = F,$$

and hence

$$\mathcal{W}(\Phi') - \mathcal{H}(\Phi') + \mathcal{I}_p(\Phi') = \mathcal{W}(\Phi'') - \mathcal{H}(\Phi'') + \mathcal{I}_p(\Phi''),$$

where $\mathcal{I}_p(\Phi')$ [$\mathcal{I}_p(\Phi'')$] denotes the number of crosses parallel to α' in Φ' [Φ''].

Next, consider any cross l perpendicular to α' . If l is not across α' then l is also a cross in Φ'' . Otherwise, the intersection of l and α' is a corner in Φ' , and l is cut by α' into two crosses in Φ'' . Hence,

$$\mathcal{I}_o(\Phi') + \mathcal{C}(\Phi') = \mathcal{I}_o(\Phi'') + \mathcal{C}(\Phi''),$$

where $\mathcal{I}_o(\Phi')$ [$\mathcal{I}_o(\Phi'')$] denotes the number of crosses perpendicular to α' in Φ' [Φ''].

Since $\mathcal{I}(\Phi') = \mathcal{I}_p(\Phi') + \mathcal{I}_o(\Phi')$ and $\mathcal{I}(\Phi'') = \mathcal{I}_p(\Phi'') + \mathcal{I}_o(\Phi'')$, we conclude that

$$\begin{aligned} \mathcal{W}(\Phi') - \mathcal{H}(\Phi') + \mathcal{I}(\Phi') + \mathcal{C}(\Phi') \\ = \mathcal{W}(\Phi'') - \mathcal{H}(\Phi'') + \mathcal{I}(\Phi'') + \mathcal{C}(\Phi''). \end{aligned}$$

□

Consider two rectangular solid dissections Ψ_1 and Ψ_2 obtained from Φ'' by cutting the entire rectangular solid into two rectangular solids by α' (Fig. 12 (d)). Since $\mathcal{R}(\Psi_1) + \mathcal{R}(\Psi_2) = \mathcal{R}(\Phi) + r$, $r < \mathcal{R}(\Psi_1) < \mathcal{R}(\Phi)$, and $r < \mathcal{R}(\Psi_2) < \mathcal{R}(\Phi)$, we obtain by the inductive hypothesis that

$$\begin{aligned} \mathcal{R}(\Psi_1) &= \mathcal{W}(\Psi_1) - \mathcal{H}(\Psi_1) + \mathcal{I}(\Psi_1) \\ &\quad + \mathcal{C}(\Psi_1) + \mathcal{T}(\Psi_1) + 1 \text{ and} \\ \mathcal{R}(\Psi_2) &= \mathcal{W}(\Psi_2) - \mathcal{H}(\Psi_2) + \mathcal{I}(\Psi_2) \\ &\quad + \mathcal{C}(\Psi_2) + \mathcal{T}(\Psi_2) + 1, \end{aligned}$$

that is

$$\begin{aligned} \mathcal{R}(\Phi) + r &= \mathcal{R}(\Psi_1) + \mathcal{R}(\Psi_2) \\ &= (\mathcal{W}(\Psi_1) + \mathcal{W}(\Psi_2)) - (\mathcal{H}(\Psi_1) + \mathcal{H}(\Psi_2)) \\ &\quad + (\mathcal{I}(\Psi_1) + \mathcal{I}(\Psi_2)) + (\mathcal{C}(\Psi_1) + \mathcal{C}(\Psi_2)) \\ &\quad + (\mathcal{T}(\Psi_1) + \mathcal{T}(\Psi_2)) + 2. \end{aligned} \tag{6}$$

Notice that $\mathcal{W}(\Psi_1) + \mathcal{W}(\Psi_2) = \mathcal{W}(\Phi'') - 1$ since α' is an internal wall in Φ'' but not in Ψ_1 or Ψ_2 . By Lemma 2,

$$\begin{aligned} (\mathcal{W}(\Phi) - 1) - (\mathcal{H}(\Phi) - h) + (\mathcal{I}(\Phi) + s) + (\mathcal{C}(\Phi) + c) \\ = (\mathcal{W}(\Psi_1) + \mathcal{W}(\Psi_2)) - (\mathcal{H}(\Psi_1) + \mathcal{H}(\Psi_2)) \\ + (\mathcal{I}(\Psi_1) + \mathcal{I}(\Psi_2)) + (\mathcal{C}(\Psi_1) + \mathcal{C}(\Psi_2)), \end{aligned} \tag{7}$$

and

$$\mathcal{T}(\Phi) - t = \mathcal{T}(\Psi_1) + \mathcal{T}(\Psi_2). \tag{8}$$

By Lemma 1 and Eqs. (6)–(8), we conclude that

$$\begin{aligned} \mathcal{R}(\Phi) &= (\mathcal{W}(\Phi) - 1) - (\mathcal{H}(\Phi) - h) + (\mathcal{I}(\Phi) + s) \\ &\quad + (\mathcal{C}(\Phi) + c) + (\mathcal{T}(\Phi) - t) - r + 2 \\ &= \mathcal{W}(\Phi) - \mathcal{H}(\Phi) + \mathcal{I}(\Phi) + \mathcal{C}(\Phi) + \mathcal{T}(\Phi) \\ &\quad - (r - h - s - c + t) + 1 \\ &= \mathcal{W}(\Phi) - \mathcal{H}(\Phi) + \mathcal{I}(\Phi) + \mathcal{C}(\Phi) + \mathcal{T}(\Phi) + 1. \end{aligned}$$

□

4. Conclusion

In this paper, we estimated the number of rooms in a rectangular solid dissection, and clarified the relation between “the number of rooms” and “the number of walls, the shape of walls and the structure of walls.” Our future work is an invention of a representation method of any rectangular solid dissection, which will be based on the above property.

References

- 1) Murata, H., Fujiyoshi, K., Watanabe, T. and Kajitani, Y.: A Mapping from Sequence-Pair to Rectangular Dissection, *Proc. IEEE ASP-DAC*, pp.625–633 (1997).
- 2) Sakanushi, K. and Kajitani, Y.: The Quarter-State Sequence to Represent the Floorplan and Applications to Layout Optimization, *Proc. IEEE APC-CAS 2000*, pp.829–832 (2000).
- 3) Yamazaki, H., Sakanushi, K., Nakatake, S. and Kajitani, Y.: The 3D-Packing by Meta Data Structure and Packing Heuristics, *IEICE Trans. Fundamentals*, Vol.E83-A, pp.639–645 (2000).
- 4) Yuh, P.H., Yang, C.L., Chang, Y.W. and Chen, H.L.: Temporal Floorplanning Using 3D-subTCG, *Proc. ASP-DAC*, pp.725–730 (2004).
- 5) Kohira, Y., Kodama, C., Fujiyoshi, K. and Takahashi, A.: Evaluation of 3D-

Packing Representations for Scheduling of Dynamically Reconfigurable Systems, *Proc. IEEE ISCAS 2006*, pp.4487–4490 (2006).

- 6) Fujiyoshi, K., Kawai, H. and Ishihara, K.: DTS: A Tree Based Representation for 3D-Block Packing, *Proc. IEEE ISCAS 2007*, pp.1045–1048 (2007).
- 7) Cheng, L., Deng, L., Wong, M.D.F.: Floorplan Design for 3-D ICs, *Proc. SASIMI*, pp.395–401 (2004).
- 8) Ohta, H., Yamada, T., Kodama, C. and Fujiyoshi, K.: The O-Sequence: Representation of 3D-Dissection, *IEICE Trans. Fundamentals*, Vol.E91-A, No.8, pp.2111–2119 (2008).

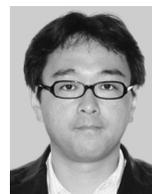
(Received April 19, 2009)

(Accepted December 17, 2009)

(Original version of this article can be found in the Journal of Information Processing Vol.18, pp.138–146.)



Hidenori Ohta received his B.E. and M.E. degrees in electrical and electronic engineering from Tokyo University of Agriculture & Technology, Tokyo, Japan, in 2004 and 2006, respectively. Currently, he is studying toward the D.E. degree in the Department of Electronic and Information Engineering from Tokyo University of Agriculture & Technology. His research interests are VLSI design and combinatorial algorithms. He is a student member of IEICE.



Toshinori Yamada earned his B.E., M.E., and D.E. degrees in electrical and electronic engineering at Tokyo Institute of Technology in Tokyo, Japan, in 1993, 1995, and 1998, respectively. From 1998 to 2003, he was a research associate in Tokyo Institute of Technology. Since 2003 he has been with Saitama University, where he is now an associate professor of the Division of Mathematics, Electronics and Informatics in the Graduate School of Science and Engineering. His research interests are in the theory of parallel and VLSI computation. In 2000, he received the Best Paper Award of IEEE Asia Pacific Conference on Circuits and Systems. He is a member of ACM, IEEE, SIAM, IEICE, and IPSJ.



Kunihiro Fujiyoshi received his B.E., M.E. and D.E. degrees in electrical and electronic engineering from Tokyo Institute of Technology, Tokyo, Japan, in 1987, 1989 and 1994, respectively. From 1992 to 1996, he was a research associate of School of Information Science at Japan Advanced Institute of Science and Technology, Ishikawa. He was with Tokyo University of Agriculture and Technology as a lecturer from 1997 and has been an associate professor since 2000 of Department of Electrical and Electronic Engineering. His research interests are in combinatorial algorithms and VLSI layout design. He is a member of IEEE, IEICE, and IPSJ.