Vol.2010-MPS-77 No.25
2010/3/5

情報処理学会研究報告
IPSJ SIG Technical Report

完全グラフの均衡型 (C_5, C_{16})-$2t$-Foil 分解

潮　　和　彦

グラフ理論において、グラフの分解問題は主要な研究テーマである。C_5 を５点を通るサイクル、C_{16} を１６点を通るサイクルとする。１点を共有する辺素な t 個の C_5 と t 個の C_{16} からなるグラフを (C_5, C_{16})-$2t$-foil という。本研究では、完全グラフ K_n を 均衡的に (C_5, C_{16})-$2t$-foil 部分グラフに分解する組合せデザインについて述べる。

Balanced (C_5, C_{16})-$2t$-Foil Decomposition of Complete Graphs

Kazuhiko Ushio

In graph theory, the decomposition problem of graphs is a very important topic. Various type of decompositions of many graphs can be seen in the literature of graph theory. This paper gives a balanced (C_5, C_{16})-$2t$-foil decomposition of complete graph K_n.

1. Introduction

Let K_n denote *the complete graph* of n vertices. Let C_5 and C_{16} be *the* 5-*cycle* and *the* 16-*cycle*, respectively. *The* (C_5, C_{16})-$2t$-*foil* is a graph of t edge-disjoint C_5's and t edge-disjoint C_{16}'s with a common vertex and the common vertex is called *the center of the* (C_5, C_{16})-$2t$-*foil*. In particular, the (C_5, C_{16})-2-foil is called *the* (C_5, C_{16})-*bowtie*. When K_n is decomposed into edge-disjoint sum of (C_5, C_{16})-$2t$-foils, we say that K_n *has a* (C_5, C_{16})-$2t$-*foil decomposition*. Moreover, when every vertex of K_n appears in the same number of (C_5, C_{16})-$2t$-foils, we say that K_n *has a balanced* (C_5, C_{16})-$2t$-*foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced* (C_5, C_{16})-$2t$-*foil system*.

†1 近畿大学理工学部情報学科
Department of Informatics, Faculty of Science and Technology, Kinki University

2. Balanced (C_5, C_{16})-$2t$-foil decomposition of K_n

Theorem. K_n has a balanced (C_5, C_{16})-$2t$-foil decomposition if and only if $n \equiv 1 \pmod{42t}$.

Proof. **(Necessity)** Suppose that K_n has a balanced (C_5, C_{16})-$2t$-foil decomposition. Let b be the number of (C_5, C_{16})-$2t$-foils and r be the replication number. Then $b = n(n-1)/42t$ and $r = (19t+1)(n-1)/42t$. Among r (C_5, C_{16})-$2t$-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_5, C_{16})-$2t$-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/42t$ and $r_2 = 19(n-1)/42$. Therefore, $n \equiv 1 \pmod{42t}$ is necessary.
(Sufficiency) Put $n = 42st + 1$ and $T = st$. Then $n = 42T + 1$.
Construct a (C_5, C_{16})-$2T$-foil as follows:
$\{(42T+1, 1, 18T+2, 37T+3, 20T+2), (42T+1, T+1, 4T+2, 15T+2, 22T+3, 7T+2, 13T+3, 25T+3, 11T+3, 26T+3, 20T+3, 16T+2, 32T+3, 24T+2, 11T+2, 2T+1)\}$
$\cup$
$\{(42T+1, 2, 18T+4, 37T+6, 17T+2), (42T+1, T+2, 4T+4, 15T+3, 22T+5, 7T+3, 13T+5, 25T+4, 11T+5, 26T+4, 20T+5, 16T+3, 32T+5, 24T+3, 11T+4, 2T+2)\}$
$\cup$
$\{(42T+1, 3, 18T+6, 37T+9, 17T+3), (42T+1, T+3, 4T+6, 15T+4, 22T+7, 7T+4, 13T+7, 25T+5, 11T+7, 26T+5, 20T+7, 16T+4, 32T+7, 24T+4, 11T+6, 2T+3)\}$
$\cup \ldots \cup$
$\{(42T+1, T, 20T, 40T, 18T), (42T+1, 2T, 6T, 16T+1, 24T+1, 8T+1, 15T+1, 26T+2, 13T+1, 27T+2, 22T+1, 17T+1, 34T+1, 25T+1, 13T, 3T)\}$.
Decompose the (C_5, C_{16})-$2T$-foil into s (C_5, C_{16})-$2t$-foils. Then these starters comprise a balanced (C_5, C_{16})-$2t$-foil decomposition of K_n.

Corollary. K_n has a balanced (C_5, C_{16})-bowtie decomposition if and only if $n \equiv 1 \pmod{42}$.

Example 1. Balanced (C_5, C_{16})-2-foil decomposition of K_{43}.
$\{(43, 1, 20, 40, 22), (43, 2, 6, 17, 25, 9, 16, 28, 14, 29, 23, 18, 35, 26, 13, 3)\}$.
This starter comprises a balanced (C_5, C_{16})-2-foil decomposition of K_{43}.

Example 2. Balanced (C_5, C_{16})-4-foil decomposition of K_{85}.
$\{(85, 1, 38, 77, 42)$,
$(85, 2, 40, 80, 36)\}$
$\cup$
$\{(85, 3, 10, 32, 47, 16, 29, 53, 25, 55, 43, 34, 67, 50, 24, 5)$,
$(85, 4, 12, 33, 49, 17, 31, 54, 27, 56, 45, 35, 69, 51, 26, 6)\}$.
This starter comprises a balanced (C_5, C_{16})-4-foil decomposition of K_{85}.

Example 3. Balanced (C_5, C_{16})-6-foil decomposition of K_{127}.
$\{(127, 1, 56, 114, 62)$,
$(127, 2, 58, 117, 53)$,
$(127, 3, 60, 120, 54)\}$
$\cup$
$\{(127, 4, 14, 47, 69, 23, 42, 78, 36, 81, 63, 50, 99, 74, 35, 7)$,
$(127, 5, 16, 48, 71, 24, 44, 79, 38, 82, 65, 51, 101, 75, 37, 8)$,
$(127, 6, 18, 49, 73, 25, 46, 80, 40, 83, 67, 52, 103, 76, 39, 9)\}$.
This starter comprises a balanced (C_5, C_{16})-6-foil decomposition of K_{127}.

Example 4. Balanced (C_5, C_{16})-8-foil decomposition of K_{169}.
$\{(169, 1, 74, 151, 82)$,
$(169, 2, 76, 154, 70)$,
$(169, 3, 78, 157, 71)$,
$(169, 4, 80, 160, 72)\}$
$\cup$
$\{(169, 5, 18, 62, 91, 30, 55, 103, 47, 107, 83, 66, 131, 98, 46, 9)$,
$(169, 6, 20, 63, 93, 31, 57, 104, 49, 108, 85, 67, 133, 99, 48, 10)$,
$(169, 7, 22, 64, 95, 32, 59, 105, 51, 109, 87, 68, 135, 100, 50, 11)$,
$(169, 8, 24, 65, 97, 33, 61, 106, 53, 110, 89, 69, 137, 101, 52, 12)\}$.
This starter comprises a balanced (C_5, C_{16})-8-foil decomposition of K_{169}.

Example 5. Balanced (C_5, C_{16})-10-foil decomposition of K_{211}.
$\{(211, 1, 92, 188, 102)$,
$(211, 2, 94, 191, 87)$,
$(211, 3, 96, 194, 88)$,
$(211, 4, 98, 197, 89)$,
$(211, 5, 100, 200, 90)\}$
$\cup$
$\{(211, 6, 22, 77, 113, 37, 68, 128, 58, 133, 103, 82, 163, 122, 57, 11)$,
$(211, 7, 24, 78, 115, 38, 70, 129, 60, 134, 105, 83, 165, 123, 59, 12)$,
$(211, 8, 26, 79, 117, 39, 72, 130, 62, 135, 107, 84, 167, 124, 61, 13)$,
$(211, 9, 28, 80, 119, 40, 74, 131, 64, 136, 109, 85, 169, 125, 63, 14)$,
$(211, 10, 30, 81, 121, 41, 76, 132, 66, 137, 111, 86, 171, 126, 65, 15)\}$.
This starter comprises a balanced (C_5, C_{16})-10-foil decomposition of K_{211}.

Example 6. Balanced (C_5, C_{16})-12-foil decomposition of K_{253}.
$\{(253, 1, 110, 225, 122)$,
$(253, 2, 112, 228, 104)$,
$(253, 3, 114, 231, 105)$,
$(253, 4, 116, 234, 106)$,
$(253, 5, 118, 237, 107)$,
$(253, 6, 120, 240, 108)\}$
$\cup$
$\{(253, 7, 26, 92, 135, 44, 81, 153, 69, 159, 123, 98, 195, 146, 68, 13)$,
$(253, 8, 28, 93, 137, 45, 83, 154, 71, 160, 125, 99, 197, 147, 70, 14)$,

$(253, 9, 30, 94, 139, 46, 85, 155, 73, 161, 127, 100, 199, 148, 72, 15)$,
$(253, 10, 32, 95, 141, 47, 87, 156, 75, 162, 129, 101, 201, 149, 74, 16)$,
$(253, 11, 34, 96, 143, 48, 89, 157, 77, 163, 131, 102, 203, 150, 76, 17)$,
$(253, 12, 36, 97, 145, 49, 91, 158, 79, 164, 133, 103, 205, 151, 78, 18)\}$.
This starter comprises a balanced (C_5, C_{16})-12-foil decomposition of K_{253}.

Example 7. Balanced (C_5, C_{16})-14-foil decomposition of K_{295}.
$\{(295, 1, 128, 262, 142)$,
$(295, 2, 130, 265, 121)$,
$(295, 3, 132, 268, 122)$,
$(295, 4, 134, 271, 123)$,
$(295, 5, 136, 274, 124)$,
$(295, 6, 138, 277, 125)$,
$(295, 7, 140, 280, 126)\}$
$\cup$
$\{(295, 8, 30, 107, 157, 51, 94, 178, 80, 185, 143, 114, 227, 170, 79, 15)$,
$(295, 9, 32, 108, 159, 52, 96, 179, 82, 186, 145, 115, 229, 171, 81, 16)$,
$(295, 10, 34, 109, 161, 53, 98, 180, 84, 187, 147, 116, 231, 172, 83, 17)$,
$(295, 11, 36, 110, 163, 54, 100, 181, 86, 188, 149, 117, 233, 173, 85, 18)$,
$(295, 12, 38, 111, 165, 55, 102, 182, 88, 189, 151, 118, 235, 174, 87, 19)$,
$(295, 13, 40, 112, 167, 56, 104, 183, 90, 190, 153, 119, 237, 175, 89, 20)$,
$(295, 14, 42, 113, 169, 57, 106, 184, 92, 191, 155, 120, 239, 176, 91, 21)\}$.
This starter comprises a balanced (C_5, C_{16})-14-foil decomposition of K_{295}.

参 考 文 献

1) Ushio, K.: Balanced bowtie and trefoil decomposition of symmetric complete tripartite digraphs, *Information and Communication Studies of The Faculty of Information and Communication Bunkyo University*, Vol.25, pp.19–24 (2000).
2) Ushio, K. and Fujimoto, H.: Balanced bowtie and trefoil decomposition of complete tripartite multigraphs, *IEICE Trans. Fundamentals*, Vol. E84-A, No. 3, pp. 839–844 (2001).
3) Ushio, K. and Fujimoto, H.: Balanced foil decomposition of complete graphs, *IEICE Trans. Fundamentals*, Vol.E84-A, No.12, pp.3132–3137 (2001).
4) Ushio, K. and Fujimoto, H.: Balanced bowtie decomposition of complete multigraphs, *IEICE Trans. Fundamentals*, Vol.E86-A, No.9, pp.2360–2365 (2003).
5) Ushio, K. and Fujimoto, H.: Balanced bowtie decomposition of symmetric complete multi-digraphs, *IEICE Trans. Fundamentals*, Vol.E87-A, No.10, pp.2769–2773 (2004).
6) Ushio, K. and Fujimoto, H.: Balanced quatrefoil decomposition of complete multigraphs, *IEICE Trans. Information and Systems, Special Section on Foundations of Computer Science*, Vol.E88-D, No.1, pp.19–22 (2005).
7) Ushio, K. and Fujimoto, H.: Balanced C_4-bowtie decomposition of complete multigraphs, *IEICE Trans. Fundamentals, Special Section on Discrete Mathematics and Its Applications*, Vol.E88-A, No.5, pp.1148–1154 (2005).
8) Ushio, K. and Fujimoto, H.: Balanced C_4-trefoil decomposition of complete multigraphs, *IEICE Trans. Fundamentals, Special Section on Discrete Mathematics and Its Applications*, Vol.E89-A, No.5, pp.1173–1180 (2006).