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完全グラフの均衡型 (C_5, C_{14})-$2t$-Foil 分解アルゴリズム

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グラフ理論において、グラフの分解問題は主要な研究テーマである。C_5、C_{14} をそれぞれ５点、１４点を通るサイクルとする。１点を共有する辺素な t 個の C_5 と t 個の C_{14} からなるグラフを (C_5, C_{14})-$2t$-Foil という。本研究では、完全グラフ K_n を均衡的に (C_5, C_{14})-$2t$-Foil 部分グラフに分解する分解アルゴリズムについて述べる。

Balanced (C_5, C_{14})-$2t$-Foil Decomposition Algorithm of Complete Graphs

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In graph theory, the decomposition problem of graphs is a very important topic. Various types of decompositions of many graphs can be seen in the literature of graph theory. This paper gives a balanced (C_5, C_{14})-$2t$-Foil decomposition of the complete graph K_n.

1. Introduction

Let K_n denote *the complete graph* of n vertices. Let C_5 and C_{14} be *the* 5-*cycle* and *the* 14-*cycle*, respectively. *The* (C_5, C_{14})-$2t$-*foil* is a graph of t edge-disjoint C_5's and t edge-disjoint C_{14}'s with a common vertex and the common vertex is called *the center of the* (C_5, C_{14})-$2t$-*foil*. In particular, the (C_5, C_{14})-2-foil is called *the* (C_5, C_{14})-*bowtie*. When K_n is decomposed into edge-disjoint sum of (C_5, C_{14})-$2t$-foils, we say that K_n *has a* (C_5, C_{14})-$2t$-*foil decomposition*. Moreover, when every vertex of K_n appears in the same number of (C_5, C_{14})-$2t$-foils, we say that K_n *has a balanced* (C_5, C_{14})-$2t$-*foil decomposition* and this number is called *the replication number*. This decomposition is to be known as *a balanced* (C_5, C_{14})-$2t$-*foil system*.

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2. Balanced (C_5, C_{14})-$2t$-foil decomposition of K_n

Theorem. K_n has a balanced (C_5, C_{14})-$2t$-foil decomposition if and only if $n \equiv 1 \pmod{38t}$.

Proof. **(Necessity)** Suppose that K_n has a balanced (C_5, C_{14})-$2t$-foil decomposition. Let b be the number of (C_5, C_{14})-$2t$-foils and r be the replication number. Then $b = n(n-1)/38t$ and $r = (17t+1)(n-1)/38t$. Among r (C_5, C_{14})-$2t$-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_5, C_{14})-$2t$-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/38t$ and $r_2 = 17(n-1)/38$. Therefore, $n \equiv 1 \pmod{38t}$ is necessary.
(Sufficiency) Put $n = 38st + 1$ and $T = st$. Then $n = 38T + 1$.
Construct a (C_5, C_{14})-$2T$-foil as follows:
$\{(38T+1, 1, 14T+2, 34T+3, 16T+1), (38T+1, T+1, 6T+2, 11T+2, 21T+3, 29T+3, 6T+3, 18T+3, 8T+3, 5T+2, 30T+3, 24T+2, 21T+2, 13T+1)\} \cup$
$\{(38T+1, 2, 14T+4, 34T+6, 16T+2), (38T+1, T+2, 6T+4, 11T+3, 21T+5, 29T+4, 6T+5, 18T+4, 8T+5, 5T+3, 30T+5, 24T+3, 21T+4, 13T+2)\} \cup$
$\{(38T+1, 3, 14T+6, 34T+9, 16T+3), (38T+1, T+3, 6T+6, 11T+4, 21T+7, 29T+5, 6T+7, 18T+5, 8T+7, 5T+4, 30T+7, 24T+4, 21T+6, 13T+3)\} \cup$
... $\cup$
$\{(38T+1, T, 16T, 37T, 17T), (38T+1, 2T, 8T, 12T+1, 23T+1, 30T+2, 8T+1, 19T+2, 10T+1, 6T+1, 32T+1, 25T+1, 23T, 14T)\}$.
Decompose the (C_5, C_{14})-$2T$-foil into s (C_5, C_{14})-$2t$-foils. Then these starters comprise a balanced (C_5, C_{14})-$2t$-foil decomposition of K_n.

Corollary. K_n has a balanced (C_5, C_{14})-bowtie decomposition if and only if $n \equiv 1 \pmod{38}$.

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Example 1. Balanced (C_5, C_{14})-2-foil decomposition of K_{39}.
$\{(39, 1, 16, 37, 17), (39, 2, 8, 13, 24, 32, 9, 21, 11, 7, 33, 26, 23, 14)\}$.
This starter comprises a balanced (C_5, C_{14})-2-foil decomposition of K_{39}.

Example 2. Balanced (C_5, C_{14})-4-foil decomposition of K_{77}.
$\{(77, 1, 30, 71, 33), (77, 3, 14, 24, 45, 61, 15, 39, 19, 12, 63, 50, 44, 27)\} \cup$
$\{(77, 2, 32, 74, 34), (77, 4, 16, 25, 47, 62, 17, 40, 21, 13, 65, 51, 46, 28)\}$.
This starter comprises a balanced (C_5, C_{14})-4-foil decomposition of K_{77}.

Example 3. Balanced (C_5, C_{14})-6-foil decomposition of K_{115}.
$\{(115, 1, 44, 105, 49), (115, 4, 20, 35, 66, 90, 21, 57, 27, 17, 93, 74, 65, 40)\} \cup$
$\{(115, 2, 46, 108, 50), (115, 5, 22, 36, 68, 91, 23, 58, 29, 18, 95, 75, 67, 41)\} \cup$
$\{(115, 3, 48, 111, 51), (115, 6, 24, 37, 70, 92, 25, 59, 31, 19, 97, 76, 69, 42)\}$.
This starter comprises a balanced (C_5, C_{14})-6-foil decomposition of K_{115}.

Example 4. Balanced (C_5, C_{14})-8-foil decomposition of K_{153}.
$\{(153, 1, 58, 139, 65), (153, 5, 26, 46, 87, 119, 27, 75, 35, 22, 123, 98, 86, 53)\} \cup$
$\{(153, 2, 60, 142, 66), (153, 6, 28, 47, 89, 120, 29, 76, 37, 23, 125, 99, 88, 54)\} \cup$
$\{(153, 3, 62, 145, 67), (153, 7, 30, 48, 91, 121, 31, 77, 39, 24, 127, 100, 90, 55)\} \cup$
$\{(153, 4, 64, 148, 68), (153, 8, 32, 49, 93, 122, 33, 78, 41, 25, 129, 101, 92, 56)\}$.
This starter comprises a balanced (C_5, C_{14})-8-foil decomposition of K_{153}.

Example 5. Balanced (C_5, C_{14})-10-foil decomposition of K_{191}.
$\{(191, 1, 72, 173, 81), (191, 6, 32, 57, 108, 148, 33, 93, 43, 27, 153, 122, 107, 66)\} \cup$
$\{(191, 2, 74, 176, 82), (191, 7, 34, 58, 110, 149, 35, 94, 45, 28, 155, 123, 109, 67)\} \cup$
$\{(191, 3, 76, 179, 83), (191, 8, 36, 59, 112, 150, 37, 95, 47, 29, 157, 124, 111, 68)\} \cup$
$\{(191, 4, 78, 182, 84), (191, 9, 38, 60, 114, 151, 39, 96, 49, 30, 159, 125, 113, 69)\} \cup$
$\{(191, 5, 80, 185, 85), (191, 10, 40, 61, 116, 152, 41, 97, 51, 31, 161, 126, 115, 70)\}$.
This starter comprises a balanced (C_5, C_{14})-10-foil decomposition of K_{191}.

Example 6. Balanced (C_5, C_{14})-12-foil decomposition of K_{229}.
$\{(229, 1, 86, 207, 97), (229, 7, 38, 68, 129, 177, 39, 111, 51, 32, 183, 146, 128, 79)\} \cup$
$\{(229, 2, 88, 210, 98), (229, 8, 40, 69, 131, 178, 41, 112, 53, 33, 185, 147, 130, 80)\} \cup$
$\{(229, 3, 90, 213, 99), (229, 9, 42, 70, 133, 179, 43, 113, 55, 34, 187, 148, 132, 81)\} \cup$
$\{(229, 4, 92, 216, 100), (229, 10, 44, 71, 135, 180, 45, 114, 57, 35, 189, 149, 134, 82)\} \cup$
$\{(229, 5, 94, 219, 101), (229, 11, 46, 72, 137, 181, 47, 115, 59, 36, 191, 150, 136, 83)\} \cup$
$\{(229, 6, 96, 222, 102), (229, 12, 48, 73, 139, 182, 49, 116, 61, 37, 193, 151, 138, 84)\}$.
This starter comprises a balanced (C_5, C_{14})-12-foil decomposition of K_{229}.

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