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ハミルトン C_k-Tenfoil デザイン

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グラフ理論において、グラフの分解問題は主要な研究テーマである。C_k を k 点を通るサイクルとする。1点を共有する辺素な10個の C_k からなるグラフを C_k-Tenfoil という。本研究では、完全多重グラフ λK_n を ハミルトン C_k-Tenfoil 全域部分グラフに分解する組合せデザインについて述べる。

Hamilton C_k-Tenfoil Designs

Kazuhiko Ushio

In graph theory, the decomposition problem of graphs is a very important topic. Various type of decompositions of many graphs can be seen in the literature of graph theory. This paper gives a Hamilton C_k-tenfoil decomposition of the complete multi-graph λK_n.

1. Introduction

The complete multi-graph λK_n is the complete graph K_n in which every edge is taken λ times. Let C_k be *the k-cycle* (or *the cycle* on k vertices). *The* C_k*-tenfoil* is a graph of 10 edge-disjoint C_k's with a common vertex and the common vertex is called *the center of the* C_k*-tenfoil*. In particular, a C_k-tenfoil satisfying $n = 10(k-1)+1$ is called *the Hamilton* C_k*-tenfoil* because the C_k-tenfoil spans λK_n.
When λK_n is decomposed into edge-disjoint sum of Hamilton C_k-tenfoils, we say that λK_n *has a Hamilton* C_k*-tenfoil decomposition*. This decomposition is called *a Hamilton* C_k*-tenfoil design.*

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2. Hamilton C_k-tenfoil decomposition of λK_n

Theorem 1. If λK_n has a Hamilton C_k-tenfoil decomposition, then (i) $n = 10(k-1)+1$ and (ii) $\lambda \equiv 0 \pmod{2k}$ for $k \equiv 2,4,8,10,14,16 \pmod{18}$, $\lambda \equiv 0 \pmod{k}$ for $k \equiv 1,5,7,11,13,17 \pmod{18}$, $\lambda \equiv 0 \pmod{2k/3}$ for $k \equiv 0,6,12 \pmod{18}$, $\lambda \equiv 0 \pmod{k/3}$ for $k \equiv 3,15 \pmod{18}$, $\lambda \equiv 0 \pmod{k/9}$ for $k \equiv 9 \pmod{18}$.
Proof. When $n = 10(k-1)+1$, suppose that λK_n is decomposed into b Hamilton C_k-tenfoils. Then $b = \lambda n(n-1)/20k = \lambda(10k-9)(k-1)/2k$. Thus, (i), (ii) hold.

Theorem 2. If λK_n has a Hamilton C_k-tenfoil decomposition, then $(s\lambda)K_n$ has a Hamilton C_k-tenfoil decomposition for every s.

Theorem 3. Let n be prime. When $n = 10(k-1)+1$, $\lambda \equiv 0 \pmod{2k}$, and $k \equiv 2,4,8,10,14,16 \pmod{18}$, λK_n has a Hamilton C_k-tenfoil decomposition.

Example 3.1. Hamilton C_4-tenfoil of $8K_{31}$.
$(n,g) = (31,3)$ n-orbit : $1,3,9,27,19,26,16,17,20,29,25,13,8,24,10,30,28,22,4,12,5,15,14,11,2,6,18,23,7,21,1$.
Hamilton C_4-tenfoil $= (31,1,3,9) \cup (31,27,19,26) \cup (31,16,17,20) \cup (31,29,25,13) \cup (31,8,24,10) \cup (31,30,28,22) \cup (31,4,12,5) \cup (31,15,14,11) \cup (31,2,6,18) \cup (31,23,7,21)$
Hamilton C_4-tenfoil $= (31,3,9,27) \cup (31,19,26,16) \cup (31,17,20,29) \cup (31,25,13,8) \cup (31,24,10,30) \cup (31,28,22,4) \cup (31,12,5,15) \cup (31,14,11,2) \cup (31,6,18,23) \cup (31,7,21,1)$
Hamilton C_4-tenfoil $= (31,9,27,19) \cup (31,26,16,17) \cup (31,20,29,25) \cup (31,13,8,24) \cup (31,10,30,28) \cup (31,22,4,12) \cup (31,5,15,14) \cup (31,11,2,6) \cup (31,18,23,7) \cup (31,21,1,3)$.
(120 edges = (15 all lengths) * 8 times)
These 3 starters comprise a Hamilton C_4-tenfoil decomposition of $8K_{31}$.

Example 3.2. Hamilton C_8-tenfoil of $16K_{71}$.

$(n, g) = (71, 7)$ n-orbit : 1, 7, 49, 59, 58, 51, 2, 14, 27, 47, 45, 31, 4, 28, 54, 23, 19, 62, 8, 56, 37, 46, 38, 53, 16, 41, 3, 21, 5, 35, 32, 11, 6, 42, 10, 70, 64, 22, 12, 13, 20, 69, 57, 44, 24, 26, 40, 67, 43, 17, 48, 52, 9, 63, 15, 34, 25, 33, 18, 55, 30, 68, 50, 66, 36, 39, 60, 65, 29, 61, 1.

Hamilton C_8-tenfoil = (71, 1, 7, 49, 59, 58, 51, 2) ∪ (71, 14, 27, 47, 45, 31, 4, 28)
∪ (71, 54, 23, 19, 62, 8, 56, 37) ∪ (71, 46, 38, 53, 16, 41, 3, 21) ∪ (71, 5, 35, 32, 11, 6, 42, 10)
∪ (71, 70, 64, 22, 12, 13, 20, 69) ∪ (71, 57, 44, 24, 26, 40, 67, 43) ∪ (71, 17, 48, 52, 9, 63, 15, 34)
∪ (71, 25, 33, 18, 55, 30, 68, 50) ∪ (71, 66, 36, 39, 60, 65, 29, 61)

Hamilton C_8-tenfoil = (71, 7, 49, 59, 58, 51, 2, 14) ∪ (71, 27, 47, 45, 31, 4, 28, 54)
∪ (71, 23, 19, 62, 8, 56, 37, 46) ∪ (71, 38, 53, 16, 41, 3, 21, 5) ∪ (71, 35, 32, 11, 6, 42, 10, 70)
∪ (71, 64, 22, 12, 13, 20, 69, 57) ∪ (71, 44, 24, 26, 40, 67, 43, 17) ∪ (71, 48, 52, 9, 63, 15, 34, 25)
∪ (71, 33, 18, 55, 30, 68, 50, 66) ∪ (71, 36, 39, 60, 65, 29, 61, 1)

Hamilton C_8-tenfoil = (71, 49, 59, 58, 51, 2, 14, 27) ∪ (71, 47, 45, 31, 4, 28, 54, 23)
∪ (71, 19, 62, 8, 56, 37, 46, 38) ∪ (71, 53, 16, 41, 3, 21, 5, 35) ∪ (71, 32, 11, 6, 42, 10, 70, 64)
∪ (71, 22, 12, 13, 20, 69, 57, 44) ∪ (71, 24, 26, 40, 67, 43, 17, 48) ∪ (71, 52, 9, 63, 15, 34, 25, 33)
∪ (71, 18, 55, 30, 68, 50, 66, 36) ∪ (71, 39, 60, 65, 29, 61, 1, 7)

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Hamilton C_8-tenfoil = (71, 2, 14, 27, 47, 45, 31, 4) ∪ (71, 28, 54, 23, 19, 62, 8, 56)
∪ (71, 37, 46, 38, 53, 16, 41, 3) ∪ (71, 21, 5, 35, 32, 11, 6, 42) ∪ (71, 10, 70, 64, 22, 12, 13, 20)
∪ (71, 69, 57, 44, 24, 26, 40, 67) ∪ (71, 43, 17, 48, 52, 9, 63, 15) ∪ (71, 34, 25, 33, 18, 55, 30, 68)
∪ (71, 50, 66, 36, 39, 60, 65, 29) ∪ (71, 61, 1, 7, 49, 59, 58, 51).

(560 edges = (35 all lengths) * 16 times)

These 7 starters comprise a Hamilton C_8-tenfoil decomposition of $16K_{71}$.

Example 3.3. Hamilton C_{14}-tenfoil of $28K_{131}$.

$(n, g) = (131, 2)$ n-orbit : 1, 2, 4, 8, 16, 32, 64, 128, 125, 119, 107, 83, 35, 70, 9, 18, 36, 72, 13, 26, 52, 104, 77, 23, 46, 92, 53, 106, 81, 31, 62, 124, 117, 103, 75, 19, 38, 76, 21, 42, 84, 37, 74, 17, 34, 68, 5, 10, 20, 40, 80, 29, 58, 116, 101, 71, 11, 22, 44, 88, 45, 90, 49, 98, 65, 130, 129, 127, 123, 115, 99, 67, 3, 6, 12, 24, 48, 96, 61, 122, 113, 95, 59, 118, 105, 79, 27, 54, 108, 85, 39, 78, 25, 50, 100, 69, 7, 14, 28, 56, 112, 93, 55, 110, 89, 47, 94, 57, 114, 97, 63, 126, 121, 111, 91, 51, 102, 73, 15, 30, 60, 120, 109, 87, 43, 86, 41, 82, 33, 66, 1.

Hamilton C_{14}-tenfoil = (131, 1, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...)
∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...)

Hamilton C_{14}-tenfoil = (131, 2, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...)
∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...)

Hamilton C_{14}-tenfoil = (131, 4, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...)
∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...)

...

Hamilton C_{14}-tenfoil = (131, 35, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...)
∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...) ∪ (131, ...).

(1820 edges = (65 all lengths) * 28 times)

These 13 starters comprise a Hamilton C_{14}-tenfoil decomposition of $28K_{131}$.

Example 3.4. Hamilton C_{16}-tenfoil of $32K_{151}$.

$(n, g) = (151, 6)$ n-orbit : 1, 6, 36, 65, 88, 75, 148, 133, 43, 107, 38, 77, 9, 54, 22, 132, 37, 71, 124, 140, 85, 57, 40, 89, 81, 33, 47, 131, 31, 35, 59, 52, 10, 60, 58, 46, 125, 146, 121, 122, 128, 13, 78, 15, 90, 87, 69, 112, 68, 106, 32, 41, 95, 117, 98, 135, 55, 28, 17, 102, 8, 48, 137, 67, 100, 147, 127, 7, 42, 101, 2, 12, 72, 130, 25, 150, 145, 115, 86, 63, 76, 3, 18, 108, 44, 113, 74, 142, 97, 129, 19, 114, 80, 27, 11, 66, 94, 111, 62, 70, 118, 104, 20, 120, 116, 92, 99, 141, 91, 93, 105, 26, 5, 30, 29, 23, 138, 73, 136, 61, 64, 82, 39, 83, 45, 119, 110, 56, 34, 53, 16, 96, 123, 134, 49, 143, 103, 14, 84, 51, 4, 24, 144, 109, 50, 149, 139, 79, 21, 126, 1.

Hamilton C_{16}-tenfoil = (151, 1, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...)
∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...)

Hamilton C_{16}-tenfoil = (151, 6, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...)
∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...)

Hamilton C_{16}-tenfoil = (151, 36, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...)
∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...)

...

Hamilton C_{16}-tenfoil = (151, 22, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...)
∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...) ∪ (151, ...).

(2400 edges = (75 all lengths) * 32 times)

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These 15 starters comprise a Hamilton C_{16}-tenfoil decomposition of $32K_{151}$.

Example 3.5. Hamilton C_{20}-tenfoil of $40K_{191}$.
$(n, g) = (191, 19)$ n-orbit : 1, 19, 170, 174, 59, 166, 98, 143, 43, 53, 52, 33, 54, 71, 12, 37, 130, 178, 135, 82, 30, 188, 134, 63, 51, 14, 75, 88, 144, 62, 32, 35, 92, 29, 169, 155, 80, 183, 39, 168, 136, 101, 9, 171, 2, 38, 149, 157, 118, 141, 5, 95, 86, 106, 104, 66, 108, 142, 24, 74, 69, 165, 79, 164, 60, 185, 77, 126, 102, 28, 150, 176, 97, 124, 64, 70, 184, 58, 147, 119, 160, 175, 78, 145, 81, 11, 18, 151, 4, 76, 107, 123, 45, 91, 10, 190, 172, 21, 17, 132, 25, 93, 48, 148, 138, 139, 158, 137, 120, 179, 154, 61, 13, 56, 109, 161, 3, 57, 128, 140, 177, 116, 103, 47, 129, 159, 156, 99, 162, 22, 36, 111, 8, 152, 23, 55, 90, 182, 20, 189, 153, 42, 34, 73, 50, 186, 96, 105, 85, 87, 125, 83, 49, 167, 117, 122, 26, 112, 27, 131, 6, 114, 65, 89, 163, 41, 15, 94, 67, 127, 121, 7, 133, 44, 72, 31, 16, 113, 46, 110, 180, 173, 40, 187, 115, 84, 68, 146, 100, 181, 1.
Hamilton C_{20}-tenfoil = (191, 1, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...)
Hamilton C_{20}-tenfoil = (191, 19, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...)
Hamilton C_{20}-tenfoil = (191, 170, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...)
...
Hamilton C_{20}-tenfoil = (191, 135, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...) ∪ (191, ...).
(3800 edges = (95 all lengths) * 40 times)
These 19 starters comprise a Hamilton C_{20}-tenfoil decomposition of $40K_{191}$.

Theorem 4. Let n be prime. When $n = 10(k-1)+1$, $\lambda \equiv 0 \pmod{k}$, and $k \equiv 1, 5, 7, 11, 13, 17 \pmod{18}$, λK_n has a Hamilton C_k-tenfoil decomposition.

Example 4.1. Hamilton C_5-tenfoil of $5K_{41}$.
$(n, g) = (41, 6)$ n-orbit : 1, 6, 36, 11, 25, 27, 39, 29, 10, 19, 32, 28, 4, 24, 21, 3, 18, 26, 33, 34, 40, 35, 5, 30, 16, 14, 2, 12, 31, 22, 9, 13, 37, 17, 20, 38, 23, 15, 8, 7, 1.
L_1 : 1, 32, 40, 9, 1
L_2 : 6, 28, 35, 13, 6
L_3 : 36, 4, 5, 37, 36
L_4 : 11, 24, 30, 17, 11
L_5 : 25, 21, 16, 20, 25
L_6 : 27, 3, 14, 38, 27
L_7 : 39, 18, 2, 23, 39
L_8 : 29, 26, 12, 15, 29
L_9 : 10, 33, 31, 8, 10
L_{10} : 19, 34, 22, 7, 19.
Hamilton C_5-tenfoil = (41, 1, 32, 40, 9) ∪ (41, 6, 28, 35, 13) ∪ (41, 36, 4, 5, 37) ∪ (41, 11, 24, 30, 17) ∪ (41, 25, 21, 16, 20) ∪ (41, 27, 3, 14, 38) ∪ (41, 39, 18, 2, 23) ∪ (41, 29, 26, 12, 15) ∪ (41, 10, 33, 31, 8) ∪ (41, 19, 34, 22, 7)
Hamilton C_5-tenfoil = (41, 32, 40, 9, 1) ∪ (41, 28, 35, 13, 6) ∪ (41, 4, 5, 37, 36) ∪ (41, 24, 30, 17, 11) ∪ (41, 21, 16, 20, 25) ∪ (41, 3, 14, 38, 27) ∪ (41, 18, 2, 23, 39) ∪ (41, 26, 12, 15, 29) ∪ (41, 33, 31, 8, 10) ∪ (41, 34, 22, 7, 19).
(100 edges = (20 all lengths) * 5 times)
These 2 starters comprise a Hamilton C_5-tenfoil decomposition of $5K_{41}$.

Example 4.2. Hamilton C_7-tenfoil of $7K_{61}$.
$(n, g) = (61, 2)$ n-orbit : 1, 2, 4, 8, 16, 32, 3, 6, 12, 24, 48, 35, 9, 18, 36, 11, 22, 44, 27, 54, 47, 33, 5, 10, 20, 40, 19, 38, 15, 30, 60, 59, 57, 53, 45, 29, 58, 55, 49, 37, 13, 26, 52, 43, 25, 50, 39, 17, 34, 7, 14, 28, 56, 51, 41, 21, 42, 23, 46, 31, 1.
L_1 : 1, 48, 47, 60, 13, 14, 1
L_2 : 2, 35, 33, 59, 26, 28, 2
L_3 : 4, 9, 5, 57, 52, 56, 4
L_4 : 8, 18, 10, 53, 43, 51, 8
L_5 : 16, 36, 20, 45, 25, 41, 16
L_6 : 32, 11, 40, 29, 50, 21, 32
L_7 : 3, 22, 19, 58, 39, 42, 3
L_8 : 6, 44, 38, 55, 17, 23, 6
L_9 : 12, 27, 15, 49, 34, 46, 12

L_{10} : 24, 54, 30, 37, 7, 31, 24.

Hamilton C_7-tenfoil = (61, 1, 48, 47, 60, 13, 14) ∪ (61, 2, 35, 33, 59, 26, 28)
∪ (61, 4, 9, 5, 57, 52, 56) ∪ (61, 8, 18, 10, 53, 43, 51) ∪ (61, 16, 36, 20, 45, 25, 41)
∪ (61, 32, 11, 40, 29, 50, 21) ∪ (61, 3, 22, 19, 58, 39, 42) ∪ (61, 6, 44, 38, 55, 17, 23)
∪ (61, 12, 27, 15, 49, 34, 46) ∪ (61, 24, 54, 30, 37, 7, 31)

Hamilton C_7-tenfoil = (61, 48, 47, 60, 13, 14, 1) ∪ (61, 35, 33, 59, 26, 28, 2)
∪ (61, 9, 5, 57, 52, 56, 4) ∪ (61, 18, 10, 53, 43, 51, 8) ∪ (61, 36, 20, 45, 25, 41, 16)
∪ (61, 11, 40, 29, 50, 21, 32) ∪ (61, 22, 19, 58, 39, 42, 3) ∪ (61, 44, 38, 55, 17, 23, 6)
∪ (61, 27, 15, 49, 34, 46, 12) ∪ (61, 54, 30, 37, 7, 31, 24)

Hamilton C_7-tenfoil = (61, 47, 60, 13, 14, 1, 48) ∪ (61, 33, 59, 26, 28, 2, 35)
∪ (61, 5, 57, 52, 56, 4, 9) ∪ (61, 10, 53, 43, 51, 8, 18) ∪ (61, 20, 45, 25, 41, 16, 36)
∪ (61, 40, 29, 50, 21, 32, 11) ∪ (61, 19, 58, 39, 42, 3, 22) ∪ (61, 38, 55, 17, 23, 6, 44)
∪ (61, 15, 49, 34, 46, 12, 27) ∪ (61, 30, 37, 7, 31, 24, 54).

(210 edges = (30 all lengths) * 7 times)

These 3 starters comprise a Hamilton C_7-tenfoil decomposition of $7K_{61}$.

Example 4.3. Hamilton C_{11}-tenfoil of $11K_{101}$.

$(n, g) = (101, 2)$ n-orbit : 1, 2, 4, 8, 16, 32, 64, 27, 54, 7, 14, 28, 56, 11, 22, 44, 88, 75, 49, 98, 95, 89, 77, 53, 5, 10, 20, 40, 80, 59, 17, 34, 68, 35, 70, 39, 78, 55, 9, 18, 36, 72, 43, 86, 71, 41, 82, 63, 25, 50, 100, 99, 97, 93, 85, 69, 37, 74, 47, 94, 87, 73, 45, 90, 79, 57, 13, 26, 52, 3, 6, 12, 24, 48, 96, 91, 81, 61, 21, 42, 84, 67, 33, 66, 31, 62, 23, 46, 92, 83, 65, 29, 58, 15, 30, 60, 19, 38, 76, 51, 1.

L_1 : 1, 14, 95, 17, 36, 100, 87, 6, 84, 65, 1

L_2 : 2, 28, 89, 34, 72, 99, 73, 12, 67, 29, 2

L_3 : 4, 56, 77, 68, 43, 97, 45, 24, 33, 58, 4

...

L_{10} : 7, 98, 59, 18, 50, 94, 3, 42, 83, 51, 7.

Hamilton C_{11}-tenfoil = (101, 1, ...) ∪ (101, 2, ...) ∪ (101, 4, ...) ∪ (101, 8, ...)
∪ (101, 16, ...) ∪ (101, 32, ...) ∪ (101, 64, ...) ∪ (101, 27, ...) ∪ (101, 54, ...) ∪ (101, 7, ...)

Hamilton C_{11}-tenfoil = (101, 14, ...) ∪ (101, 28, ...) ∪ (101, 56, ...) ∪ (101, 11, ...)
∪ (101, 22, ...) ∪ (101, 44, ...) ∪ (101, 88, ...) ∪ (101, 75, ...) ∪ (101, 49, ...) ∪ (101, 98, ...)

Hamilton C_{11}-tenfoil = (101, 95, ...) ∪ (101, 89, ...) ∪ (101, 77, ...) ∪ (101, 53, ...)
∪ (101, 5, ...) ∪ (101, 10, ...) ∪ (101, 20, ...) ∪ (101, 40, ...) ∪ (101, 80, ...) ∪ (101, 59, ...)

...

Hamilton C_{11}-tenfoil = (101, 36, ...) ∪ (101, 72, ...) ∪ (101, 43, ...) ∪ (101, 86, ...)
∪ (101, 71, ...) ∪ (101, 41, ...) ∪ (101, 82, ...) ∪ (101, 63, ...) ∪ (101, 25, ...) ∪ (101, 50, ...).

(550 edges = (50 all lengths) * 11 times)

These 5 starters comprise a Hamilton C_{11}-tenfoil decomposition of $11K_{101}$.

Example 4.4. Hamilton C_{19}-tenfoil of $19K_{181}$.

$(n, g) = (181, 2)$ n-orbit : 1, 2, 4, 8, 16, 32, 64, 128, 75, 150, 119, 57, 114, 47, 94, 7, 14, 28, 56, 112, 43, 86, 172, 163, 145, 109, 37, 74, 148, 115, 49, 98, 15, 30, 60, 120, 59, 118, 55, 110, 39, 78, 156, 131, 81, 162, 143, 105, 29, 58, 116, 51, 102, 23, 46, 92, 3, 6, 12, 24, 48, 96, 11, 22, 44, 88, 176, 171, 161, 141, 101, 21, 42, 84, 168, 155, 129, 77, 154, 127, 73, 146, 111, 41, 82, 164, 147, 113, 45, 90, 180, 179, 177, 173, 165, 149, 117, 53, 106, 31, 62, 124, 67, 134, 87, 174, 167, 153, 125, 69, 138, 95, 9, 18, 36, 72, 144, 107, 33, 66, 132, 83, 166, 151, 121, 61, 122, 63, 126, 71, 142, 103, 25, 50, 100, 19, 38, 76, 152, 123, 65, 130, 79, 158, 135, 89, 178, 175, 169, 157, 133, 85, 170, 159, 137, 93, 5, 10, 20, 40, 80, 160, 139, 97, 13, 26, 52, 104, 27, 54, 108, 35, 70, 140,
99, 17, 34, 68, 136, 91, 1.

L_1 : 1, 119, 43, 49, 39, 116, 48, 101, 73, 180, 62, 138, 132, 142, 65, 133, 80, 108, 1

L_2 : 2, 57, 86, 98, 78, 51, 96, 21, 146, 179, 124, 95, 83, 103, 130, 85, 160, 35, 2

L_3 : 4, 114, 172, 15, 156, 102, 11, 42, 111, 177, 67, 9, 166, 25, 79, 170, 139, 70, 4

...

L_{10} : 150, 112, 115, 110, 58, 24, 141, 127, 90, 31, 69, 66, 71, 123, 157, 40, 54, 91, 150.

Hamilton C_{19}-tenfoil = (181, 1, ...) ∪ (181, 2, ...) ∪ (181, 4, ...) ∪ (181, 8, ...)
∪ (181, 16, ...) ∪ (181, 32, ...) ∪ (181, 64, ...) ∪ (181, 128, ...) ∪ (181, 75, ...) ∪ (181, 150, ...)

Hamilton C_{19}-tenfoil = (181, 119, ...) ∪ (181, 57, ...) ∪ (181, 114, ...) ∪ (181, 47, ...)
∪ (181, 94, ...) ∪ (181, 7, ...) ∪ (181, 14, ...) ∪ (181, 28, ...) ∪ (181, 56, ...) ∪ (181, 112, ...)

Hamilton C_{19}-tenfoil = (181, 43, ...) ∪ (181, 86, ...) ∪ (181, 172, ...) ∪ (181, 163, ...)
∪ (181, 145, ...) ∪ (181, 109, ...) ∪ (181, 37, ...) ∪ (181, 74, ...) ∪ (181, 148, ...) ∪ (181, 115, ...)

...

Hamilton C_{19}-tenfoil = (181, 73, ...) ∪ (181, 146, ...) ∪ (181, 111, ...) ∪ (181, 41, ...)

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$\cup$ $(181, 82, ...)$ $\cup$ $(181, 164, ...)$ $\cup$ $(181, 147, ...)$ $\cup$ $(181, 113, ...)$ $\cup$ $(181, 45, ...)$ $\cup$ $(181, 90, ...)$.

(1710 edges = (90 all lengths) * 19 times)

These 9 starters comprise a Hamilton C_{19}-tenfoil decomposition of $19K_{181}$.

参 考 文 献

1) Ushio, K.: Balanced bowtie and trefoil decomposition of symmetric complete tripartite digraphs, *Information and Communication Studies of The Faculty of Information and Communication Bunkyo University*, Vol.25, pp.19–24 (2000).
2) Ushio, K. and Fujimoto, H.: Balanced bowtie and trefoil decomposition of complete tripartite multigraphs, *IEICE Trans. Fundamentals*, Vol. E84-A, No. 3, pp. 839–844 (2001).
3) Ushio, K. and Fujimoto, H.: Balanced foil decomposition of complete graphs, *IEICE Trans. Fundamentals*, Vol.E84-A, No.12, pp.3132–3137 (2001).
4) Ushio, K. and Fujimoto, H.: Balanced bowtie decomposition of complete multigraphs, *IEICE Trans. Fundamentals*, Vol.E86-A, No.9, pp.2360–2365 (2003).
5) Ushio, K. and Fujimoto, H.: Balanced bowtie decomposition of symmetric complete multi-digraphs, *IEICE Trans. Fundamentals*, Vol.E87-A, No.10, pp.2769–2773 (2004).
6) Ushio, K. and Fujimoto, H.: Balanced quatrefoil decomposition of complete multigraphs, *IEICE Trans. Information and Systems, Special Section on Foundations of Computer Science*, Vol.E88-D, No.1, pp.19–22 (2005).
7) Ushio, K. and Fujimoto, H.: Balanced C_4-bowtie decomposition of complete multigraphs, *IEICE Trans. Fundamentals, Special Section on Discrete Mathematics and Its Applications*, Vol.E88-A, No.5, pp.1148–1154 (2005).
8) Ushio, K. and Fujimoto, H.: Balanced C_4-trefoil decomposition of complete multigraphs, *IEICE Trans. Fundamentals, Special Section on Discrete Mathematics and Its Applications*, Vol.E89-A, No.5, pp.1173–1180 (2006).