A Preparation Program for the Time Table Using Random Number

KIICHI FUJINO*

1. *Introduction*

This Program was developed by the author to produce the time table of Waseda University high school in 1962 and 1963, and it was written for the digital computer TOSBAC 3121 installed in Waseda University Electronic Computation Center.

This method contains following characteristics.

(1) We defined 16 kinds of basic patterns which constitute every real teaching pattern, and then even if a change occurs in the teaching pattern of the subject, the modification of the program is not need, so far as the teaching pattern of each subject is represented by the suitable patterns.

(2) We defined the L-assignment conditions LC corresponding to each basic pattern.

(3) We used random number in the process of assignment.

(4) We simplified the form of representation of input data.

2. *Preparation for the Program*

We suppose a school has m classes and has m_i classes in the i-th grade $(1 \leqq i \leqq 3)$, then $m=\sum_{i=1}^{3} m_i$. We define the class number (CN) of the j-th class in the i-th grade as $m_{i,j}=\sum_{k=1}^{i-1} m_k + j (1 \leqq j \leqq m_i)$. We suppose that the school has n_i hours in the i-th day and that n is the total hours in a week, then we have $n=\sum_{i=1}^{6} n_i$. Thus the period number (PN) of the j-th period in the i-th day is defined as $\sum_{k=1}^{i-1} n_k + j$ $(1 \leqq i \leqq 6,\ 1 \leqq j \leqq n_i)$. And also subject number (SN) corresponds to each subject as follows: if a major group S_i contains several minor subjects $S_{i,j}$, then the subject number of $S_{i,j}$ is $SN(S_i)+j$, where $SN(S_i)$ represents the basic number of S_i and is given a suitable number. For example, S_2 represents a major subject Mathematics and $S_{2,2}$ represents a minor subject Geometry, then SN of $S_{2,2}$ is $SN(S_2)+2=20+2=22$.

When the school has P teachers, each teacher is given a sequencial number which is called teacher number (TN) satisfying $1 \leqq TN \leqq P$.

Therefore a school time table can be considered as a matrix which is called the

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* Electronic Computation Center, Waseda University, Tokyo.

time table matrix T with m rows and n columns. The contents of each element $T(u,v)$ $(1 \leqq u \leqq m,\ 1 \leqq v \leqq n)$ represent the information about the lesson taught in the v-th period of the u-th class. This information consists of (i) Subject number (SN) (ii) The number of teacher who covers this lesson (TN). And if a class is divided into two sub-classes and another teacher (called a partner) covers the other subclass at the same time, then $T(u,v)$ needs two more informations: (iii) Subject number covered by the partner (PSN) (iv) Teacher number of the partner (PTN).

3. *Input Data*

Available Time Vector (ATV) We define the p-th available time vector $ATV(p)$ as an n-dimension vector, whose i-th element is represented by $ATV(p)_i$. If the i-th period of the teacher (p) is available, $ATV(p)_i$ is 00 otherwise $ATV(p)_i$ is 99. When the element $T(u,v)$ is assigned, u is written into $ATV(p)_v$, so $ATV(p)_v$ becomes non zero. After this, any period of the teacher (p) is no longer assigned into the v-th column of the matrix T. When the process finishes completely, $ATV(p)$ is considered as the personal time table.

Relations between Classes and Teacher (RCT) Let $RCT(p)$ denote a set of relations between class number and subject numbers covered by the teacher p. $RCT(p)$ contains at most J relations $RCT(p)_j$ $(1 \leqq j \leqq J)$. Each relation consists of the following four informations: CN_j^p (class nmber) $Time_j^p$ (the number of hours to teach this subject) SN_j^p (subject number) PTN_j^p (teacher number of the partner).

In order to simplify the construction of $RCT(p)$, if $j(p)<J$, we put $RCT(p)_j$ $=$all zero $(j(p)<j \leqq J)$. We call the set of $RCT(p)$ $(1 \leqq p \leqq P)$ $RCTS$ and then $RCT(p)=\bigcup_{j=1}^{J} RCTS(J(p-1)+j-1)$ holds. If $RCT(p)_j$ and $RCT(q)_k$ represent the lesson of the two divided sub-classes of a class, they are processed independently of the order of p and q and that at the same time. For this purpose, we put $PTN_j^p=q$ and $PTN_k^q=p$.

Initial condition for time table matrix. If we assume that all the periods except the period h for home room hour are available for the assignment, the initial conditions for the matrix T can be written as

$T(i,j)=$all zero $(1 \leqq i \leqq m,\ 1 \leqq j \leqq n,\ j \neq h)$ and

$T(i,h)=999999999999$ $(1 \leqq i \leqq m)$

When every $T(i,j)$ is produced, the u-th row of the matrix T gives the time table of the u-th class. If T_0 is the initial location of the matrix T, the location of $T(u,v)$ is $T_0+(u-1)n+v-1$, where it is assumed that each $T(u,v)$ occupies one word.

System condition (1) There are several restrictions which are required from the point of the school management, and may be changed every year, so it is better to give these restrictions to the program as the system condition (1). They are

(i) PN of the home room hour h

(ii) Maximum number of the hours covered by each teacher MAXT

(iii) Maximum number of the morning hour (the 1-st period of the day) which each teacher can cover in a week MAX(MLT)

(iv) Maximum number of hours of a subject covered in a class a day MAX(D1)

4. *Relations between Subjects and Teaching Patterns*

Sixteen kinds of bacic patterns $L(j)$ $(1 \leqq j \leqq 16)$ correspond to states defined by the values of four variables a_1, a_2, a_3 and a_4 which have the following meaning. When a binary number $a_1a_2a_3a_4$ equals a number $(a_1a_2a_3a_4)d$ in the decimal representation, the decimal number j of $L(j)$ defined by a_1, a_2, a_3 and a_4 equals $(a_1a_2a_3a_4)d+1$.

Variale \ Value	0	1
a_1	Covers one hour	Covers two consecutive hours
a_2	Covered by single teacher	Covered by two teachers
a_3	No using special room	Using special room
a_4	Not affected by grade	Affected by grade

Teaching pattern $F(i)$ of a subject $(SN=i)$ is written as

$$F(i)=\sum_{j=1}^{16} a_{ij} \times L(j)$$

where $L(j)$ is the j-th basic pattern, a_{ij} is 0 or a finite positive integer and

$$a_{ij} \begin{cases} =0: F(i) \text{ does not contain } L(j) \\ \neq 0: F(i) \text{ contains } L(j) \ a_{ij} \text{ times} \end{cases}$$

The total hour of subject i is $\sum_{j=1}^{16} a_{ij} \times u(L(j))$, where $u(L(j))$ denotes the number of hour contained in $L(j)$. The relations of teaching pattern and basic patterns are represented as the following matrix A.

MATRIX A	Basic Pattern $L(j)$															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
General subject	a															
Mathematics (III)	3								1							
Physics (I, II)												1				
Physics (III)			1							1						
Chemistry (I, II)												1				
Chemistry (III)			1							1						
Biology				2												
Geography			1						1							
Gymnastics			1													
Foreign Language (II)	2															
Foreign Language (II) (divided class)					2											
Art															1	

Note "a" of general subject may be different corresponding to subject number.

System condition (2) Matrix A may be changed every year, so we had better give it to the program as the system condition (2).

5. *Assignment Condition*

When a pair of TN and SN of $RCT(p)_j$ is assigned to $T(u, v)$, several conditions which must be satisfied with u, v, TN and SN are called assignment conditions. The number of a teaching pattern which $RCT(p)_j$ represents is given by CN_j^p and SN_j^p, thus from the matrix A we can decide the coefficients $a_{i,l}$ satisfying $F(CN_j^p, SN_j^p)=\sum_{l=1}^{16} a_{i,l}L(l)$. When the assignment of $L(l)$ for $a_{i,l} \neq 0$ is repeated $a_{i,l}$ times, it is completed to assign $RCT(p)_j$ into the time table. Therefore, if we define L-assignment condition $LC(l)$ for each basic pattern $L(l)$ $(1 \leqq l \leqq 16)$, any teaching pattern may be processable. In order to assign the hours of $RCT(p)_j$ with suitable spread in the time table, additional conditions (system condition (1)) are checked.

$LC(l)$ required by $L(l)$ is represented by the boolean product of sub-conditions $LC^{(i)}(a_i)$ $(i=1, 2, 3, 4)$ that is $LC(l)=\bigwedge_{i=1}^{4} LC^{(i)}(a_i)$.

$LC^{(1)}(a_1)$

a_1	$LC^{(1)}(a_1)$
0	i) $T(CN, \alpha)=0$
1	ii) $T(CN, \alpha)=T(CN, \alpha+1)=0$ and $\alpha \not\equiv 4$ or $\alpha \not\equiv 6 \pmod 6$ or $T(CN, \alpha-1)=T(CN, \alpha)=0$ and $\alpha \not\equiv 1$ or $\alpha \not\equiv 5 \pmod 6$

Note i) means that $T(CN, \alpha)$ is available for assignment.
ii) means that two consecutive hours are available and they do not extend over two days and a recess after lunch is not put between them.

$LC^{(2)}(a_2)$

a_2	$LC^{(2)}(a_2)$	Note
0	$ATV(p)_\alpha=0$	The period α of teacher p is available.
1	$ATV(p)_\alpha=ATV(PTN)_\alpha=0$	The period α of both teacher p and his partner teacher is available.

$LC^{(3)}(a_3)$

a_3	$LC^{(3)}(a_3)$
0	Unconditional
1	$N[AT_k(\alpha)]<S_k$ Refer note

Note AT_k is a table of the information which represents whether k-th special room (k-th S.R.) is used in each period or not. S_k is the number of k-th S.R. and $N[AT_k(\alpha)]$ is the number of the classes using k-th S.R. in the α-th period.

$LC^{(4)}(a_4)$

a_4	$LC^{(4)}(a_4)$
0	Unconditional
1	Grade$\neq$grade$[AT_k(\alpha)]$ Refer note

Note The grade of the class (CN) to be assigned must not be coincident with the grade of other classes already recorded in $AT_k(\alpha)$.

$LC(l)$ ($l=1, 2,\cdots, 16$) can be written as following Fig. 1.

Fig. 1 Table of L-assignment condition.

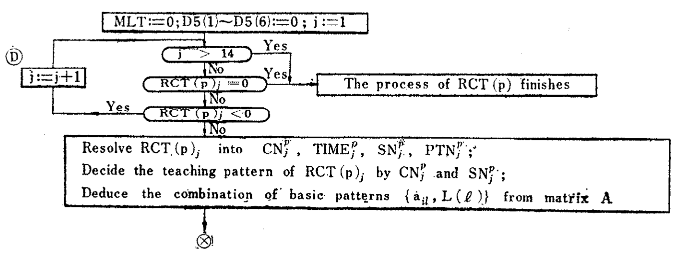
		Time Table	ATV	$N[ATK]$	$G[ATK]$
l	$a_1a_2a_3a_4$	a_1	a_2	a_3	a_4
1	0 0 0 0*	$T(CN, \alpha)=0$	$ATV(p)_\alpha=0$		
2	0 0 0 1				$G\neq G[ATK(\alpha)]$
3	0 0 1 0*			$N[ATK(\alpha)]<S_K$	
4	0 0 1 1*				$G\neq G[ATK(\alpha)]$
5	0 1 0 0*		$ATV(p)_\alpha=0$ and $ATV(q)_\alpha=0$		
6	0 1 0 1				$G\neq G[ATK(\alpha)]$
7	0 1 1 0			$N[ATK(\alpha)]<S_K$	
8	0 1 1 1				$G\neq G[ATK(\alpha)]$
9	1 0 0 0*	$T(CN, \alpha)$ $=T(CN, \alpha+1)$ $=0$ and $\alpha\not\equiv 4, 6$ (mod 6) or $T(CN, \alpha-1)$ $=T(CN, \alpha)$ $=0$ and $\alpha\not\equiv 1, 5$ (mod 6)	$ATV(p)_\alpha=ATV(p)_{\alpha+1}=0$ or $ATV(p)_{\alpha-1}=ATV(p)_\alpha=0$		
10	1 0 0 1*				NOTE (2)
11	1 0 1 0			$N[ATK(\alpha)]$, $N(ATK(\alpha+1)]<S_K$ or $N[ATK(\alpha-1)]$, $N[ATK(\alpha)]<S_K$	
12	1 0 1 1*				NOTE (2)
13	1 1 0 0		$ATV(p)_\alpha=ATV(p)_{\alpha+1}=0$ $ATV(q)_\alpha=ATV(q)_{\alpha+1}=0$ or $ATV(p)_{\alpha-1}=ATV(p)_\alpha=0$ $ATV(q)_{\alpha-1}=ATV(q)_\alpha=0$		
14	1 1 0 1				NOTE (2)
15	1 1 1 0*			$N[ATK(\alpha)]$, $N[ATK(\alpha+1)]<S_K$ or $N[ATK(\alpha-1)]$, $N[ATK(\alpha)]<S_K$	
16	1 1 1 1				NOTE (2)

Note (1) $L(l)$s marked by * are used in our experiment.

Note. (2) $G\neq G[ATK(\alpha)]$ and $G\neq G[ATK(\alpha+1)]$ or $G\neq G[ATK(\alpha-1)]$ and $G\neq G[ATK(\alpha)]$.

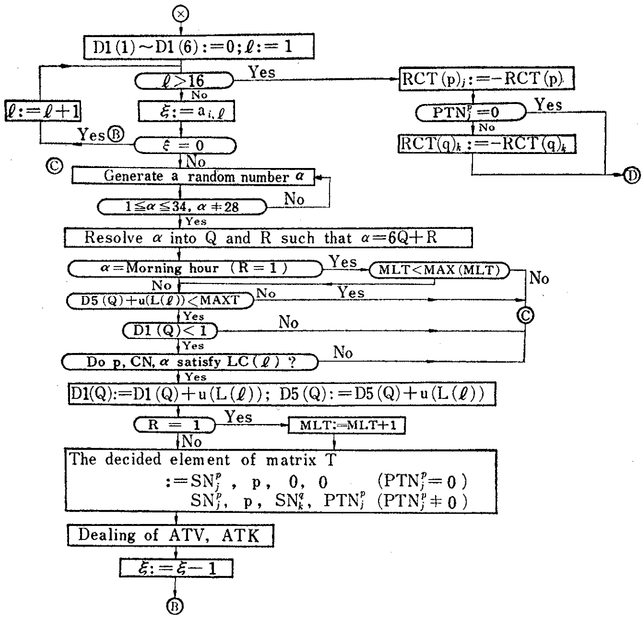
6. *Outline of the Assignment Process*

The assignment process of this program is performed with random numbers. In this part we shall explain the outline of the process.



(1) If a random number α satisfies $1 \leqq \alpha \leqq P$ and $IPT[\alpha]=0$ (the teacher α is not processed yet), let p denote α, and the teacher p receives the process.

(2) We shall describe the process of RCT (p) by the following flow chart.



Note D5-table is a set of the counters to check the number of hours covered by each teacher a day.
D1-table is a set of the counters to check the number of hours to teach a subject a day.

(3) If the process of $RCT(p)$ finishes, we put 1 into $IPT[p]$, and add 1 to a (the number of processed teacher). If $a<P$, we repeat (1), otherwise the assignment process finishes completely.

Example of Processing $RCT(p)_j$ We explain the process by examples.

(A) *The case of Mathematics III* We suppose its teaching pattern is represented as $F=3\times L(1)+1\times L(9)$. In the first place, $1\times L(9)$ is assigned. Let the random number routine generate a number α repeatedly until α satisfies $LC(9)$ and the additional conditions. Then two consecutive hours of $L(9)$ are decided and assigned into $T(CN_j{}^p, \alpha)$ and $T(CN_j{}^p, \alpha+1)$ or $T(CN_j{}^p, \alpha-1)$ and $T(CN_j{}^p, \alpha)$. The informations Ⓐ$=[SN_j{}^p, p, 0, 0]$ (no partner) are written into the above decided elements of T, and $CN_j{}^p$ into $ATV(p)\alpha$ and $ATV(p)_{\alpha+1}$ or into $ATV(p)_{\alpha-1}$ and $ATV(p)\alpha$. In the next place let the random number routine generate α repeatedly until α satisfies $LC(1)$ and the additional conditions, and we write Ⓐ into $T(CN_j{}^p, \alpha)$ and $CN_j{}^p$ into $ATV(p)\alpha$. This process for $L(1)$ is repeated three times. Therefore five hours of Mathematics III are necessarily assigned extending over four days (one of them is consecutive two hours lesson).

(B) *The case of Art* Let its teaching pattern be $F=1\times L(15)$. We decide a random number α satisfying $LC(15)$ and the additional conditions. $L(15)$ represents a lesson of consecutive two hours, covered by two teachers and using two special rooms. The data $[SN_j^p,\ p,\ SN_k^q,\ PTN_j^p]$ are put into the decided two hours, where SN_k^q is SN of the subject covered by the partner and is the SN part of $RCT(q)_k$ such that $CN_k^q=CN_j^p$, $TIME_k^q=TIME_j^p$, $PTN_k^q=p$. And then, CN_j^p is written into the decided elements of $ATV(p)$ and $ATV(q)$. And also, $N[AT_k(\alpha)]$ and $N[AT_k(\alpha+1)]$ or $N[AT_k(\alpha-1)]$ and $N[AT_k(\alpha)]$ are added by 1. $RCT(q)_k$ will be skipped by its negative sign given here, in the process of $RCT(q)$, so the double assignment does not occur.

Examples of Input data and Results

Teacher No. 28

RCT (28)

1	0	2	8	0	0	5	0	2	4	0	0	0
2	0	0	8	0	0	2	0	2	1	0	0	0
8	0	0	4	0	0	2	0	2	1	0	0	0
4	0	0	5	0	0	2	0	2	2	0	0	0
5	0	0	6	0	0	2	0	2	2	0	0	0
6	0	2	3	0	0	2	0	2	4	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0

Content

Class	Time	Subject	Partner
3G	5	Mathematics III	....
1C	2	Algebra	....
1D	2	Algebra	....
1E	2	Geometry	....
1F	2	Geometry	....
3B	2	Mathematics III	....

ATV (28)

99	99	99	99	99	99
			99		
				99	99

Content

Mon.	Holiday for Research					
Tues.						
Wed.						
Thur.						
Fri.				H.R.		
Sat.					/	/

Personal Time Table (28)

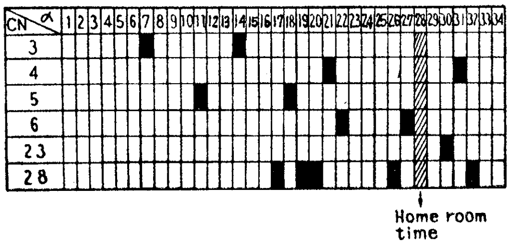
99	99	99	99	99	99
3		23		5	
	3			23	5
28	28	4	6		
	28	6	99		23
5	23			99	99

Output Form of Personal Time Table

Mon.	Study					
Tues.	1C	:	3B	:	1E	:
Wed.	:	1C	:	:	3G	1E
Thur.	3G	3G	1D	1F	:	:
Fri.	:	3G	1F	H.R.	:	3B
Sat.	1D	3G	:	:	:	:

" : " means free time.
" STUDY " means the holiday for research.

Corresponding Part of Time Matrix T



Since Monday is a holiday for research for the teacher 28, there is no assignment on Monday

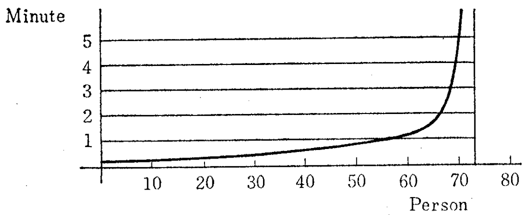
7. *Conclusion*

Problems on the design of program Almost all the teaching patterns in the high school can be constructed by 16 kinds of basic $L(1)\sim L(16)$. However, the subject containing the complicated basic patterns gives a great effect to the processing of assignment, and moreover affects the assignment of the other subject. So, enough consideration is necessary in the selection of basic pattern. Generally speaking, if each $ATV(p)$ is simple and basic patterns are selected from $L(1)\sim L(8)$, the computation will be easy.

TOSBAC 3121 used in the computation is a decimal number machine and its word consists 12 digits, and on the other hand $1 \leqq TN$, SN, $CN < 99$ and the number of the hours in a day is six except Saturday. So we could simplify the input data.

Remarks about the result of experiment In the case of part-time teacher, the number of available hours in his ATV vector and his covering hours are mostly coincident, so the assignment is difficult. Thus, they are processed at the beginning of the process.

In the case of full-time teacher, if the assignment of $RCT(p)_j$ requires too much time, $RCT(p)_j$ is stopped and $RCT(p)_{j+1}$ is processed. In our experiment, we had 73 full-time teachers and the number of their covering hours was 921, and we succeeded in assignment of 869 hours (that is 94.5%). The following graph shows the mean time required by the assignment for each teacher.



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