Statistical Properties of Translation-Noise of Character Figure

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1. *Introduction*

One of the optimal coordinate systems for representing a original character pattern effectively can be obtained by solving the eigen value problem. By means of this coordinate system, a pattern can be represented as a vector in a Euclidian space. By making a linear transformation of these optimal coordinate system, we construct a new space for the convenience of classification procedure.

So as to investigate a significant pattern of noise, the difference between the input pattern and the standard one is assumed as a noise pattern, and the difference between the element of the input and that of the standard pattern is also considered as a noise element in this paper. The noise element caused by the parallel shift of the standard pattern on the plane is particularly named as the *translation-noise element*. Among the various kinds of noise, the translation-noise occures most frequent and it occupies the largest part of noise. So it is necessary to examine the property of this type of noise.

Although the set of expansion coefficients is uniquely determined for an input pattern in each space, but the range of distribution of expansion coefficients is not known. In order to make it clear the structure of pattern in each space, the distribution of expansion coefficients must be investigated. For this reason, we assume the expansion coefficient of pattern as a random variable. Following this assumption, each expansion coefficient can be considered as the sample drawn from some statistical population. On the basis of this statistical model, we can set up various kinds of tests of hypothesis with respect to the set of projections of patterns that are treated with the preprocessing.

The original character figure considered in this paper is the full set of Japanese Katakana whose stroke width is 0.50 mm in average.

2. *Preprocessing and Representation of Character*

Let $e(x_i, y_j)$ be the quantized character, let $f(x_i, y_j)$ be positionally and brightly normalized pattern, then quantized character is transformed as

$$f(x_i, y_j)=e(x_i+[x_a], y_j+[y_a])-k_0, \qquad (1)$$

where k_0 is defined as the integral value of $e(x_i, y_j)$, and as for $[x_a]$, $[y_a]$ see (3).

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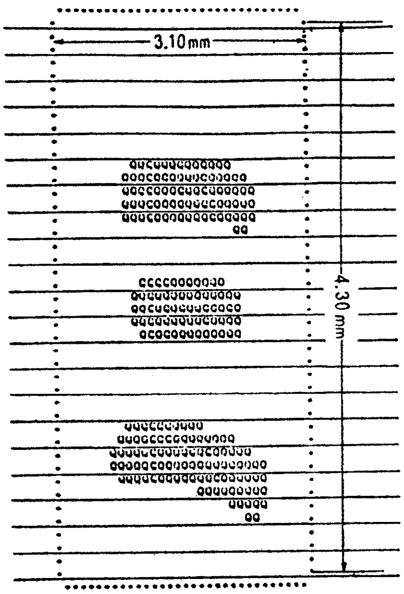

Fig. 1. Standard quantized character with 31×43 meshes.

Let $g(x_i, y_j)$ be the sampled pattern transformed from $f(x_i, y_j)$, then

$$g(x_i, y_j)=\sum_{t=-4}^{4}\sum_{s=-4}^{4}\left[e-\frac{(x_i-u_{i+s})^2+(y_j-v_{j+t})^2}{2\sigma^2}\times f(u_{i+s}, v_{j+t})\times\frac{c}{\sigma}\right], \quad (2)$$

where c is constant, $\sigma=0.20$, $1\leq i\leq 11$ and $1\leq j\leq 15$.

Let $\boldsymbol{g}_r$ be the sampled standard character which belong to the r-th category among the set of 46 categories, let $\boldsymbol{\tau}$ be the average pattern of the sampled standard characters $\{\boldsymbol{g}_r\}$, and let $\boldsymbol{h}_r$ be the canonicalized standard pattern, then the optimal coordinate system can be obtained by means of solving the eigenvalue equation:

$$\lambda_m\varphi_m=\boldsymbol{A}\varphi_m\,;\ a_{ij}=\frac{1}{46}(\boldsymbol{h}_i, \boldsymbol{h}_j), \quad (3)$$

where a_{ij} is an element of matrix A. In this coordinate system, a pattern can be expressed as follows,

$$\boldsymbol{h}_r=\sum_{m=1}^{45}\beta_{rm}\varphi_m\,;\ \beta_{rm}=(\boldsymbol{h}_r, \varphi_m), \quad (4)$$

where $\{\varphi_m\}$ are normalized eigen vectors. The space defined by this coordinate system is named as the *feature space* and the expansion coefficient β_{rm} is named as the *feature coefficient* in this paper.

From the N-dimensional subspace of this feature space, we construct the new space by using the Affinian transformation. Let $\boldsymbol{\chi}_k$ be the new k-th coordinate axis, and let $\{\boldsymbol{\chi}^l;\ l=1, 2, \cdots, N\}$ be the reciprocal system of $\{\boldsymbol{\chi}_k\}$, then the new coordinate axis is defined as

$$\boldsymbol{\chi}_k=\sum_{m=1}^{N} u_{rm}\frac{\varphi_m}{\sqrt{\lambda_m}}, \tag{5}$$

subjected to the constraints,

$$\begin{cases}(\boldsymbol{\chi}_k,\ \boldsymbol{\chi}^l)=\delta_k{}^l\\ \|\boldsymbol{\chi}_k\|^2=\dfrac{1}{N}\displaystyle\sum_{m=1}^{N}\dfrac{1}{\lambda_m}\equiv\mu_0.\end{cases} \tag{6}$$

In this space, the pattern within the N-dimensional subspace of the feature space is expressed as

$$\boldsymbol{h}_{Nr}=\sum_{k=1}^{N}\gamma_r{}^k\boldsymbol{\chi}_k\ ;\ \gamma_r{}^k=(\boldsymbol{h}_r,\ \boldsymbol{\chi}^k). \tag{7}$$

This new space is named as the *equi-variance space*, and $\gamma_r{}^k$ is called as the *equi-variance characteristic parameter*.

3. *Distribution of Equi-Variance Characteristic Parameters*

In order to examine the grobal property of the N characteristic parameters of a pattern, we suppose the parameters as the samples drawn from a population, although the parameters are uniquely determined for an input character. At first by means of *run-test*, we examine whether the given N measurements are randomly drawn or not. Calculated numbers of *run* are shown in the Table 1. Secondary, we make clear the type of the population. The distribution of the measurements seems to be a normal distribution, then we carry out the test for goodness of fit assuming hypothetical distribution to be a normal distribution.

Table 1. Total Number of RUN.

Category	Stand. Patt.		Noise Patt. (right direction)		Category	Stand. Patt.		Noise Patt. (right direction)		Category	Stand. Patt.		Noise Patt. (right direction)	
	N=8	N=36	N=8	N=36		N=8	N=36	N=8	N=36		N=8	N=36	N=8	N=36
ア	7	23	8	29	チ	5	21	7	24	ム	5	19	5	26
イ	4	16	6	29	ツ	5	21	4	28	メ	4	20	3	26
ウ	6	16	6	26	テ	3	21	7	26	モ	6	18	5	30
エ	6	18	6	29	ト	7	19	6	22	ヤ	5	20	8	24
オ	5	16	7	25	ナ	5	20	7	30	ユ	2	24	5	26
カ	3	19	7	24	ニ	6	19	6	27	ヨ	5	20	6	25
キ	5	22	7	24	ヌ	5	18	4	27	ラ	6	20	4	27
ク	5	15	4	24	ネ	5	19	5	26	リ	5	18	6	24
ケ	7	19	6	26	ノ	6	18	6	29	ル	5	16	4	31
コ	6	18	5	27	ハ	5	15	6	26	レ	6	12	7	32
サ	7	23	8	24	ヒ	4	17	7	28	ロ	7	26	6	28
シ	6	20	4	32	フ	4	17	3	26	ワ	3	23	2	28
ス	6	21	4	27	ヘ	6	13	8	23	ヲ	6	20	4	26
セ	4	19	7	28	ホ	3	20	3	28	ン	4	22	4	28
ソ	6	20	4	26	マ	5	18	4	30					
タ	5	19	4	27	ミ	4	16	4	30					

(displacement: 0.10 mm)

Table 2. k-statistics of standard patterns. (N=36)

	Skewness	Kurtosis		Skewness	Kurtosis		Skewness	Kurtosis
ア	.2011	-.2085	チ	.1723	-.3091	ム	.0513	-.0287
イ	-.1322	-.5019	ツ	-.1076	-.7103	メ	.3441	-.7622
ウ	.3142	-.2955	テ	.0536	.0999	モ	.0357	-.5105
エ	-.0020	.5795	ト	.5482	.2022	ヤ	.4599	.2140
オ	.0040	-.6533	ナ	.5084	-.1380	ユ	-.3323	-.3108
カ	-.4631	-.1928	ニ	.3305	.5331	ヨ	-.1532	-.0688
キ	.0110	-.1826	ヌ	-.4143	-.4695	ラ	.4166	-.5098
ク	-.0383	-.5487	ネ	-.0782	-.7856	リ	.3931	-.5528
ケ	.4175	1.2065	ノ	.1977	-.3304	ル	.1459	-.3000
コ	-.3515	.2090	ハ	-.6821	.3292	レ	-.5417	-.5679
サ	.1208	-.2449	ヒ	.1358	-.1474	ロ	-.3209	1.4280
シ	.1937	-.4683	フ	.0417	.6467	ワ	-.0848	-.4442
ス	-.0837	-.8855	ヘ	.4670	-.2693	ヲ	.2487	-.2434
セ	.0558	.1283	ホ	-.4709	.5595	ン	-.6363	.4826
ソ	-.1732	-.5284	マ	-.4018	-.4746	σ_G	.3925	.7681
タ	-.1084	-.2791	ミ	.4381	-.5047			

Table 3. k-statistics of translation-noise.

	Right direction		Left direction			Right direction		Left direction	
	Skewness	Kurtosis	Skewness	Kurtosis		Skewness	Kurtosis	Skewness	Kurtosis
ア	.2494	-.7477	-.0339	-.4045	ノ	.0516	-.5135	.0407	-.3897
イ	.2172	-.4341	.1561	-.3305	ハ	-.0201	-.7632	.2681	-.7206
ウ	.0649	-.6272	.0411	-.8000	ヒ	.0472	-.2707	.2313	-.2075
エ	.1440	-.7300	-.1081	-.9473	フ	.2238	-.1212	-.1332	-.4097
オ	.0937	-.4724	.3694	-.6112	ヘ	-.1647	-.5157	.2532	-.0781
カ	-.4732	-.5667	.1897	-.3238	ホ	.0178	-1.2094	.1191	-1.2283
キ	-.2338	-.9446	.5915	-.4604	マ	-.0522	-.5795	.2955	-.1260
ク	.1275	-1.2194	-.3203	-.5473	ミ	-.0109	.2200	.1748	-.0178
ケ	.2995	-.6034	.0339	-.4757	ム	-.3604	-.7924	.3262	-.3116
コ	-.3211	.0808	.5763	.6554	メ	.1123	-.6633	.1393	.0422
サ	.2863	-.3622	-.0162	-.7237	モ	.2342	-.4210	.0820	-.7761
シ	.1046	-1.4865	.0994	-1.3500	ヤ	.0698	.0029	-.0924	-.6284
ス	-.3147	-.7049	.2152	-.8023	ユ	.0272	-.3620	.0516	-.8885
セ	-.0105	-.8333	.2125	-.7756	ヨ	-.1698	.4266	.4399	.7062
ソ	.1188	-.3368	.0865	-.7128	ラ	.1141	.5780	.2752	.2707
タ	-.0803	-.6701	.1777	-.0805	リ	.2373	-.4142	.2336	-.6150
チ	.2704	-.2977	.2731	.5466	ル	.1063	-1.0324	.2091	-1.2644
ツ	.1226	-1.0627	.0691	-1.3709	レ	.0571	-1.4556	.0967	-1.3614
テ	.2131	-.4636	.0334	-.1041	ロ	.1419	-.7464	.3022	-.2685
ト	.0291	-1.0253	.3162	-.0376	ワ	.3162	-.6904	.0557	-.9679
ナ	.2704	-.7723	-.0889	-.6273	ヲ	.2700	.1499	.1405	.0974
ニ	-.1912	.0202	.4039	.7119	ン	-.0682	-1.2670	.4542	-.1335
ヌ	-.1430	-.7959	.1923	-.6793	σ_G	0.3925	0.7681	0.3925	0.7681
ネ	.1074	-.7434	.1496	-.5457					

(N=36; number of classes: 13; class width: 0.40; displacement: 0.10 mm)

Table 4. F-statistics of noise. (Testing homogeneity of population mean.)

	$N=8$	$N=36$		$N=8$	$N=36$		$N=8$	$N=36$
ア	1.475	.1464	チ	.749	0.288	ム	.915	.0412
イ	.016	.0007	ツ	.447	0.248	メ	.381	.0274
ウ	.272	.0186	テ	.707	0.411	モ	.736	.0303
エ	1.190	.0947	ト	.190	0.194	ヤ	.204	.0149
オ	.641	.0168	ナ	1.671	0.954	ユ	1.896	.1428
カ	.787	.0572	ニ	.842	.1223	ヨ	.614	.0411
キ	.031	.0009	ヌ	.352	.0223	ラ	.246	.0203
ク	2.708	.1839	ネ	.711	.0140	リ	.379	.0328
ケ	.851	.0315	ノ	.813	.0953	ル	.611	.0085
コ	2.153	.1968	ハ	.089	.0123	レ	.068	.0044
サ	.170	.0093	ヒ	.535	.0455	ロ	.383	.0399
シ	.496	.0192	フ	2.538	.3553	ワ	.402	.0411
ス	.440	.0260	ヘ	.036	.0031	ヲ	.239	.0230
セ	1.068	.0530	ホ	.140	.0020	ン	1.561	.1048
ソ	1.239	.0679	マ	1.144	.0753			
タ	2.528	.1164	ミ	.802	.0538			

($z=4$; displacement: 0.10 mm)

Table 5. F_{max}-statistics of noise. (Testing homogeneity of population variance)

	$N=8$	$N=36$		$N=8$	$N=36$		$N=8$	$N=36$
ア	1.69	2.78	チ	2.67	1.24	ム	2.53	1.67
イ	1.67	1.60	ツ	1.24	2.86	メ	1.41	1.65
ウ	1.16	1.75	テ	4.25	1.85	モ	3.66	1.27
エ	1.75	1.16	ト	15.19	1.25	ヤ	2.02	1.76
オ	4.49	1.50	ナ	3.90	1.68	ユ	1.81	1.30
カ	1.57	1.57	ニ	13.88	1.55	ヨ	2.74	1.57
キ	2.75	1.95	ヌ	1.60	1.40	ラ	2.78	1.38
ク	2.89	1.93	ネ	2.28	1.10	リ	3.24	3.15
ケ	1.49	2.33	ノ	1.85	2.40	ル	1.34	1.45
コ	2.01	1.29	ハ	17.58	2.17	レ	6.80	2.39
サ	1.87	2.14	ヒ	2.04	1.64	ロ	1.51	1.41
シ	1.61	1.34	フ	1.88	2.06	ワ	1.57	3.07
ス	2.24	1.23	ヘ	1.16	1.63	ヲ	4.41	1.84
セ	1.54	1.73	ホ	4.94	1.99	ン	1.34	1.19
ソ	4.89	2.67	マ	2.16	1.73			
タ	2.81	1.43	ミ	28.70	1.15			

($z=4$; displacement: 0.10 mm)

We calculate the *k-statistics* from the set of N characteristic parameters and compute the *skewness* and the *kurtosis* using these *k-statistics*. These statistics are illustrated in Table 2 and Table 3 with respect to each standard pattern and each translation-noise pattern. In case of translation--noise pattern, the population from which parameters are drawn seems to be different according to the variation of direction of the translation. Then, in order to examine the homogeneity of the populations which are corresponding to each of the categories, we test the homogeneity of population mean and homogeneity of population variance employing *F-test* and *Hartly-test*, respectively. Calculated statistics with respect to these tests are shown in Table 4 and Table 5.

On the other hand, we can assume the population with respect to the set

Table 6. *k*-statistics of standard elements around axis.

Axis No.	Skewness	Kurtosis	Axis No.	Skewness	Kurtosis	Axis No.	Skewness	Kurtosis
1	.2011	-.2085	14	.0558	.1283	27	.1358	-.1474
2	-.1322	-.5019	15	-.1732	-.5284	28	.0417	.6467
3	.3142	-.2955	16	-.1084	-.2791	29	.4670	-.2693
4	-.0020	.5795	17	.1723	-.3091	30	-.4709	.5595
5	.0040	-.6533	18	-.1076	-.7103	31	-.4018	-.4746
6	-.4631	-.1928	19	.0536	.1000	32	.4381	-.5047
7	.0110	-.1826	20	.5482	.2022	33	.0513	-.0287
8	-.0383	-.5487	21	.5084	-.1380	34	.3441	-.7623
9	.4175	1.2065	22	.3305	.5331	35	.0357	-.5105
10	-.3515	.2090	23	-.4143	-.4695	36	.4596	.2140
11	.1208	-.2449	24	-.0782	-.7856	σ_G	.3501	.6876
12	.1937	-.4684	25	.1977	-.3306			
13	-.0837	-.8855	26	-.6821	.3291			

(number of classes: 13; class width: 0.40)

Table 7. *k*-statistics of noise elements around axis.

Axis No.	Skewness	Kurtosis	Axis No.	Skewness	Kurtosis	Axis No.	Skewness	Kurtosis
1	.0331	-.6088	14	.0440	-.3182	27	.3149	.4034
2	.0175	-.8241	15	.0733	-.3261	28	.0449	-.6870
3	.0641	-.6509	16	.1569	-.2993	29	-.0491	-.5591
4	.0296	-.5017	17	.0999	-.2470	30	.0562	-.3294
5	.0740	-.2131	18	-.1027	-.0394	31	-.1261	.0769
6	.2749	-.3645	19	.0598	-.3960	32	.2058	.3949
7	.1525	-.5893	20	.0660	-.3785	33	-.0437	-.5755
8	.0965	.5064	21	-.0457	-.9454	34	-.0976	-.6023
9	-.1019	-.3909	22	.0751	-.8951	35	.0673	-.8233
10	.1975	-.0511	23	-.1202	-.7858	36	.0136	-.7065
11	-.0832	-.3524	24	-.1639	-.0022	σ_G	.1791	.3564
12	.1293	-.2957	25	.1417	-.5985			
13	-.0106	-.7636	26	-.0514	-.3137			

(number of classes: 13; class width: 0.50; displacement: 0.10 mm)

of projections of the patterns on the certain coordinate axis. On the basis of this population, we carry out the test of goodness of fit under the hypothetical distribution being normal. Calculated *skewness* and *kurtosis* are shown in Table 6 and Table 7. Moreover, by means of *T-statistics* and *W-statistics*, we test the null hypothesis H: $\mu_0=0$ and the null hypothesis H: $\sigma_0^2=c_d$. The obtained these statistics are shown in Table 8.

As the result of these hypothesis testings, a characteristic parameter of the translation-noise pattern at any axis can be assumed as a sample drawn from $N(0, c_d)$, and that of the standard pattern can be assumed as a sample drawn from $N(0, 1)$.

Table 8. T, W-statistics of noise around axis. (displacement: 0.10 mm)

Axis No.	$\lvert T \rvert$	W	Axis No.	$\lvert T \rvert$	W	Axis No.	$\lvert T \rvert$	W
1	.3675	156.24	13	.0437	147.50	25	.2309	293.35
2	.0817	277.61	14	.0546	193.30	26	.2207	162.31
3	.7198	157.20	15	.2775	271.42	27	1.2102	114.16
4	.5019	159.99	16	.5490	160.98	28	1.6610	112.18
5	.2122	179.85	17	.0383	116.68	29	1.5137	110.94
6	.1759	175.06	18	.1824	191.01	30	.3492	115.60
7	.2533	139.95	19	.5376	226.74	31	.7838	147.71
8	.3902	118.06	20	1.0939	184.16	32	.4725	119.73
9	.0001	107.75	21	.4919	404.97	33	.9243	156.77
10	.2420	92.49	22	.2419	455.09	34	.1496	232.73
11	.3419	169.54	23	.6072	209.68	35	.1750	302.46
12	.2116	127.03	24	.5559	149.72	36	.4927	173.90

($\mu_0=0.000$; $\sigma_0^2=1.353$)

4. *Probability Density Function of Translation-Noise*

In this section, using the results of preceding section, we shall try to obtain the practical probability density function of translation-noise elements in the feature space. A characteristic parameter γ^k is related to the set of feature coefficients $\{\beta_m\}$ by the following formula,

$$\gamma^k=\sum_{m=1}^{N}\frac{u_{km}}{\sqrt{\lambda_m}}\beta_m. \tag{8}$$

Therefore if γ^k is distributed according to the normal distribution, β_m is also distributed according to the normal distribution. As it became clear that the parameters $\gamma^1, \gamma^2, \cdots, \gamma^N$ have mutually no relation, let us assume that $\{\beta_m\}$ are mutually independent for convenience.

On the other hand, so long as the displacement of pattern is small, translation-noise pattern has a linear structure with respect to displacement. Hence the distribution of feature coefficients around the coordinate axis of the feature space has zero by mean. With respect to the variance of feature coefficient, it can be assumed to be constant compared with eigen values. See Fig. 2. Let Δ^2 be this variance, then it is defined as follows;

$$\begin{cases} V_m=\dfrac{1}{46\,z}\displaystyle\sum_{i=1}^{z}\sum_{r=1}^{46}{}_i\beta^2{}_{rm} \\ \Delta^2=\dfrac{1}{N}\displaystyle\sum_{m=1}^{N}V_m\,. \end{cases} \tag{9}$$

As the results of these considerations, we can make it clear the probability density function of noise elements in the feature space, and let it be $w(\boldsymbol{\beta})$ then

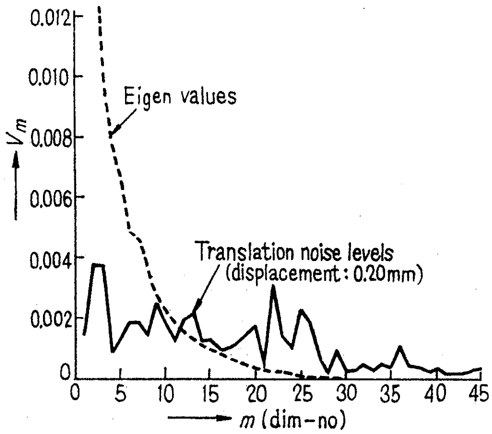

Fig. 2. Comparison of Average power-spectra between the standard characters and the translation noise.

$$\begin{cases} w(\boldsymbol{\beta})=\prod_{m=1}^{N} p(\beta_m, \Delta^2) \\ p(\beta, \Delta^2)=\dfrac{1}{\sqrt{2\pi}\,\Delta}e^{-\frac{\beta^2}{2\Delta^2}} \end{cases} \tag{10}$$

In order to examine the availability of this density function, we investigate the distribution of characteristic parameters. Let γ be a noise pattern, let $\bar{\gamma}^k$ be the k-th component of the average noise pattern, and let $\overline{\gamma^k\gamma^l}$ be an element of covariance matrix of γ, then according to the formulas (8) and (10), $\bar{\gamma}^k$ becomes to be zero, and $\overline{\gamma^k\gamma^l}$ is calculated as follows:

$$\begin{aligned}\overline{\gamma^k\gamma^l}&=\sum_{m=1}^{N}\sum_{n=1}^{N}\frac{u_{km}}{\sqrt{\lambda_m}}\frac{u_{ln}}{\sqrt{\lambda_n}}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}w(\boldsymbol{\beta})\beta_m\beta_n d\boldsymbol{\beta}\\ &=\Delta^2\sum_{m=1}^{N}\frac{u_{kn}u_{ln}}{\lambda_m}.\end{aligned} \tag{11}$$

With respect to this reciprocal system, the following formula is satisfied,

$$(\boldsymbol{x}^k, \boldsymbol{x}^l)=\sum_{m=1}^{N}\frac{u_{km}u_{lm}}{\lambda_m}. \tag{12}$$

Using the above formula, We see that

$$(\bar{\gamma}^k)^2=\Delta^2\mu_0 \quad (k=1, 2, \cdots, N) \tag{13}$$

We name the value calculated according to the formula (13) as the theoretical value, and we name the mean square value with respect to the characteristic parameters as the experimental value. The theoretical value and the experimental value are shown in Table 9.

Furthermore, in order to investigate the usefulness of this density fun., we also examine the distribution of Humming distance of code system of a pattern. The N-bits code system of a pattern can be obtained by encoding the set of N characteristic parameters as follows: the code symbol is 1, if parameter's value is positive, and the code symbol is -1, if it is negative.

Table 9. Variance of character parameters around axis.

displacement	theoretical value $\overline{\gamma}_k^2$		experimental value $C_d=\frac{1}{N}\sum_{k=1}^{N}\frac{1}{46}\sum_{r=1}^{46}\beta_{k(r)2}$	
	$N=8$	$N=36$	$N=8$	$N=36$
0.10 mm	0.080	1.36	0.078	1.35
0.20 mm	0.315	5.10	0.300	4.89

Let q be the sign inversion probability, and let the noise be additive, then q is defined as follows:

$$q=2\int_0^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}\int_{-\infty}^{0}\frac{1}{\sqrt{2\pi(\overline{\gamma}^k)^2}}e^{-\frac{(x-y)^2}{2(\overline{\gamma}^k)^2}}dxdy$$

$$=\frac{1}{\pi}\tan^{-1}\sqrt{(\overline{\gamma}^k)^2}\,. \tag{14}$$

Stillmore it became clear that there was no relation between the characteristic parameters at each axis. Therefore, let Q_m be the probability of m bits sign inversion in N bits code, then

$$Q_m=\binom{N}{m}q^m(1-q)^{N-m}. \tag{15}$$

The experimental results of this distribution are shown in Fig. 3. As for the probability q, we name the value which is defined by the formulas (9), (13) and (14) as the theoretical value, and we name the value which agrees well with this distribution as the experimental value. The comparison of the experimental value with the theoretical value is shown in Table 10. Validity of the formula (10) as the density function can be read from Table 10, and Fig. 4.

5. *Conclusions*

It mainly depends on the appropriate preprocessing that the characteristic

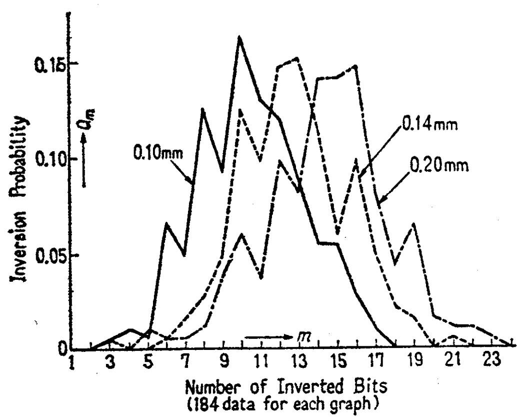

Fig. 3. Variation of Sign-Inversion Probability.

Table 10. Probability of sign inversion.

displacement	q (N=36)	
	theoretical value	experimental value
0.10 mm	0.27	0.28
0.14 mm		0.34
0.20 mm	0.37	0.41

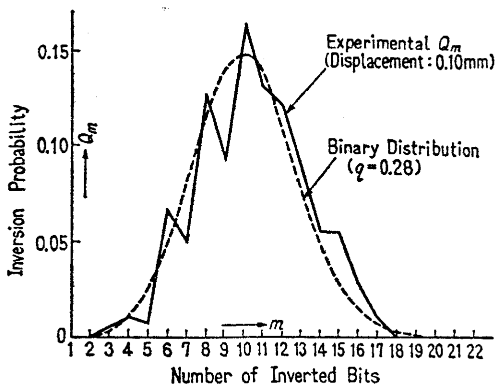

Fig. 4. Comparison binary distribution with experimental Q_m distribution.

parameters of standard patterns have a good statistical property. And it also depends on the smallness of displacement that the statistical property of the noise elements is like that of standard characteristic parameters. As the results of this study, the distribution of the coefficients of the translation-noise pattern is shown to be Gaussian, if the character is properly normalized.

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