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A Synthetic Model of Handwriting Letters

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1. *Introduction*

K. N. Stevens first proposed the analysis by synthesis method (henceforth abbreviated to A-b-S) as a guiding principle of his speech analysis and recognition. This is one of the typical and practical techniques that execute the idea of active analysis. The first attempt to apply A-b-S scheme to handwriting and pattern recognition was made by M. Eden at MIT. He considered two alternative models of the description of cursive scripts. One was based on a set of defined primitive stroke segments analogous to the distinctive feature in speech analysis, and the other was based on a sinusoidal model of practiced writing. However, neither of these models are considered to be best to simulate the actual human handwriting. The author, first of all, examines and refines the simple dynamic model proposed by Denier van der Gon and proposes an active method for handwriting analysis and recognition.

2. *A-b-S Model of Handwriting Process*

Before describing the structure of the A-b-S model of handwriting process shown in Fig. 1, there must be some comments on the signal representation at each stage of the model. We consider two levels called roughly as "muscular" and "stroke". The representations at the muscular level are in terms of the

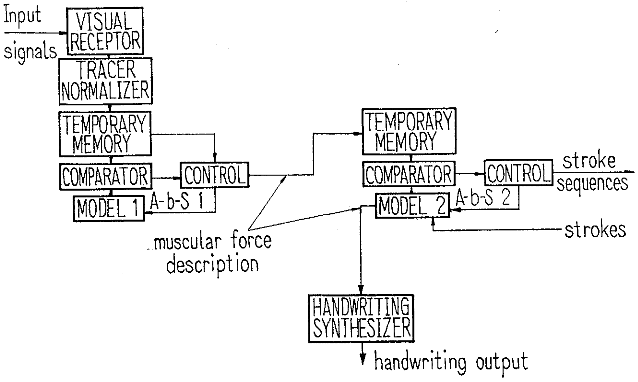

Fig. 1. The system block diagram of the proposed A-b-S model of handwriting.

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actual muscular forces applied to hand-pen couple to generate a sequence of strokes. We express these as $F_x(t)$, $F_y(t)$ in the following sections.

The first stage "A-b-S 1" in Fig. 1 is for extracting muscular force signals from a handwritten or a handwriting sample. The input samples, which may be placed in temporary storage, are compared in the comparator with signals synthesized by the model. When the best match is obtained in the comparator, the control component reads out the muscular forces which produced that match.

Since the active analysis procedure seems to have certain attractive features when applied to extracting the muscular force description, it is reasonable to examine whether the same general approach can be applied to the higher analysis stage in the model, that is, the conversion of muscular force descriptions to representations in terms of a sequence of discrete symbols—strokes. The "model 2" stores rules that relate a sequence of strokes to muscular force descriptions. The box labeled "control" determines the order in which different sequence of strokes are selected and converted to muscular force descriptions for the comparison with input data. The output is a sequence of strokes that produces a minimum error at the comparator.

Once the handwriting sample has been decoded into a sequence of strokes, these strokes must be converted to letters and grouped into words and sentences in the following stages. We shall not be concerned here with the transformation into letters, words and sentences.

3. *The Dynamic Model of Handwriting Process*

Cursive handwriting may be considered as a highly skilled process which is executed by means of a rapid sequence of motion. An important principle of the process is position feedback. Although this principle works doubtlessly in guiding human movements to a certain extent, it is well-known that it does not work for quick and well-practiced movements. Thus, Lashley concluded that in these cases, an effector mechanism can be primed to discharge at a given intensity or for a given duration, independent of any sensory control. Thus, Denier van der Gon postulated the following assumptions.

(1) The effects of position feedback can be neglected for cursive handwriting.

(2) The timings of muscle contraction determine the shape of the pattern to be generated and the magnitude of the applied muscular force does not play an important role.

He proposed the simple model of human handwriting process, in which hand-pen couple was assumed as a mass point that moved according to the following simple equation,

$$\ddot{d}+R_d\dot{d}=F_d(t) \qquad (d=x,\ y) \tag{1}$$

where R_d represents time invariant model parameter. The handwriting patterns are considered as the resultant loci of the movement of a equivalent mass point.

In order to have more refined and generalized discussions on the dynamics of human handwriting movement, consider the process schematized in Fig. 2,

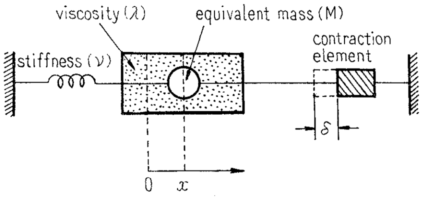


Fig. 2. The dynamic model of handwriting process.

in which the effects of stiffness, internal friction of muscle and friction force between the surface of paper and a pencil point are taken into account. The motion of a mass point, when muscle contracts by δ, is expressed by the following dynamic equation,

$$\ddot{d}+r_d\dot{d}+n_d d=h_d\delta_d=F_d(t) \qquad (d=x,\ y) \tag{2}$$

where,

$F_d(t)=f_d(t)/M$	normalized muscular force	
$r_d=l_d+m_dP(t)/v$		(3)
$l_d=\lambda_d/M$	viscosity coeff. of muscle	
$m_d=\mu_d/M$	friction coeff.	
$n_d=\nu_d/M$	stiffness coeff. of muscle	
$v=(\dot{x}^2+\dot{y}^2)^{1/2}$	writing speed	
$P(t)$	writing pressure	
M	equivalent mass.	

If the writing pressure $P(t)$ and the writing speed v are time invariant, eq. (3) become

$$r_d=l_d+m_dP/V=R_d. \tag{4}$$

Consequently, the normalized effective viscosity R_d is proved to be time invariant. And further by neglecting the stiffness term of the muscle which is actually considered to be of less importance compared with the effective viscosity term eq. (2) become

$$\ddot{d}+R_d\dot{d}=F_d(t) \qquad (d=x,\ y)$$

This is exactly identical to the model proposed by van der Gon. It is clear, therefore, that the tacit assumptions have been introduced in van der Gon's model, that the speed and pressure of a pencil point during handwriting movements are time-independent and the stiffness term of muscle is small enough to be neglected compared with the effective viscosity term.

4. *Experimental Studies of Handwriting Movements*

Handwriting analyzer is deviced to measure (x, y) position and pressure of a pencil point during handwriting movement as functions of time. Further, in an effort to gain some knowledge of muscular force during movements, some electromyographic (EMG) records of muscle activity in the forearm are taken. A pair of surface electrode is placed on the dorsal part of the forearm, that is, directly over the muscle extensor carpi ulnaris (EMG 1) and the other is placed over the muscle abductor pollicis longus (EMG 2).

Fig. 3 shows one of the records of the observed left-right displacement $x(t)$ and pressure waveform $P(t)$ of a pencil point together with 2 channels of the corresponding EMG's for a subject (a 29 years old male).

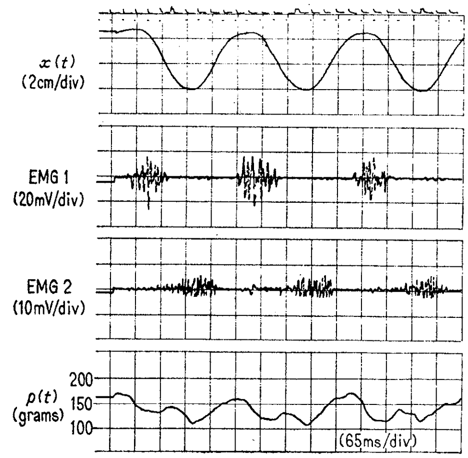


Fig. 3. Simple repetitive left-right wrist motion.

CASE A First of all, in order to estimate the optimum value of parameter R_d appearing in the van der Gon's model, eq. (1), we define the approximation error $\varepsilon_A(R_x)$ as follows,

$$\varepsilon_A(R_x)=1/T\int_T\{F_x{}^A(t,\ R_x)-M_x(t)\}^2dt \tag{5}$$

where,

$$F_x{}^A(t,\ R_x)=\ddot{x}+R_x\dot{x}$$

$M_x(t)$ is derived from the corresponding EMG's. By minimizing ε_A with respect to R_x, we can estimate the optimum value $R_x{}^*$ which is considered to be the intended parameter value. Thus, $R_x{}^*=18(T^{-1})$, $\varepsilon_A{}^*=2.5\times10^{-2}$ were obtained for the data shown in Fig. 3.

CASE B In order to evaluate the effects on handwriting movements of a pencil point pressure, consider the following model

$$\ddot{d}+m_dP(t)/v\,\dot{d}=F_d(t) \qquad (d=x,\ y) \tag{6}$$

which is derived from eq. (2) by neglecting both the stiffness and viscosity terms of the muscle. In the case of simple left-right motions, eq. (6) become

$$\ddot{x}\pm m_x P(t)=F_x{}^B(t,\ m_x) \qquad \dot{x}\gtrless 0$$

since

$$v=|\dot{x}|.$$

By minimizing the expression

$$\varepsilon_B(m_x)=1/T\int_T\{F_x{}^B(t,\ m_x)-M_x(t)\}^2dt \tag{7}$$

$m_x{}^*=2\times10^{-3}(M^{-1})$, $\varepsilon_B{}^*=1.3\times10^{-2}$ were obtained for the same data as in case A.

Fig. 4 shows the finally estimated waveferms of muscular force $F_x{}^A(t)$ and $F_x{}^B(t)$ together with the corresponding EMG signals. It might be recognized, as a whole, that although the estimated muscular force waveforms have rather good correspondence with EMG patterns for both cases, the fine structures and found different between the two. The difference between $\varepsilon_A{}^*$ and $\varepsilon_B{}^*$ for case A and for case B tells fluently the fact that the effects on handwriting movements of the writing pressure are not so small to be ignored.

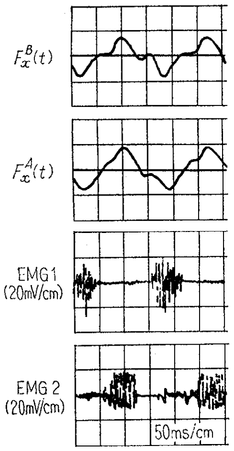


Fig. 4. Estimated waveforms of muscular force.

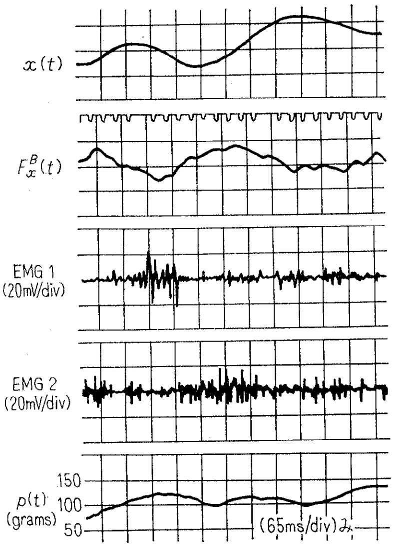


Fig. 5. Muscular force waveform calculated for writing the Japanese character "mi".

5. *Synthetic Studies of Handwriting*

It can be regarded that handwriting signals are composed of a sequence of strokes that are digital time segments. The stroke is defined as the time eliminated locus of a pencil point movement caused by a muscle contraction.

Fig. 5 shows the muscular force waveforms calculated from

$$F_x(t)=\ddot{x}+m_x P(t)/v\ \dot{x}$$

where $m_x = 2\times 10^{-3}$ for the same subject as in case A writing the Japanese letter "み", together with the corresponding EMG's. It might safely be seen from the inspections of these data that the muscular contraction causes the force with an exponential increase and decrease in amplitude. The introduction of this idea makes it possible to determine uniquely the force function by only giving the contraction timings $\tau_i's$ of muscle, at which handwriting signals are divided into a sequence of stroke segments.

Subjects are asked to write certain Japanese letter without any constraints, and the samples of (x, y) position and pressure of a pencil point during handwriting are transferred to the computer. First of all, from the data obtained by above processes, a rough guess of timings

$$\boldsymbol{\tau}_x{}^0(\tau_{x0}{}^0,\ \tau_{x1}{}^0,\ \tau_{x2}{}^0,\ \cdots),\qquad \boldsymbol{\tau}_y{}^0(\tau_{y0}{}^0,\ \tau_{y1}{}^0,\ \tau_{y2}{}^0,\ \cdots)$$

which are considered to correspond to those of muscle contractions, is made by extracting zero crossing timings of $F_d(t)$ calculated from

$$F_d(t) = \ddot{d} + m_d P(t)/v\,\dot{d} \qquad (d = x,\ y) \tag{8}$$

which is the copy of eq. (6) and the parameter m_d is assumed pre-known for each subject.

Now let $x_s(t)$, $y_s(t)$ be the synthesized position functions by the proposed model of handwriting. The integral functional $J_d(d = x,\ y)$ is introduced which evaluates the distance between the actual handwriting and the synthesized handwriting as follows,

$$J_d(\boldsymbol{\tau}_d = 1/T\int_T [\{d(t,\ \boldsymbol{\tau}_d) - d_s(t)\}^2 + \{\dot{d}(t,\ \boldsymbol{\tau}_d) - \dot{d}_s(t)\}^2]dt \tag{9}$$

Our concern here is to find the optimum timing $\boldsymbol{\tau}_d{}^*$ which minimizes the functional J_d. Iterations of A-b-S algorithms can be executed by setting the previously guessed $\boldsymbol{\tau}_d{}^0$ as a initial value of $\boldsymbol{\tau}_d$. Thus the optimally estimated value $\boldsymbol{\tau}_d{}^*$ will be obtained and its corresponding force function $F_d{}^*(t)$ will also be calculated.

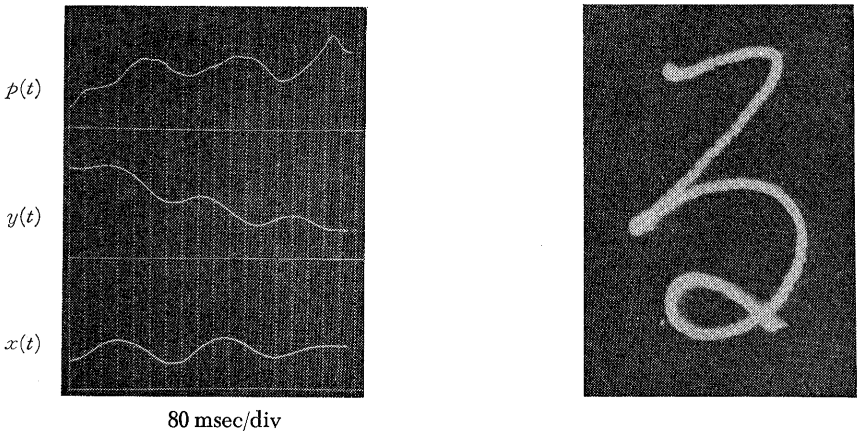
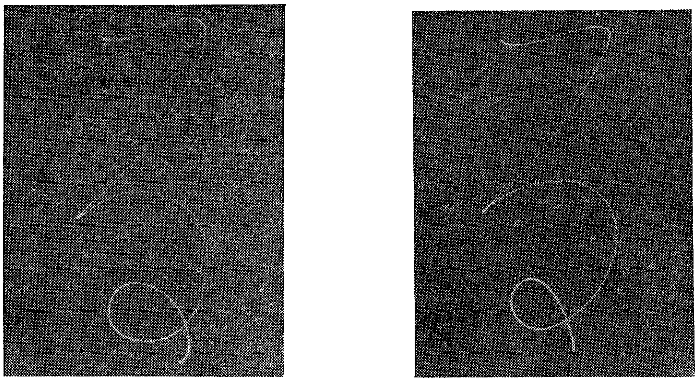


Fig. 6. Original handwriting sample.



A B

Fig. 7. Synthesized patterns.

Fig. 6 is the original handwriting that the synthesized output is supposed to match. Fig. 7 shows the synthesized patterns in which the writing speed and pressure are assumed constant in case A, but not in case B. The synthesis in case A is quite successful except for the tail of "る" where the remarkably high writing pressure is observed. Further it was found that the last stroke on the tail could not be made to have the same shape as the sample without changing the rest part so that it did not agree with the sample. On the other hand, the example in case B, for which the independent variations of the pressure are provided, is found to be matched quite well. These reveal that the constant pressure model is not completely compatible with an arbitrary sample of cursive script, that is, the writing pressure plays an important role in the human handwriting.

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