On Computational Methods for the Centroid and Spread of Character Images

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1. *Introduction*

In this paper the center line of stroke is represented by straight lines and circular arcs, and a character image is represented by thickenning this center line. In our method the centroid and spread are calculated directly from the information concerning center lines.

2. *Construction of Character Image by Segments*

Let Ω be the entire two dimensional $x-y$ plane and represent its coordinates by a column vector $\boldsymbol{r}$. A two-valued black and white character image given in the domain Ω is expressed by a function $\Phi(\boldsymbol{r})$ which takes value 1 for black portion and 0 for white portion.

Bending point, inflection point, branching point and intersection point of the center line are called junction points. A character image is decomposed into segments at junction points as shown in Fig. 1. We introduce the notations defined as follows:

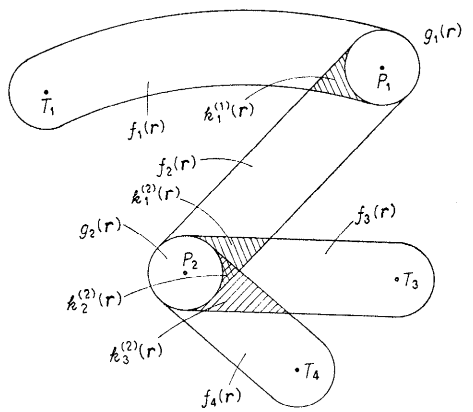

Fig. 1 Disjointing a character image at junction points

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$f_i(\boldsymbol{r})$: The i-th segment (the center line is thickened with stroke width $2w$, and end points or junction points are rounded with radius w).

$g_j(\boldsymbol{r})$: Disk with radius w, and with center at the j-th junction point.

$h_j(\boldsymbol{r})$: When segments overlap at the j-th junction point, the whole overlapped portion outside of the disk.

$k_l^{(j)}(\boldsymbol{r})$: When segments overlap at the j-th junction point, the overlapped portion outside of the disk. The symbol l is the parameter denoting its number.

M: The number of the segments composing a character.

N: The number of the junction points of a character.

n : The number of segments decomposed at the j-th junction points.

A character image $F_0(\boldsymbol{r})$ given in Ω is represented as

$$F_0(\boldsymbol{r})=\sum_{i=1}^{M} f_i(\boldsymbol{r})-\sum_{j=1}^{N}[(n_j-1)g_j(\boldsymbol{r})+h_j(\boldsymbol{r})], \tag{1}$$

where

$$h_j(\boldsymbol{r})\leqq \sum_{l=1}^{\binom{n_j}{2}} k_l^{(j)}(\boldsymbol{r}). \tag{2}$$

If we can assume that the overlapped portion of the segments outside of the disk is sufficiently small, the character image $F(\boldsymbol{r})$ can be written as

$$F(\boldsymbol{r})=\sum_{i=1}^{M} f_i(\boldsymbol{r})-\sum_{j=1}^{N}(n_j-1)g_j(\boldsymbol{r}). \tag{3}$$

In the general case of character image we can assume the overlapped portion outside of the disk to be small, so that in the following we base our discussion on the above $F(\boldsymbol{r})$.

3. *Calculation of the Centroid of Character Image*

The centroid $\boldsymbol{a}$ of a character image $F(\boldsymbol{r})$ given in the domain Ω is defined by:

$$\boldsymbol{a}\equiv\iint_{\Omega}\boldsymbol{r}F(\boldsymbol{r})d\boldsymbol{r}\Big/\iint_{\Omega}F(\boldsymbol{r})d\boldsymbol{r}.$$

Substitution of Eq. (3) into this expression yields

$$\boldsymbol{a}=\sum_{i=1}^{M}\mu_i\boldsymbol{a}_i-\nu\sum_{j=1}^{N}(n_j-1)\boldsymbol{b}_j, \tag{4}$$

where

$$\begin{cases} \boldsymbol{a}_i\equiv\iint_{\Omega}\boldsymbol{r}f_i(\boldsymbol{r})d\boldsymbol{r}\Big/\iint_{\Omega}f_i(\boldsymbol{r})d\boldsymbol{r}, \quad \boldsymbol{b}_j\equiv\iint_{\Omega}\boldsymbol{r}g_j(\boldsymbol{r})d\boldsymbol{r}\Big/\iint_{\Omega}g_j(\boldsymbol{r})d\boldsymbol{r}, \\ \mu_i\equiv\iint_{\Omega}f_i(\boldsymbol{r})d\boldsymbol{r}\Big/\iint_{\Omega}F(\boldsymbol{r})d\boldsymbol{r}, \quad \nu\equiv\iint_{\Omega}g_j(\boldsymbol{r})d\boldsymbol{r}\Big/\iint_{\Omega}F(\boldsymbol{r})d\boldsymbol{r}=\pi w^2\Big/\iint_{\Omega}F(\boldsymbol{r})d\boldsymbol{r}. \end{cases}$$

The coordinates (a_x, a_y) of the centroid of the straight line segment $f_i(x, y)$ given in the domain Ω with end points or junction points (x_m, y_m) and (x_n, y_n), are given by

$$a_x=(x_m+x_n)/2, \quad a_y=(y_m+y_n)/2. \tag{5}$$

The coordinates (a_x', a_y') of the centroid of the circular segment $f_i(x, y)$ given in the domain Ω having (x_m, y_m) and (x_n, y_n) as end points or junction points and (x_0, y_0) as the center of curvature with radius R, as shown in Fig. 2,

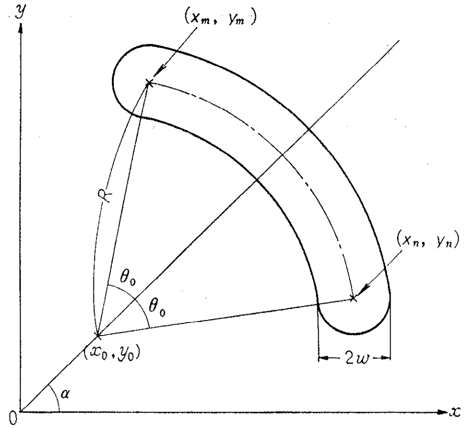

Fig. 2 Circular segment

are given by:

$$\begin{cases} a_x' = \{R(4R\sin\theta_0+\pi w\cos\theta_0)\cos\alpha/(4\theta_0 R+\pi w)\}+x_0, \\ a_y' = \{R(4R\sin\theta_0+\pi w\cos\theta_0)\sin\alpha/(4\theta_0 R+\pi w)\}+y_0. \end{cases} \tag{6}$$

4. *Calculation of the Spread of Character Image*

The spread σ^2 of a character image $F(\boldsymbol{r})$ given in a domain Ω of $x-y$ plane, is defined as

$$\sigma^2 \equiv \iint_\Omega (\boldsymbol{r}-\boldsymbol{a})^T(\boldsymbol{r}-\boldsymbol{a})F(\boldsymbol{r})d\boldsymbol{r} \Big/ \iint_\Omega F(\boldsymbol{r})d\boldsymbol{r}, \tag{7}$$

where T denotes transposition.

Transforming this expression by Eq. (3), we have

$$\sigma^2 = \sum_{i=1}^{M} \mu_i \{\sigma_i^2+(\boldsymbol{a}-\boldsymbol{a}_i)^T(\boldsymbol{a}-\boldsymbol{a}_i)\} - \nu \sum_{j=1}^{N} (n_j-1)\{\sigma^2+(\boldsymbol{a}-\boldsymbol{b}_j)^T(\boldsymbol{a}-\boldsymbol{b}_j)\}, \tag{8}$$

where $\boldsymbol{a}_i, \boldsymbol{b}_i, \mu_i$ and ν are as given in section 3, and

$$\begin{cases} \sigma_i^2 \equiv \iint_\Omega (\boldsymbol{r}-\boldsymbol{a}_i)^T(\boldsymbol{r}-\boldsymbol{a}_i) f_i(\boldsymbol{r})d\boldsymbol{r} \Big/ \iint_\Omega f_i(\boldsymbol{r})d\boldsymbol{r}, \\ \sigma^2 \equiv \iint_\Omega (\boldsymbol{r}-\boldsymbol{b}_j)^T(\boldsymbol{r}-\boldsymbol{b}_j) g_j(\boldsymbol{r})d\boldsymbol{r} \Big/ \iint_\Omega g_j(\boldsymbol{r})d\boldsymbol{r} = w^2/2. \end{cases}$$

The spread σ_i^2 of the straight line segment $f_i(x, y)$ which is given in the domain Ω with end points or junction points (x_m, y_m) and (x_n, y_n), is given by:

$$\sigma_i^2 = \{(\pi w^3)/2+4w^2 l+\pi w l^2+(4 l^3)/3\}/(\pi w+4 l). \tag{9}$$

The spread ρ_i^2 of the circular segment $f_i(x, y)$ given in the domain Ω having (x_m, y_m) and (x_n, y_n) as end points or junction points and (x_0, y_0) as the center of curvature with radius R, is given by:

$$\rho_i^2=R^2\left[1-\left(\frac{\sin\theta_0^2}{\theta_0}\right)\left(1+\left(\frac{\pi}{4\theta_0}\right)\left(\frac{w}{R}\right)\right)^{-1}\cdot\left\{1-\left(\frac{\pi}{4\theta_0}\right)\left(\frac{w}{R}\right)\right.\right.$$
$$\left.\times\left\{(1-2\theta_0\cot\theta_0)-(\theta_0\cot\theta_0-1)^2\left(\frac{\pi}{4\theta_0}\right)\left(\frac{w}{R}\right)\left(1+\left(\frac{\pi}{4\theta_0}\right)\left(\frac{w}{R}\right)\right)\right\}\right\}$$
$$\left.+\left(\frac{w}{R}\right)^2\left\{\left(1+\frac{1}{2}\left(\frac{w}{4\theta_0}\right)\left(\frac{w}{R}\right)\right)\left(1+\left(\frac{w}{4\theta_0}\right)\left(\frac{w}{R}\right)\right)^{-1}\right\}\right]. \tag{10}$$

5. *Error Caused by Neglecting the Overlap of Segments*

5.1 *Error in Centroid*

When the overlapped portion cannot be neglected, the centroid of a character image $F_0(\boldsymbol{r})$ is given by $\boldsymbol{a}_0$. The error in centroid is defined by $\varepsilon_a=|\boldsymbol{a}_0-\boldsymbol{a}|$, and becomes

$$\varepsilon_a\leqq\sum_{j=1}^{N}\sum_{l=1}^{\binom{n_j}{2}}(|\boldsymbol{a}|+\lambda)\iint_\Omega k_l^{(j)}(\boldsymbol{r})d\boldsymbol{r}\Big/\iint_\Omega F(\boldsymbol{r})d\boldsymbol{r}, \tag{11}$$

where λ is the maximum of absolute values of centroid of the overlapped portion.

There are three types of segment decomposition at junction points: (straight line-straight line), (straight line-circular arc) and (circular arc-circular arc). Let us consider the first case.

Let two center lines join at point (x_j, y_j) with angle γ as shown in Fig. 3,

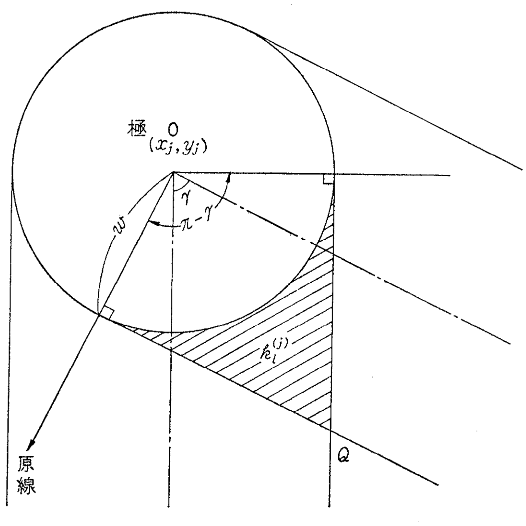

Fig. 3 Piled part of segments

and let $k_l^{(j)}(\boldsymbol{r})$ be the overlapped portion. Let $\psi(r,\theta)$ be the function $k_l^{(j)}(\boldsymbol{r})$ expressed in polar coordinates (r,θ) with the pole at (x_j, y_j) and the origin line as shown in the figure. Since the overlapped portion is symmetric with respect to the line $\overline{OQ}$, we have

$$\iint_{\Omega} k_l^{(j)}(\boldsymbol{r})d\boldsymbol{r} = 2\int_0^{\frac{\pi-\gamma}{2}} d\theta \int_0^{B(\theta)} r dr = w^2\left(\cot\frac{\gamma}{2} - \frac{\pi-\gamma}{2}\right), \tag{12}$$

where

$$B(\theta) = \begin{cases} r/\cos\theta : \ 0 \leqq \theta \leqq (\pi-\gamma)/2, \\ r/\cos(\pi-\gamma-\theta) : \ (\pi-\gamma)/2 \leqq \theta \leqq \pi-\gamma. \end{cases}$$

5.2 *Error in Spread*

When the overlapped portion cannot be neglected, the spread of a character image $F_0(\boldsymbol{r})$ is given by σ_0^2. The error ε_{σ^2} in the spread is defined by $\varepsilon_{\sigma^2} \equiv |\sigma_0^2 - \sigma^2|$.

Let K be the maximum value of the spread of the overlapped portion, and let D be the maximum value of the distance between the centroid $\boldsymbol{a}$ of $F(\boldsymbol{r})$ and the centroid $\boldsymbol{c}_l$ of the overlapped portion. Then we have, assuming that $\boldsymbol{a}_0 \equiv \boldsymbol{a}$,

$$\varepsilon_{\sigma^2} \leqq \frac{1}{\iint_{\Omega} F(\boldsymbol{r})d\boldsymbol{r}} \sum_{j=1}^{N} \sum_{l=1}^{\binom{n_j}{2}} (\sigma^2 + K + D^2) \iint_{\Omega} k_l^{(j)}(\boldsymbol{r})d\boldsymbol{r}. \tag{13}$$

6. *Conclusion*

The present method of calculation of the centroid and the spread of a character image seems to give more exact values in shorter time than the traditional method in which the centroid and the spread are computed after the character pattern is generated in advance.