# Pattern Recognition by the Codes of 3×3 Elements with Karhunen-Lóeve Orthogonal System

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*Abstract*

A method is developed for pattern recongnition by the use of Karhunen-Lóeve orthogonal system applied to 3×3 element codes. Computer-simulated experiments are carried into effect in order to test the usefulness of the present method.

## 1. *Introduction*

A method is proposed for pattern recognition, in which 3×3 element codes are used and Karhunen-Lóeve orthogonal system are applied to them. Karhunen-Lóeve system have been studied as a method for pattern recognition in [1]~[3], and some experimental results were reported in [4]~[5]. A new method, however, is proposed here, of which procedures are as follow. (1) Input patterns are lowered in dimension by a preprocessor to give 3×3 codes. (2) Feature-coefficients are given by applying Karhunen-Lóeve method to these codes, where the concept of auto-correlation is used. (3) When an unknown pattern is given, it is also lowered in dimension and then recognized by projecting it to the patterns of feature-coefficients thus obtained. It is essential for the present method to eliminate unnecessary part of information from a given pattern and to reformed it into an intrinsic pattern or code of a lower dimension. The degree of dependence is studied for the set of 3×3 codes, since the percentage of correct recognition is governed by the method of their constitution. The relation between the recognition rate and the degree of dependence is also given.

## 2. *Pre-Processing*

When a given pattern is written in a 20×20 square for example, it becomes 400 dimensional vector so that its autocorrelation is represented by a 400×400 matrix. According to this cause, a great deal of memory-capacity and processing time are required to obtain the feature-coefficients from this matrix. For this reason, a method of calculating the auto-correlation matrix in much lower dimension should be developed, even if the dimensionality of the original pattern is large.

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Now, a pattern of a 15×15 quantized mesh is scanned and its geometric parts (edge points, turning points) which coincide with prescribed 3×3 code patterns are extracted, where original patterns and code patterns take on the value 1 or 0 on the respective meshes. The set of 3×3 codes are generally selected by designer's experience. The reader who wants to know the details of the adopted code patterns can refer to the reference [6]. Now, let $C$ be a set of extracted features, denoted by

$$C=\{C^{(1)}, C^{(2)}, \cdots, C^{(S)}, \cdots, C^{(q)}\} \tag{1}$$

where $C^{(S)}$ is a 3×3 matrix, $q$ is the number of extracted code elements, $C$ containing the inherent information of the pattern. By a direct rearrangement, the code $C^{(S)}$ may be expressed as follow,

$$y_S=(C_{11}^{(S)}, C_{12}^{(S)}, C_{13}^{(S)}, \cdots, C_{31}^{(S)}, C_{32}^{(S)}, C_{33}^{(S)}) \tag{2}$$

$y_S$ containing the same information as $C^{(S)}$. Still more, if a matrix Y is formed by these $y_S$ 's as follow.

$$y_S=\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_q \end{bmatrix}=\begin{bmatrix} C_{11}^{(1)}, C_{12}^{(1)}, \cdots\cdots, C_{33}^{(1)} \\ C_{11}^{(2)}, C_{12}^{(2)}, \cdots\cdots, C_{33}^{(2)} \\ \vdots \quad \vdots \quad \vdots \\ C_{11}^{(q)}, C_{12}^{(q)}, \cdots\cdots, C_{33}^{(q)} \end{bmatrix}=\begin{bmatrix} y_{11}, y_{12}, \cdots\cdots, y_{1l} \\ y_{21}, y_{22}, \cdots\cdots, y_{2l} \\ \vdots \quad \vdots \quad \vdots \\ y_{q1}, y_{q2}, \cdots\cdots, y_{ql} \end{bmatrix} \tag{3}$$

where $y_{s,\, 3(i-1)+j}=C_{ij}^{(S)}(i, j=1\sim3)$, then this matrix $Y$ becomes an inherent matrix whose elements take on the value 1 or 0. Furthermore, a simplified pattern should be reformed from matrix $Y$ so as not to lose the characteristics or tendency of the pattern as much as possible. If component $x_i$ of the simplified pattern is defined by

$$x_i=\sum_{s=1}^{q} y_{si} \tag{4}$$

the $X$ is denoted by

$$X=[x_1\ x_2, \cdots, x_l]=\left[\sum_{s=1}^{q} y_{s1}, \sum_{s=1}^{q} y_{s2}, \cdots, \sum_{s=1}^{q} y_{sl}\right] \tag{5}$$

where $l$ is the dimensionality of $X$. This reformed vector $X$ which is obtained by summing up the features in $Y$ seems to preserve the characteristic configuration and tendency of the pattern. In other word, the vector represents a version of the pattern which is lowered in dimension. The vector will contain sufficient information of Eq. (1). Next, by a simple normalization, a normalized low-dimensional pattern vector $X$ is obtained and this vector is called the reformed pattern vector.

## 3. *Theory*

The basic idea of Karhunen-Lóeve system consists in expanding a pattern vector with a set of orthonormal vectors in order to extract the necessary information as much as possible. Let $M$ and $N$ be the numbers of the pattern classes and the typical pattern, respectively. Then, the set of typical input pattern is denoted by

$$X_N = \{X_m^{(n)} \mid m=1\sim M,\ n=1\sim N\} \tag{6}$$

Moreover, let $B_m$ be a set of $k$ orthonormal vectors

$$B_m = \{\beta_m^{(i)} \mid i=1,\ 2\cdots\cdots k\} \tag{7}$$

$$(\beta_m^{(i)},\ \beta_m^{(j)}) = \delta_m^{(i,j)},\ \delta_m^{(i,j)}:\ Kronecker's\ \delta \tag{8}$$

defined for each pattern class $m$, where $k$ is the number of dimension of vectors. An input pattern is generally expanded by this $B_m$ as follow,

$$X_m^{(n)} = am_1^{(n)}\cdot\beta_m^{(1)} + a_{m2}^{(n)}\cdot\beta_m^{(2)} + \cdots + a_{mk}^{(k)} \tag{9}$$

where

$$a_{mi}^{(n)} = (\beta_m^{(i)}\cdot X_m^{(n)}),\ i=1\sim k \tag{10}$$

In the Karhunen-Lóeve expansion, the base $B_m$ is chosen with reference to the stochastic properties of patterns belonging to class $m$. Let $P_m(X)$ be probability of occurrence of $X$ in calss $m$. Then the auto-correlation matrix is given by

$$\mathrm{G}_m = \sum_{n=1}^{N} P_m(X_m^{(n)})\cdot X_m^{(n)}\cdot X_m^{(n)t} \tag{11}$$

for the pattern of class $m$. The set $B_m$ of the eigen vectors of Eq. (11) is called the Karhunen-Lóeve orthogonal system or shortly *K-L* system.

## 4. *Method for Decision*

In the expansion Eq. (9) of a pattern belong to class $m$ by the *K-L* system $\{\beta_m^{(i)}\}$, the coefficient $a_{ms}^{(i)}$ are expected to vanish approximately,

$$a_{m,\,r+1}^{(n)} = a_{m,\,r+2}^{(n)} = \cdots = a_{m,\,k}^{(n)} = 0 \tag{12}$$

for $s$ larger than $r$, where $r$ is a certain integer ($r \geq 1$). But this is not the case, if a pattern $X_m^{(n)}$ is expanded by the system $B'(m' \neq m)$. Therefore, the absolute value of an unknown pattern $X$

$$T_m(X) = \sum_{i=1}^{k} (a_{mi})^2 \qquad a_{mi} = (\beta_m{}^i\cdot X) \tag{13}$$

is nearly equal to

$$T_m(X) = \sum_{i=1}^{r} (a_{mi})^2 \tag{14}$$

if $X$ belongs to class $m$, but is much less than $T_m(X)$, if $X$ does not belong to class $m$. Hence, $T_m(X)$ plays the role of a discriminant function for class $m$. Let

$$d(X) = \max_{m} \{T_m(X) \mid m=1\sim M\} \tag{15}$$

An unknown pattern $X$ is categorized into class $m$, if $T_m(X)$ has the largest value $d(X)$.

## 5. *The Degree of Dependence*

The vector $y_S$, the rearranged $3\times3$ code $C^{(S)}$, has only 9 components. For this reason, it is impossible that all the $3\times3$ codes are formed independently to one another and all the necessary features are extracted by these codes. But it

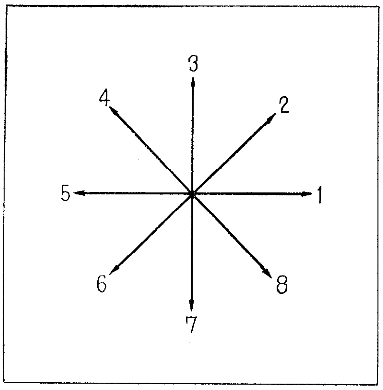


Fig. 5. 1. The direction of eight edge points.

Table 5. 1. The result of Iij.

| Case \ Iij | I($E_1$, $E_1$) | I($E_1$, $E_2$) | I($E_1$, $E_3$) | I($E_2$, $E_2$) | I($E_2$, $E_3$) | I($E_3$, $E_3$) |
|---|---|---|---|---|---|---|
| (a) | 0. 5000 | 0. 2835 | 0. 4681 | 0. 5245 | 0. 4697 | 0. 5183 |
| (b) | 0. 1786 | 0. 4328 | 0. 3770 | 0. 5245 | 0. 4697 | 0. 5183 |
| (c) | 0. 2143 | 0. 4110 | 0. 2499 | 0. 5245 | 0. 4697 | 0. 5183 |

is desirable condition for a system of pattern recognition that all features are independent, since non-redundant codes can carry more information than any other. The fact is obvious from information theory.

Now, the degree $I_{ij}$ of dependence is defined. It is explained in the below that the smaller the value becomes, the better the percentage of correct recognition becomes (see Fig. 6.1). Let us divide the set $E$ of vector $y_S$ into 3 subclasses

$$E=\{E_1, E_2, E_3\} \tag{16}$$

where $E_1$, $E_2$, $E_3$ express the sets of code vectors representing edge points, turning points of two arcs and turning points of three arcs, respectively. Then, $I_{ij}$ is defined by

$$I_{ij}=\sum_{s=1}^{l} y_{is}\cdot y_{is} \Big/ \sqrt{\sum_{s=1}^{l} y_{is}^2}\cdot\sqrt{\sum_{s=1}^{l} y_{js}^2} \tag{17}$$

From this definition, $I_{ij}$ is calculated in the following three cases (see Fig. 5. 1).

(a) All 8 edge points are extracted at the center of 3×3 codes.

(b) 4 edge points (1, 3, 5, 7) of 8 edge points are extracted in the place far from the center of 3×3 codes.

(c) All 8 edge points are extracted in the place far from the center of 3×3 codes.

The direction of edge points are shown in Fig. 5.1 and the result of $I_{ij}$ computed for the sets $E_1$, $E_2$, $E_3$ is given under notation $I(E_i, E_j)$ in Table 5.1.

## 6. *Experimental Results*

To illustrate the above method, computer-simulated experiments were carried out. Handwritten English characters $A \sim H$ were given as pattern classes. The examples of the reformed patterns, obtained by Eq. (1)-(6) are shown in Table 6.1. Next, in order to calculate the *K-L* system, the following two probability distributions are assumed.

$$(1) \quad P_m(X) = {}^1/_{10}, \; N = 10 \tag{18}$$

$$(2) \quad P_m(X) = {}^1/_{15}, \; N = 15 \tag{19}$$

where $N$ is the number of typical patterns. The *K-L* system obtained from $p_m(X)$ is given in Table 6.2, where the vectors corresponding to the largest eigenvalue only are shown ($r=1$). Next, two recognition experiments were carried out in the case of 40 and 72 unknown samples. Some experimental results are shown in Table 6.3. Furthermore the same recognition was done for six classes

Table 6.1. Examples of reformed patterns, Case (b).

| Input Pattern \ Reformed Pattern | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Normalized factor |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 4 | 2 | 2 | 4 | 4 | 2 | 0 | 2 | 64 |
| B | 2 | 1 | 0 | 3 | 3 | 2 | 2 | 0 | 1 | 32 |
| C | 1 | 0 | 0 | 0 | 2 | 0 | 1 | 0 | 0 | 6 |
| D | 1 | 1 | 2 | 1 | 0 | 1 | 1 | 1 | 1 | 11 |
| E | 5 | 3 | 0 | 2 | 2 | 1 | 3 | 1 | 2 | 57 |
| F | 3 | 3 | 2 | 1 | 2 | 2 | 2 | 0 | 0 | 35 |
| G | 1 | 1 | 3 | 2 | 2 | 2 | 0 | 2 | 1 | 28 |
| H | 0 | 1 | 3 | 3 | 2 | 4 | 3 | 0 | 1 | 49 |

Table 6.2. Eigen Vectors (KL-System), Case (b).

| | | | |
|---|---|---|---|
| +1.91801E-01 | +4.98161E-01 | +5.59614E-01 | +5.32614E-01 |
| +2.53885E-01 | +2.49365E-01 | +4.92518E-01 | +1.23169E-01 |
| +2.41413E-01 | +1.62962E-01 | +2.26123E-02 | +1.28848E-01 |
| +2.66773E-01 | +4.92527E-01 | +7.08461E-02 | +4.96246E-01 |
| +7.30277E-01 | +3.27994E-01 | +4.70579E-01 | +6.98754E-02 |
| +4.40573E-01 | +1.87574E-01 | +0.00000E-51 | +7.47139E-02 |
| +1.32524E-01 | +4.67389E-01 | +3.85785E-01 | +5.02242E-01 |
| +0.00000E-51 | +1.42994E-01 | +0.00000E-51 | +3.05037E-01 |
| +1.55851E-01 | +1.97141E-01 | +2.61637E-01 | +2.87306E-01 |
| +6.33643E-01 | +5.77212E-01 | +5.42263E-01 | +1.30987E-01 |
| +4.76904E-01 | +4.99577E-01 | +3.88006E-01 | +1.00459E-01 |
| +7.46943E-02 | +3.17013E-01 | +3.64956E-01 | +1.74188E-01 |
| +3.24157E-01 | +2.81170E-01 | +1.06935E-01 | +3.54945E-01 |
| +2.07144E-01 | +2.15659E-01 | +5.10652E-01 | +3.39288E-01 |
| +1.66305E-01 | +3.03352E-01 | +2.65469E-01 | +7.15699E-01 |
| +3.87854E-01 | +3.11123E-01 | +1.28105E-01 | +3.21576E-01 |
| +1.66619E-01 | +3.46083E-02 | +1.78933E-01 | +1.53350E-01 |
| +1.07997E-01 | +3.44624E-02 | +1.76418E-01 | +2.49329E-01 |

Table 6.3. The percentage of correct recognition.

| Case \ Pm(X) | Probability | | the number of sample patterns |
|---|---|---|---|
| | $Pm(X)={}^1/_{10}$ | $Pm(X)={}^1/_{15}$ | |
| (b) | 80 (%) | 82.5 (%) | 40 |
| (c) | 88 (%) | 93.1 (%) | 72 |

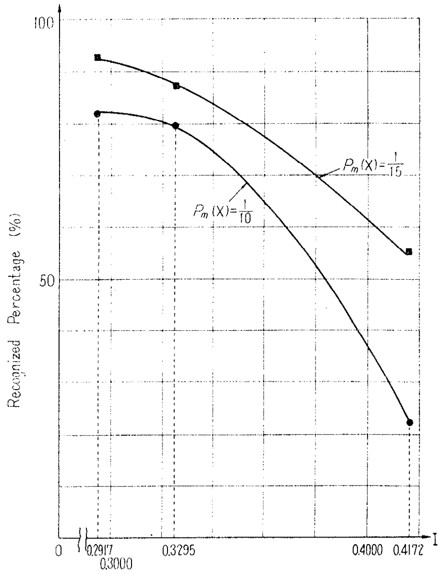


Fig. 6.1. The relation between the degree of dependence and recognized percentage.

$A \sim F$ (and 42 samples) and the correct recognition rate 94.8 per cent was obtained. This result is comparable with the correct recognition rate 98.0 per cent by human beings. Consequently, it is concluded that result is fairly good. The percentage of correct recognition is also plotted for $I_{ij}$, in Fig. 6.1, where $I = {}^1/_3 \cdot \sum_{j=1}^{3} I(E_1, E_j)$. By this Fig. 6.1, the correctness of the discussion in section 5 is proved.

## 7. *Conclusions*

By the above results, the following three conclusions were obtained.

(1) A desirable percentage of correct recognition was obtained by the present mothod, which reduced given patterns in dimension by pre-processing.

(2) The processing-time and memory capacity were reduced sufficiently.

(3) Desirable result were obtained by choosing a set of independent codes.

By conclusions (1) and (2), it is known that this method is more practical than the method studied earlier. A guide how to form a set of 3×3 codes is given by conclusion (2).

8. *Acknowledgment*

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