On a Method for Extraction of Frequency Distribution Characteristic from Nonstationary Random Process by Digital Filter

TETSUO FUNADA* AND KAZUO IKEGAYA**

As a method of vocal sound analysis, Short-time power spectrum $PS(\omega, t)$ is generally used. $PS(\omega, t)$ is figured on two dimensional plane where horizontal axis is time and vertical axis is frequency. $PS(\omega, t)$ is quite effective for describing the feature of nonstationary random signal.

In this paper, two parameters based on $PS(\omega, t)$ have been introduced to extract the feature of nonstationary random signal. One is Short-time average frequency $\nu(t)$, which is defined as the square root of the second moment of $PS(\omega, t)$ with regard to ω and it may correspond to center frequency of $PS(\omega, t)$. The other is Short-time variance frequency $\gamma(t)$, which is defined as the fourth root of the fourth moment of $PS(\omega, t)$ about $\nu(t)$, and it may correspond to band width of $PS(\omega, t)$.

The purpose of this paper is to present a method for calculating these parameters effectively by digital filter and to show the results obtained by applying this method to nonstationary random signal.

Supposing that $x_0(t)$ is an ergodic random signal, then its autocorrelation function $R_0(\tau)$ and its power spectrum density $S_0(\omega)$ are formularized, respectively, as follows (1),

$$\left.\begin{aligned} R_0(\tau) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_0(\omega)\cos\omega\tau d\omega \\ S_0(\omega) &= \int_{-\infty}^{\infty} R_0(\tau)\cos\omega\tau d\tau \end{aligned}\right\} \tag{1}$$

where

$$\left.\begin{aligned} R_0(\tau) &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x_0(t)x_0(t+\tau)dt \\ S_0(\omega) &= \lim_{T\to\infty}\frac{1}{2T}\left|\int_{-T}^{T} x_0(t)e^{-j\omega t}dt\right|^2 \end{aligned}\right\} \tag{2}$$

If the average of $x_0(t)$ is zero, expected zero crossing number with plus slope in unit time is given as (2)

This paper first appeared in Japanese in Joho-Shori (Journal of the Information Processing Society of Japan), Vol. 13, No. 1 (1972), pp. 22~28.

* Faculty of Technology, Kanazawa University
** Faculty of Engineering, Nagoya University

$$\nu_0=\frac{1}{2\pi}\sqrt{\frac{\int_{-\infty}^{\infty}\omega^2 S_0(\omega)d\omega}{\int_{-\infty}^{\infty}S_0(\omega)d\omega}}=\frac{1}{2\pi}\sqrt{\frac{R_0''(\tau)|_{\tau=0}}{R_0(\tau)|_{\tau=0}}} \tag{3}$$

Authors define γ_0 as a parameter expressing spectrum width of $x_0(t)$ about ν_0, and γ_0 can be presented as

$$\gamma_0=\frac{1}{2\pi}\sqrt{\frac{\int_{-\infty}^{\infty}(\omega^2-4\pi^2\nu_0^2)^2 S_0(\omega)d\omega}{\int_{-\infty}^{\infty}S_0(\omega)d\omega}} \tag{4}$$

Provided that $x(t)$ is nonstationary random signal, then Short-time autocorrelation $R(\tau, t)$ and Short-time power spectrum $S(\omega, t)$ are (3)

$$R(\tau, t)=\int_{-\infty}^{\infty}x(s)x(s-|\tau|)w_2(t-s)ds \tag{5}$$

$$S(\omega, t)=\left|\int_{-\infty}^{\infty}x(s)w_1(t-s)e^{-j\omega s}ds\right|^2 \tag{6}$$

where $w_1(t)$, $w_2(t)$ are window functions; setting them as follows,

$$w_1(t)=\begin{cases}0 & (t<0)\\ e^{-\alpha t} & (t\geqq 0)\end{cases},\quad w_2(t)=\begin{cases}0 & (t<0)\\ e^{-2\alpha t} & (t\geqq 0)\end{cases}\qquad \alpha>0 \tag{7}$$

the relation between $R(\tau, t)$ and $S(\omega, t)$ becomes (4)

$$R(\tau, t)=\frac{e^{\alpha|\tau|}}{2\pi}\int_{-\infty}^{\infty}S(\omega, t)\cos\omega\tau d\omega \tag{8}$$

or

$$S(\omega, t)=2\int_0^{\infty}e^{-\alpha\tau}R(\tau, t)\cos\omega\tau d\tau \tag{9}$$

Let Short-time power $P(t)$ be given as

$$P(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}S(\omega, t)d\omega \tag{10}$$

According to eqs. (5), (8), eq. (10) can be written as

$$P(t)=R(0, t)=\int_{-\infty}^{\infty}[x(s)]^2 w_2(t-s)ds \tag{11}$$

In obtaining $P(t)$ by digital operation, calculation is convenient by eq. (11) rather than eq. (10). In general, the relation between $w_1(t)$ and $w_2(t)$ is obtained by Parseval's formula as eq. (12) in order that eq. (11) hold.

$$w_2(t)=[w_1(t)]^2 \tag{12}$$

Dealing with nonstationary random signal, parameters $\nu(t)$, $\gamma(t)$ are defined as eqs. (13), (14).

They may correspond to eqs. (3), (4), respectively.

$$\nu(t)=\frac{1}{2\pi}\sqrt{\frac{\int_{-\infty}^{\infty}\omega^2 S(\omega, t)d\omega}{\int_{-\infty}^{\infty}S(\omega, t)d\omega}} \tag{13}$$

$$\gamma(t)=\frac{1}{2\pi^4}\sqrt{\frac{\int_{-\infty}^{\infty}[\omega^2-4\pi^2\nu^2(t)]^2S(\omega,t)d\omega}{\int_{-\infty}^{\infty}S(\omega,t)d\omega}}=\frac{1}{2\pi^4}\sqrt{\frac{\int_{-\infty}^{\infty}\omega^4S(\omega,t)d\omega}{\int_{-\infty}^{\infty}S(\omega,t)d\omega}-16\pi^4\nu^4(t)} \quad (14)$$

$\nu(t)$ and $\gamma(t)$ is named Short-time average frequency and Short-time variance frequency respectively. In order to calculate $\nu(t)$, $\gamma(t)$ according to eqs. (13), (14). the Short-time power spectrum $S(\omega, t)$ should be known.

Getting two dimensional $S(\omega, t)$, however, need a large amount of calculation. To avoid this complication, an approach to obtaining $\nu(t)$, $\gamma(t)$ without calculating any $S(\omega, t)$ has been developed as follows.

First, $\int S(\omega, t)d\omega$ in denominator of eqs. (13), (14) is substituted by eq. (11), if eq. (12) is satisfied.

Next, $\int\omega^2S(\omega, t)d\omega$ is expressed by following relation

$$\int_{-\infty}^{\infty}\omega^2S(\omega,t)d\omega=\int_{-\infty}^{\infty}[x'(s)]^2w_2(t-s)ds$$
$$-2\int_{-\infty}^{\infty}[x(s)x'(s)]h_1(t-s)ds+\int_{-\infty}^{\infty}[x(s)]^2h_2(t-s)ds \quad (15)$$

where

$$\left.\begin{aligned}h_1(t)&=w_1(t)w_1'(t)\\h_2(t)&=[w_1'(t)]^2\end{aligned}\right\} \quad (16)$$

Similarly,

$$\int_{-\infty}^{\infty}\omega^4S(\omega,t)d\omega=\int_{-\infty}^{\infty}[x''(s)]^2w_2(t-s)ds-4\int_{-\infty}^{\infty}[x''(s)x'(s)]h_1(t-s)ds$$
$$+4\int_{-\infty}^{\infty}[x'(s)]^2h_2(t-s)ds+2\int_{-\infty}^{\infty}[x''(s)x(s)]h_3(t-s)ds \quad (17)$$
$$-4\int_{-\infty}^{\infty}[x'(s)x(s)]h_4(t-s)ds+\int_{-\infty}^{\infty}[x(s)]^2h_5(t-s)ds$$

where

$$\left.\begin{aligned}h_3(t)&=w_1(t)w_1''(t)\\h_4(t)&=w_1'(t)w_1''(t)\\h_5(t)&=[w_1''(t)]^2\end{aligned}\right\} \quad (18)$$

The condition that $w_1(t)$ has second derivative and Fourier integral must be satisfied. Setting

$$w_1(t)=\begin{cases}0 & (t<0)\\ t^2e^{-\omega_1 t} & (t\geqq 0),\ \omega_1>0\end{cases} \quad (19)$$

$w_2(t)$ becomes eq. (20), according to eq. (12)

$$w_2(t)=\begin{cases}0 & (t<0)\\ t^4e^{-2\omega_1 t} & (t\geqq 0)\end{cases} \quad (20)$$

Let $x(n)$ to be sampling value of nonstationary signal $x(t)$ at $t=nT$, where T is sampling time. In calculating eqs. (11), (15), (17) by digital filter, digital differentiation and convolution operation are appeared.

1) Digital differentiation is executed by the digital differential filter whose transfer function $\overline{H}_D(z^{-1})$ is

$$\bar{H}_D(z^{-1})=\frac{\sum_{n=0}^{5} a_n z^{-n}}{1+b_1 z^{-1}} z^2 \tag{21}$$

where

$$\begin{cases} a_0=-a_5=-q/16, \ a_1=-a_4=7q/16 \\ a_2=-a_3=q(p+1)/2, \ b_1=e^{-\alpha T} \end{cases} \tag{22}$$

Setting $q=1/2.4$, $p=4.0$, $\alpha T=0.8$, frequency characteristic of $|\bar{H}_D(e^{-j\omega t})|$ has been presented in Fig. 1, where linear portion extends to 90% of the folding frequency $1/2T$. Time series $x(l)$ and its differentiated time series $x'(l)$ are related by following digital filter.

$$x'(l)=\sum_{n=0}^{5} a_n x(l-n)-b_1 x'(l-1) \tag{23}$$

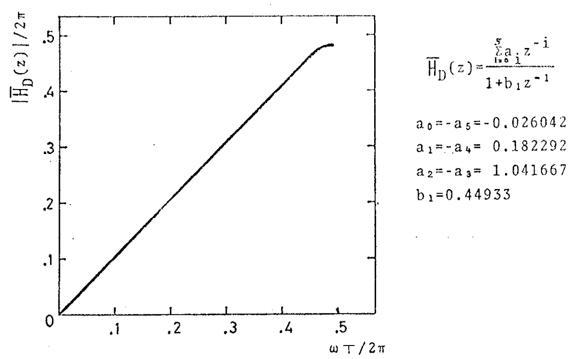

Fig. 1. Amplitude Characteristic of Digital Differential Filter.

2) The convolution operation in eqs. (11), (15), (17) are equivalent to smoothing operation by digital filter, whose transfer function $\bar{H}_1(z^{-1})\sim\bar{H}_5(z^{-1})$ and $\bar{W}_2(z^{-1})$ are obtained respectively by applying Standard-z-transform to impulse response $h_1(t)\sim h_5(t)$ and $w_2(t)$. Because each impulse response is a linear combination of the terms $t^m e^{-2\omega_1 t}$, each z-transform has been expressed as following form,

$$\frac{\sum_{i=0}^{4} A_i z^{-i}}{1+\sum_{i=1}^{5} B_i z^{-i}} \tag{24}$$

where B_i are common to each filter. Each integral in eqs. (11), (15), (17) is substituted by digital operation, for example eq. (11) becomes

$$P(l)=\sum_{i=0}^{4} A_i[x(l-i)]^2-\sum_{i=1}^{5} B_i P(l-i) \tag{25}$$

From the above investigation, digital procedure of calculating $\nu(l)$, $\gamma(l)$ which are Short-time average frequency and Short-time variance frequency, respectively, at time $t=lT$ has been obtained.

Each wave shown in Fig. 2, Fig. 3 is a shape of the signal whose spectrum distribution is single tuned and changing in center frequency or band width as the time goes.

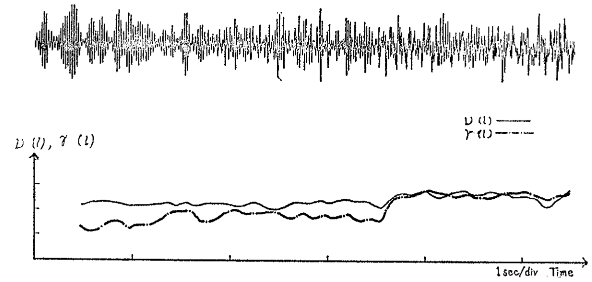

Fig. 2. Short-Time Average Frequency $\nu(l)$, Short-Time Variance Frequency $\gamma(l)$.

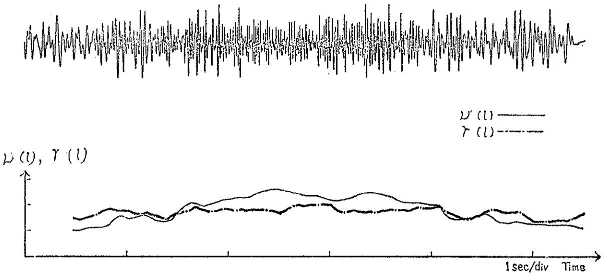

Fig. 3. Short-Time Average Frequency $\nu(l)$, Short-Time Variance Frequency $\gamma(l)$.

As for the data in Fig. 2, center frequency is fixed and band width is increasing as the time proceeds. Similarly, in the case of Fig. 3, center frequency is increasing at first and decreasing finally as the time changes and band width is constant. In both example, these features are extracted by the $\nu(l)$, $\gamma(l)$.

In processing nonstationary random signal, amplitude, envelope, zero crossing and Short-time power spectrum are usually available. In this paper, it has been shown that the Short-time average frequency and Short-time variance frequency are also effective parameters to specify the feature of frequency distribution of nonstationary random signal.

References

[1] R. B. Blackman and J. W. Tukey: The measurement of power spectra, Dover Publications Inc., New York, 1959.

[2] S. O. Rice: Mathematical analysis of random noise, *Bell Syst. Tech. J.*, Vol. 24, 1945.

[3] J. L. Flanagan: Speech analysis, synthesis and perception, Springer-Verlag, 1965.

[4] R. M. Fano: Short-time autocorrelation functions and power spectra, *J. Acoust. Soc. Am.*, Vol. 22, No. 5, 1950.