# Minimum Dissection of LSI Artwork Patterns

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Abstract

This paper is concerned with the problem of dissecting a plane figure, which is bound by horizontal and vertical lines, into a set of rectangles. A simple formula which gives the minimum number of rectangles to dissect a plane figure is presented. A computer algorithm for obtaining such a dissection is also presented. As an application to the artwork generation of LSI, this algorithm provides a simple procedure to expose a desired figure using a photohead pattern generator.

## 1. Introduction

The fabrication of LSI's requires the delineation of precisely defined patterns in various materials in order to obtain the required functional performance of the device. A pattern generator is a computer controlled lightscanning system used for the delineation of such patterns on a sensitive film called a mask. This system delineates a given pattern on a mask through a rectangular light sopt of variable sizes.[3]

As shown in Fig.1, a mask pattern is a plane figure bound by horizontal and vertical line segments. In general, its boundary consists of one external polygonal curve and a number of internal ones.

As long as a pattern generator of the above mentioned type is used, a mask pattern must be dissected into a set of non-overlapping rectangles. It should be noted here that the overexposure of sensitive film due to the overlap of rectangles can decrease the precision of masks.

In order to reduce the running time of such a pattern generator, the problem of minimizing the number of rectangles which cover a given plane figure is of great importance. This paper presents a formula which gives the minimum number of rectangles to constitute a given mask pattern unless it is "degenerate". By a "degenerate" mask pattern, we mean one whose boundary includes two or more separate line-segments having the same

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x- or y- coordinates. An algorithm for dissecting a mask pattern into the minimum number of rectangles is also presented.

## 2. Mathematical Model of Mask Patterns

A rectangular cellular complex [1,2] is a finite set of rectangular domains defined in the plane such that no two of them overlap except at their boundaries, and the image of a rectangular cellular complex is the union of all points belonging to these rectangular domains. In what follows a mask pattern refers to the image of a rectangular cellular complex as defined above. This mathematical model represents almost all plane figures that appear in LSI masks. And we assume, without loss of the generality, that a mask pattern being considered is a connected set.

For a mask pattern P, its boundary is denoted by $\partial P$. In general, $\partial P$ consists of a number of connected components, and each of them is a closed polygonal curve consisting of horizontal and vertical line segments. Thus, each component of $\partial P$ can be defined by a cyclic sequence of vertices of P. A vertex v of P is said to be convex (resp. concave), if P has a $90^\circ$ (resp. $270^\circ$) interior angle at v (see Fig.2).

A mask pattern P is degenerate, if two or more edges of P are included in the same straight line. Otherwise, it is non-degenerate. When the boundary $\partial P$ of a mask pattern P consists of two or more closed polygonal curves, one of them encircles all the others. Thus $\partial P$, in general, consists of one "external" curve and a number of "internal" ones.

## 3. Rectangular Dissection of a Mask Pattern

For a mask pattern P, a set of rectangular domains $\{r_1, r_2, \dots, r_M\}$ is called a dissection of P of order M, if

i) $P = \bigcup_{i=1}^{M} r_i$ and

ii) $(r_i \cap r_j) - (\partial r_i \cup \partial r_j) = \emptyset, \quad \forall i \neq j$

In what follows, the term "rectangles" is used for the rectangular domains determined by

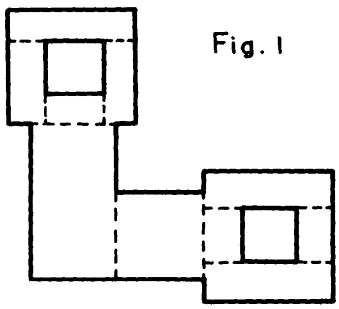


Fig. 1

An example of mask pattern

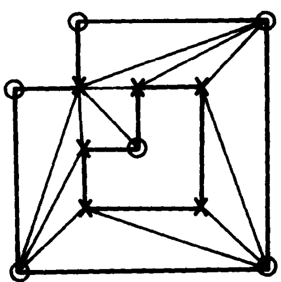


Fig. 2

$N_1 = 6$

$N_2 = 6$

$T = 12$

$E = 12$

o : convex point

x : concave point

a dissection of P. A minimum dissection of P is a dissection with a minimum order among all possible (rectangular) dissections of P.

For a mask pattern P, the line segments determining a dissection are called border-lines (see broken lines in Fig.3). One of the terminal points of a border-line is called a dissection point (see the points marked $\Delta$ in Fig.3) if it is not a vertex of P.

Theorem 1 For any mask pattern P, there exists a dissection D such that

$$M \leqq \frac{N}{2} + B - 2 \quad \cdots\cdots\cdots \quad (1)$$

where M is the order of D, N is the number of vertices of P, and B is the number of connected components of $\partial P$

Theorem 2 Any dissection of a mask pattern P satisfies

$$M \geqq \frac{N}{2} + B - 2 \quad \cdots\cdots\cdots \quad (2)$$

if P is nondegenerate, where M, N and B are as in Theorem 1.

The combination of Theorems 1 and 2 immediately implies that the order M of a minimum dissection of a nondegenerate mask pattern is given by

$$M = \frac{N}{2} + B - 2 \quad \cdots\cdots\cdots \quad (3)$$

An algorithm for obtaining such a minimum dissection is suggested by the following proof of Theorems 1 and 2.

Lemma 1 For a mask pattern P, let

$$N = N_1 + N_2 \quad \cdots\cdots\cdots \quad (4)$$

be the number of vertices of P, $N_1$ (resp. $N_2$) be the number of convex (resp. concave) vertices of P and B be the number of connected components of P. Then

$$N_1 = \frac{N}{2} + 4 - 2B \quad \cdots\cdots\cdots \quad (5)$$

$$N_2 = \frac{N}{2} - 4 + 2B \quad \cdots\cdots\cdots \quad (6) \qquad \text{(see Fig.2)}$$

(Proof of Lemma 1) Consider a triangular dissection of P such that the set of vertices of all the triangles consists only of the vertices of P. It is clear that such a dissection always exists. Let T be the number of triangles and E be the number of line segments connecting a pair of vertices of P which determine the triangular dissection. Then, from Euler's formula,

$$T - (N + E) + N = 2 - B \quad \cdots\cdots\cdots \quad (8)$$

Note that the number of vertices is equal to the number of edges of P.

Since each edge of P belongs to one triangle and each line segment determining the dissection belongs to two adjacent triangles,

$$3T = N + 2E \quad \cdots\cdots\cdots \quad (9)$$

Combinining (8) and (9), we have

$$T = N - 4 + 2B \quad \ldots\ldots\ldots \quad (10)$$

Noting that the total sum of the interior angles of all the triangles is equal to that of P, i.e.

$$(N - 4 + 2B) \cdot 180^\circ = N_1 \cdot 90^\circ + N_2 \cdot 270^\circ \quad \ldots\ldots\ldots \quad (11)$$

we have

$$N_1 + 3N_2 = 2N - 8 + 4B \quad \ldots\ldots\ldots \quad (12)$$

Now the substitution of (4) into (12) leads to (5) and (6) Q.E.D.

(Proof of Theorem 1) We assume that P has at least one concave vertex since otherwise P itself is a rectangle. We consider a dissection determined by the following sequence of border-lines. Take a concave vertex $v_1$ and, from $v_1$, draw a horizontal or vertical line passing through the inside of P until it reaches a point $w_1$ on $\partial P$. The line segment $L_1 = [v_1, w_1]$ thus drawn, is considered to be the first border line. Next we take another concave vertex $v_2$, if any, and draw the second border line $L_2 = \{v_2, w_2\}$ in the same way as above except that $w_2$ is either on $\partial P$ or on a border-line already drawn. Continuing this way until there is no more concave vertex, we have a sequence $\{L_1, L_2, \ldots\}$ of border-lines which determines a desired dissection of P. For each border-line, say $L_k = [v_k, w_k]$ of the dissection, $v_k$ is always a concave vertex of P, while $w_k$ is either a dissection point or a vertex of P, hence it is clear that

$$N_3 \leq N_2 \quad \ldots\ldots\ldots \quad (13)$$

where $N_2$ is the number of concave vertices of P and $N_3$ is the number of dissection points.

Let M be the order, (i.e., the number of rectangles) in the dissection determined above. Then, since each vertex of P is a vertex of one rectangle and each dissection point is a common vertex of two adjacent rectangles, we have

$$4M = N + 2N_3 \quad \ldots\ldots\ldots \quad (14)$$

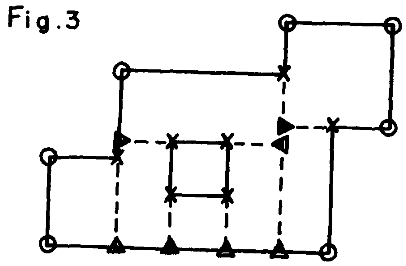
Now the combination of (13), (14) and Lemma 1 proves to the theorem. Q.E.D.

(Proof of Theorem 2) For any dissection of a non-degenerate mask pattern P, there is no border-line in which both terminal points are concave points. And, for any concave vertex, there exists at least one border-line incident with it. Therefore

$$N_3 \geq N_2 \quad \ldots\ldots\ldots \quad (15)$$

where $N_2$ and $N_3$ are as in the proof on Theorem 1 (see Fig.3)

Next we observe that each vertex of P is a vertex of at least one rectangle and that each dissection point is a vertex of at least two adjacent rectangles. Thus we have




$N_1 = 7$, $N_2 = 7$, $N_3 = 7$
$N = 14$, $B = 2$, $N/2 + B - 2 = 7$

$$4M \geqq N + 2N_3 \quad \ldots\ldots\ldots \quad (16)$$

where M is the order of the dissection.

Now, from (15), (16) and Lemma 1, we have the theorem. Q.E.D.

## 4. Computing Algorithm

In our computer program, a mask pattern P is identified by means of the set of its horizontal edges. Each horizontal edge $H_i$ is represented by an ordered sequence $H_i = (x_i, x'_i, y_i)$ of three numbers. Then it means that $v_i = (x_i, y_i)$ and $v'_i = (x'_i, y_i)$ are the terminals of $H_i$. Furthermore, the following convention is used. Namely, when $x'_i < x_i$ (resp. $x_i < x'_i$), it means that the lower (resp. upper) side of $H_i$ is in P. The data representing $H_i$'s is arranged in the descending order of $y_i$'s and packed in a table. (see Fig.4)

### Rectangular Dissection Algorithm

(1) Set $P_0 \leftarrow P$.

(2) Pick up the horizontal edge $H_1 = (x_1, x'_1, y_1)$ of $P_0$ stored in the top of the table

(3) Search the table from the top to the bottom until a horizontal edge $H_k = (x_k, x'_k, y_k)$ satisfying

$$(x'_1, x_1) \cap (x_k, x'_k) \neq \emptyset \quad \ldots\ldots\ldots \quad (17)$$

or a pair of horizontal edges $H_k = (x_k, x'_k, y_k)$ and $H_e = (x_e, x'_e, y_e)$ satisfying

$$x_1 = x'_k < x_k, \quad x'_e < x_e = x_2, \quad y_k = y_e < y_1 \quad \ldots\ldots\ldots \quad (18)$$

is found (see Fig.6)

(4) Let r be the rectangle defined by

$$r = \left\{ (x, y) \in R^2 \;\middle|\; x'_1 \leqq x \leqq x_1, \quad y_k \leqq y \leqq y_1 \right\} \quad \ldots\ldots\ldots \quad (19)$$

and update the table so that it represents $P_0 - r$ in place of $P_0$ (see Fig.5).

(5) If the table is empty, the algorithm terminates here. Otherwise, return to step(2).

In the dissection D obtained by the above algorithm, each edge of the rectangles determined at step(3)indicates a border-line unless it coincides with an edge of P. It is easily seen here that each concave vertex of P is a terminal of one, and only one, border-line and no pair of border-lines crosses. Thus, it can be shown in the same way as the proof of Theorem 1 that D satisfies (1). And, when P is non-degenerate in particular, Theorem 2 implies that the algorithm produces a minimum dissection.

The order M of D becomes less than $\frac{N}{2} + B - 2$ if an edge or its extension of a

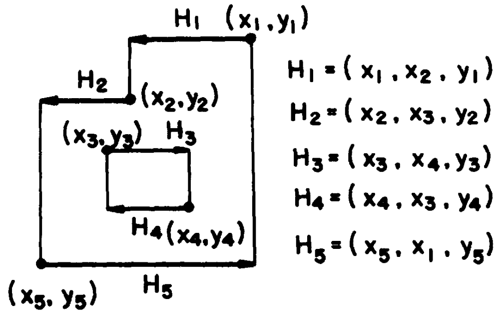



Fig.4

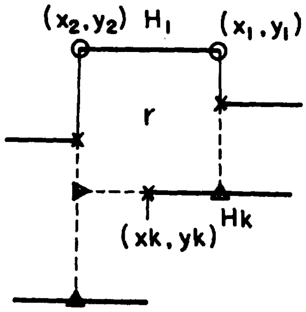



Fig.5

rectangle determined at Step(3)includes two or more edges of P. Fig.6 illustrates examples of degenerate mask patterns for which the algorithm produces a minimum dissection.

The proposed algorithm always gives a minimum dissection for non-degenerate mask patterns. And, even for degenerate ones, it is observed through our computational results that the algorithm gives a minimum dissection in almost all cases.

References

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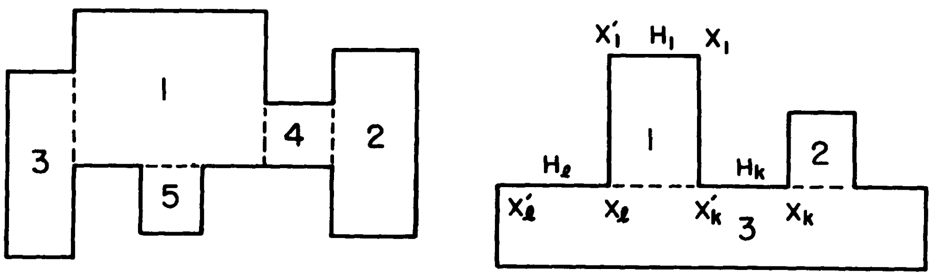


(a) Fig. 6 (b)