A Performance Analysis of Disk Based Computer Systems by Tandem Queueing Model

Masatoshi MIYAZAKI*, Shingo TOMITA**, Shoichi NOGUCHI*
and Juro OIZUMI***

Abstract

A tandem queueing model for a performance analysis of disk based computer systems which work under a multiprogramming environment is discussed. Assuming that all input/output(I/O) requests from a central processing unit(CPU) are processed by moval head disk storage devices, channels of this system can be considered as two-stage tandem channels, that is, first channels are storage units and second channels are control units. Then, the systems are modeled by use of a technique of a tandem queue.

The model is approximately represented as $M/M/S_1 \rightarrow M/S_2$ under assumptions that service times at a CPU, first channels and second channels are exponentially distributed independent random variables, where S_1 and S_2 are number of the first and the second channels respectively. Furthermore, the model has features of a finite waiting room M/M/1(N), where N is a degree of multiprogramming.

A CPU productivity is defined with a equilibrium state probability. Compared with measured and calculated data of the CPU productivity, a model validation is discussed in the case of $N \leqq S_1$.

1. Introduction

A simple queueing model with a finite waiting room to estimate a CPU productivity of multiprogrammed computer systems has been proposed[1]. The model assumes that the computer system consists of a CPU and identical I/O channels. In disk based computer systems, a disk storage system could be regarded as these channels.

A disk storage system consists of control units and disk units. An I/O request from each process being executed in a multiprogramming manner is to be processed through one of the control units and the disk units. The I/O operation takes three statuses such as seeking, waiting for positioning and transmitting of data in order[2], [3]. The waiting and transmitting operation are considered as one operation, which may be called a transfer, because the transmitting operation follows the waiting as soon as the positioning is completed. Then, an I/O operation may be devided into two discrete operations; the first is a seeking and the second is a transfer.

Consequently, for more detail discussions of a CPU productivity in disk based computer systems, an analysis of the I/O operations should be treated by using a tandem queueing model.

2. Tandem Queueing Model

An I/O operation in a disk storage system may be represented as a two-stage tandem

This paper first appeared in Japanese in Joho-Shori (Journal of the Information Processing Society of Japan), Vol. 18, No. 2 (1977), pp. 120~127.

* Tohoku University
** Yamaguchi University
*** University of Electro-Communication

queueing system in which first servers are disk storage units(first channels) and second servers are control units(second channels) as shown in Figure 1 under several assumptions.

The assumptions are as follows. (1) An I/O request to a same storage unit being in a operation will never occure while an idle storage unit exists. This assumption may be reasonable if data on storage units could be uniformly allocated. (2) A data transfer from a storage unit after completion of seeking will be performed through any control unit being idle. (3) When all control units are operating, a seeking operation for a newly arrived I/O request does not start untill any control unit becomes idle because a control unit has a function to start seeking. But, we neglect this blocking effect.

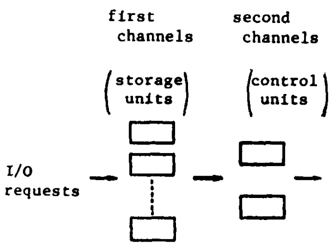

Fig.1 A tandem queueing model of a disk storage.

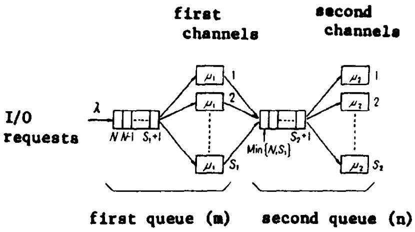

Fig.2 A tandem queueing model of disk based computer systems.

Figure 2 shows a tandem queueing model of disk based computer systems which work under a multiprogramming environment. A degree of multiprogramming is denoted by N $(1 \leqq N \leqq \infty)$, and N processes will allways exist in a system. The system has only one CPU. An I/O request will continuously arrive at first channels while a CPU is executing any process. It is assumed that arrival intervals of I/O requests are independent, identical, exponentially distributed random variable with mean $1/\lambda$. Number of first and second channels is denoted by S_1 and S_2 respectively. Usually a condition $S_1 \geqq S_2$ is needed. Furthermore, $N \geqq S_2$ is required because a possibility of concurrent opration of control units must exist. It is assumed that processing times at the first and the second channels are also exponentially distributed with mean $1/\mu_1$ and $1/\mu_2$ respectively.

The queueing model has two queues, such that a first queue at first channels and a second queue at second channels. An I/O request is to go to the second queue as soon as the processing at the first channel is completed. Number of I/O requests in the first and the second queue including under being processed is denoted by m and n respectively. As number of whole I/O requests in the system cannot exceed N, a condition $0 \leqq m+n \leqq N$ is induced. This means the queueing system with a finite waiting room.

A range of m and n is as follows. m is $0 \leqq m \leqq N$ because a new I/O request can enter the first queue when n = 0. A first channel cannot perform a next I/O request untill the I/O request performed by the first channel is completed at any second channel because the transfer operation needs both of a storage unit and a control unit. Then, number of first channels which can serve I/O requests is S_1-n, and n is $n \leqq S_1$ because of $S_1-n \geqq 0$. Consequently, n must be $0 \leqq n \leqq \mathrm{Min}\,(N, S_1)$ because n cannot exceed N as well as m.

3. Equilibrium State Probability (in the case of $N \leqq S_1$)

In this section a equilibrium equation is shown in the case of $N \leqq S_1$ and a general form of a equilibrium state probability is derived.

Let $p_{m,n}$ be a equilibrium state probability that m and n I/O requests are in the first and the second queue respectively. All conbinations of m and n under a condition of $0 \leqq m \leqq N$, $0 \leqq n \leqq N$ and $0 \leqq m + n \leqq N$ give number of equilibrium states as $(N + 1)(N + 2)/2$. A equilibrium equation is given as follows;

$$
\begin{aligned}
& -\lambda p_{0,0} + \mu_2 p_{0,1} = 0 \\
& -(\lambda + a_1\mu_2) p_{0,n} + a_2\mu_2 p_{0,n+1} + \mu_1 p_{1,n-1} = 0, \quad (m = 0,\ 1 \leqq n \leqq N-1) \\
& -S_2\mu_2 p_{0,N} + \mu_1 p_{1,N-1} = 0, \quad (m = 0,\ n = N) \\
& -(\lambda + m\mu_1) p_{m,0} + \mu_2 p_{m,1} + \lambda p_{m-1,0} = 0, \quad (1 \leqq m \leqq N-1,\ n = 0) \\
& -N\mu_1 p_{N,0} + \lambda p_{N-1,0} = 0, \quad (m = N,\ n = 0) \\
& -(\lambda + m\mu_1 + a_1\mu_2) p_{m,n} + a_2\mu_2 p_{m,n+1} + (m+1)\mu_1 p_{m+1,n-1} + \lambda p_{m-1,n} = 0, \\
& \qquad\qquad (m \neq 0,\ n \neq 0,\ m+n \leqq N-1) \\
& -(m\mu_1 + a_1\mu_2) p_{m,n} + (m+1)\mu_1 p_{m+1,n-1} + \lambda p_{m-1,n} = 0, \quad (m \neq 0,\ n \neq 0,\ m+n = N),
\end{aligned}
\tag{1}
$$

where

$$
a_1 = \begin{cases} n, & (1 \leqq n < S_2) \\ S_2, & (S_2 \leqq n \leqq N-1), \end{cases} \qquad a_2 = \begin{cases} n+1, & (1 \leqq n < S_2) \\ S_2, & (S_2 \leqq n \leqq N-1), \end{cases}
$$

and a constraint

$$\Sigma p_{m,n} = 1 \tag{2}$$

should be given.

We suppose a general form of $p_{m,n}$ as follows;

$$p_{m,n} = A_m B_n \rho_1^m \rho_2^n p_{0,0}, \tag{3}$$

where $\rho_1 = \lambda/\mu_1$, $\rho_2 = \lambda/\mu_2$ and $A_0 = 1$, $B_0 = 1$.

In the case of $1 \leqq m \leqq N$ and $n = 0$, following equations are obtained from equation (1) and (3).

$$
\begin{aligned}
& -1 + B_1 = 0 \\
& -(\lambda + \mu_1) A_1 \rho_1 + \mu_2 A_1 B_1 \rho_1 \rho_2 + \lambda = 0 \\
& -(\lambda + 2\mu_1) A_2 \rho_1^2 + \mu_2 A_2 B_1 \rho_1^2 \rho_2 + \lambda A_1 \rho_1 = 0 \\
& \qquad \vdots \\
& -(\lambda + (N-1)\mu_1) A_{N-1} \rho_1^{N-1} + \mu_2 A_{N-1} B_1 \rho_1^{N-1} \rho_2 + \lambda A_{N-2} \rho_1^{N-2} = 0 \\
& -N\mu_1 A_N \rho_1^N + \lambda A_{N-1} \rho_1^{N-1} = 0.
\end{aligned}
\tag{4}
$$

A relation of $A_m = A_{m-1}/m$ is derived by using $B_1 = 1$ which is given from first equation of (4). Then we obtain a general form of A_m as follows;

$$A_m = \frac{1}{m!} \tag{5}$$

In the case of $m = 0$ and $1 \leqq n \leqq N$, following equations are obtained.

$$-(\lambda+\mu_2)B_1\rho_2+2\mu_2B_2\rho_2^2+\mu_1A_1\rho_1=0$$
$$-(\lambda+2\mu_2)B_2\rho_2^2+3\mu_2B_3\rho_2^3+\mu_1A_1B_1\rho_1\rho_2=0$$
$$-(\lambda+3\mu_2)B_3\rho_2^3+4\mu_2B_4\rho_2^4+\mu_1A_1B_2\rho_1\rho_2^2=0$$
$$\vdots$$
$$-(\lambda+(S_2-2)\mu_2)B_{S_2-2}\rho_2^{S_2-2}+(S_2-1)\mu_2B_{S_2-1}\rho_2^{S_2-1}+\mu_1A_1B_{S_2-3}\rho_1\rho_2^{S_2-3}=0$$
$$-(\lambda+(S_2-1)\mu_2)B_{S_2-1}\rho_2^{S_2-1}+S_2\mu_2B_{S_2}\rho_2^{S_2}+\mu_1A_1B_{S_2-2}\rho_1\rho_2^{S_2-2}=0$$
$$-(\lambda+S_2\mu_2)B_{S_2}\rho_2^{S_2}+S_2\mu_2B_{S_2+1}\rho_2^{S_2+1}+\mu_1A_1B_{S_2-1}\rho_1\rho_2^{S_2-1}=0$$
$$-(\lambda+S_2\mu_2)B_{S_2+1}\rho_2^{S_2+1}+S_2\mu_2B_{S_2+2}\rho_2^{S_2+2}+\mu_1A_1B_{S_2}\rho_1\rho_2^{S_2}=0$$
$$\vdots$$
$$-(\lambda+S_2\mu_2)B_{N-1}\rho_2^{N-1}+S_2\mu_2B_N\rho_2^N+\mu_1A_1B_{N-2}\rho_1\rho_2^{N-2}=0$$
$$-S_2\mu_2B_N\rho_2^N+\mu_1A_1B_{N-1}\rho_1\rho_2^{N-1}=0. \tag{6}$$

Substituting $B_1=1$ and $A_1=1$ derived from (5), B_n is obtained as follows;

$$B_n=\begin{cases}\dfrac{1}{n!}, & (0\leqq n\leqq S_2)\\[2mm] \dfrac{1}{S_2!\,S_2^{n-S_2}}, & (S_2<n\leqq N)\end{cases} \tag{7}$$

It is easily confarmed that A_m from equation (5) and B_n from (7) can be extended to the case of $m \gneqq 0$, $n \gneqq 0$ and $m+n \leqq N-1$ and of $m+n=N$. $p_{0,0}$ is obtained from equation (2) as follows;

$$p_{0,0}=\left\{\sum_{0\leqq m+n\leqq N} A_mB_n\rho_1^m\rho_2^n\right\}^{-1} \tag{8}$$

Consequently, the equilibrium state probability $p_{m,n}$ in the case of $N\leqq S_1$ is given by equation (3), (5), (7) and (8).

4. <u>CPU Productivity</u>

As a factor of system performance, let us consider only a CPU productivity in a steady state of systems. A CPU will not become idle but continue to execute any process while m + n is less than N. Then, a CPU productivity denoted by α may be obtained as follows;

$$\alpha=\sum_{0\leqq m+n<N} p_{m,n} \tag{9}$$

Figure 3 shows a comparison between measured and calculated data of a CPU productivity for the sake of a model verification. The measured data were obtained from the system 'NEAC 2200 Model 700' at Tohoku University Computer Center. The value of ρ_1 and ρ_2 can be obtained from the case of N = 1. The calculated CPU productivity would be comparatively equal to the measured. Consequently, we could conclude that the tandem queueing model can be applied to a performance evaluation of disk based multiprogrammed computer systems.

5. Equilibrium Equation

(in the case of $N > S_1$)

Number of I/O requests which can be processed concurrently at first channels cannot exceed $S_1 - n$ when $N > S_1$. Then, a equilibrium equation is obtained as follows;

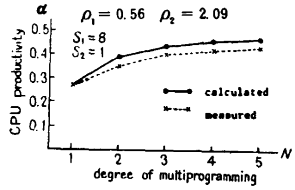

Fig. 3 A comparison between measured and caluculated data of CPU productivity.

$$
\begin{aligned}
&-\lambda p_{0,0} + \mu_2 p_{0,1} = 0 \\
&-(\lambda + a_1\mu_2)p_{0,n} + a_2\mu_2 p_{0,n+1} + \mu_1 p_{1,n-1} = 0, \quad (m = 0,\ 1 \leqq n \leqq S_1 - 1) \\
&-(\lambda + S_2\mu_2)p_{0,S_1} + \mu_1 p_{1,S_1-1} = 0, \quad (m = 0,\ n = S_1) \\
&-(\lambda + a_3\mu_1)p_{m,0} + \mu_2 p_{m,1} + \lambda p_{m-1,0} = 0, \quad (n = 0,\ 1 \leqq m \leqq N - 1) \\
&-S_1\mu_1 p_{N,0} + \lambda p_{N-1,0} = 0, \quad (n = 0,\ m = N) \\
&-(\lambda + a_4\mu_1 + a_1\mu_2)p_{m,n} + a_2\mu_2 p_{m,n+1} + a_5\mu_1 p_{m+1,n-1} + \lambda p_{m-1,n} = 0, \\
&\qquad\qquad (m \neq 0,\ m + n \leqq N - 1,\ 1 \leqq n \leqq S_1 - 1) \\
&-(a_4\mu_1 + a_1\mu_2)p_{m,n} + a_5\mu_1 p_{m+1,n-1} + \lambda p_{m-1,n} = 0, \\
&\qquad\qquad (m \neq 0,\ m + n = N,\ 1 \leqq n \leqq S_1 - 1) \\
&-(\lambda + S_2\mu_2)p_{m,S_1} + \mu_1 p_{m+1,S_1-1} + \lambda p_{m-1,S_1} = 0, \quad (m \neq 0,\ n = S_1,\ m+n \leqq N - 1) \\
&-S_2\mu_2 p_{m,S_1} + \mu_1 p_{m+1,S_1-1} + \lambda p_{m-1,S_1} = 0, \quad (m + n = N,\ n = S_1),
\end{aligned}
\tag{10}
$$

where $a_1 \sim a_5$ are given as follows;

$$
a_1 = \begin{cases} n, & (1 \leqq n < S_2) \\ S_2, & (S_2 \leqq n \leqq S_1) \end{cases}
\qquad
a_2 = \begin{cases} n + 1, & (1 \leqq n < S_2) \\ S_2, & (S_2 \leqq n \leqq S_1) \end{cases}
$$

$$
a_3 = \begin{cases} m, & (1 \leqq m \leqq S_1) \\ S_1, & (S_1 < m \leqq N - 1) \end{cases}
\qquad
a_4 = \begin{cases} m, & (m < S_1 - n) \\ S_1 - n, & (m \geqq S_1 - n) \end{cases}
$$

$$
a_5 = \begin{cases} m + 1, & (m < S_1 - n) \\ S_1 - n + 1, & (m \geqq S_1 - n). \end{cases}
\tag{11}
$$

6. Conclusions

A tandem queueing model similar to $M/M/S_1 \rightarrow M/S_2$ has been proposed for performance evaluation of disk based multiprogrammed computer systems. A reasonable correlation between a measured CPU productivity and a calculated one was confarmed, though a number of assumptions were made to simplify the analysis. The model is fairly tractable when $N \leqq S_1$. If a equilibrium state probability is needed when $N > S_1$, a simultaneous linear equation derived from equation (10) should be solved.

Acknowledgement

The authors wish to thank S.Matsuzawa and S.Obata of Tohoku University Computer Center for their measurement of the system and many constructive suggestions.

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