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偏光特性と幾何特性を利用した透明物体の表面形状計測

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あらまし　ガラスやアクリル製の物体のような透明物体に白色光を全方向から照射し、前に偏光板を置いたCCDカメラで物体を撮影し、その画像を元に物体の偏光特性を利用して、透明物体の3次元形状を計測する手法について述べる。偏光特性を利用して透明物体の表面法線を計測すると、一点につき、4つの法線候補が計算されてしまう。本論文では、4つの法線候補の中から幾何学的な知識を用いて自動的に正しい法線を選択する手法について述べる。この手法により、透明物体の3次元形状を自動的に計測する事ができる事を示す。

Shape Measurement of Transparent Objects using Polarization and Geometrical Characteristics

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Abstract We propose a new method for measuring the shape of transparent objects such as glass or acrylic ornaments. We claim that we can obtain the 3D geometric shape of transparent objects by analyzing the polarization state of lights reflected on the object surface. Using a CCD camera with polarizer in front of its lens, we take several pictures of transparent objects illuminated from all directions. We obtain four possible surface normals at each point. We propose a method for choosing a correct surface normal from those four by geometrical and topological constraints. We demonstrate that we can automatically obtain the 3D shape of transparent objects.

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1 Introduction

We propose a new method of measuring the shape of transparent objects such as glass or acrylic ornaments. We can obtain the digitized 3D object data simply by taking a picture of the object using a CCD camera with a polarizer in front of its lens.

We use the characteristics of the polarization of the object. Since transparent objects have specularity, they reflect light on their surfaces. The light refelcted from the surface of an object polarizes, depending on the characteristics of the object surface. By examining the light reflected from the object while rotating the polarizer set in front of a CCD camera, we can observe how the light is polarized. We can determine surface orientations of transparent objects by using this method.

The method described in this paper is based on the research of our previous work[1]. We proposed that we could recognize surface orientation from the polarization of transparent objects. Our improved research was published recently in [2]. We showed the method using the data regarding the polarization of infrared light in addition to the information about the polarization of visible light.

In this paper, we propose a new method from a different approach, that of using the geometric characteristic in addition to the information about the polarization of visible light.

This method allows easy measurement of the shape of the transparent objects without any contact with it. From multiple images taken by the CCD camera, the method's software automatically calculates the 3D shape of the object.

In section 2, we will show the theory of this method; the basic theory of light and reflection and the mechanism of the polarization. We describe the relationship between polarization and surface orientation.

In section 3, we will show how we calculate the surface orientation of the object from a several pictures. We will show the method of determining the azimuth angle and the elevation of the surface normal of the object.

Section 4 shows the experimental setup and experimental result.

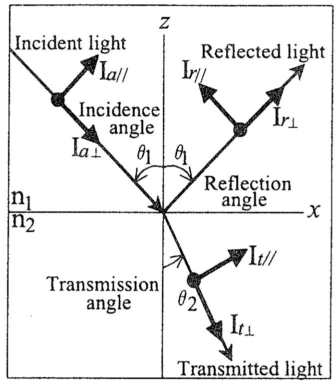

Figure 1: Reflected and transmitted lights

We will conclude this paper in section 5.

2 Theory of measurement

2.1 Fresnel's law

We define that the interface surface of medium 1 and 2, each refractive index is n_1 and n_2, is located in x-y plane as shown in Figure 1. In this case, a part of light refracts and transmits through the medium 2 and the rest of light reflects on the interface surface.

The incident, reflected and transmitted light are expressed with subscript, a, r or t respectively. And parallel or perpendicular to the x-z plane is expressed with subscript $\|$ or $\perp$ respectively.

We denote the light intensity of the incident and reflected light as $I_{a\|}$ and $I_{r\|}$ (parallel component) or $I_{a\perp}$ and $I_{r\perp}$ (perpendicular component). Reflectance of light intensity of the parallel and perpendicular component is expressed as:

$$F_{\|} = \frac{I_{r\|}}{I_{a\|}} = \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)}$$
$$F_{\perp} = \frac{I_{r\perp}}{I_{a\perp}} = \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \quad (1)$$

These intensity reflectances are referred to as the Fresnel reflection coefficients. Equation (1) indicates that there is an angle where $F_{\|}$ is 0. This incidence angle is called the Brewster angle θ_B. From $\theta_1 + \theta_2 = \pi/2$ given where $F_{\|} = 0$ and from

Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$, we obtain the following equation:

$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right) \tag{2}$$

When the incidence angle is equal to Brewster angle, reflected light will be the linear polarized light of the perpendicular component, since all of the parallel component is transmitted.

2.2 Polarization

Since transparent objects has specularity, we can observe highlights (light reflected on an object surface) where the angle of the incident light equals that of the reflected light.

Natural light is unpolarized, i.e., it is a light that has equal magnitude of polarization components in all directions. Suppose that a natural light reflects on an object we observe the reflected light through a polarizer. The intensity of the observed light oscillates sinusoidally as a function of the angular orientation of the polarizer. The light intensity oscillates between maximum light intensity $I_{\max}$ and minimum light intensity $I_{\min}$.

We define the degree of polarization as:

$$\rho = \frac{I_{\max} - I_{min}}{I_{\max} + I_{min}} \tag{3}$$

We call it a degree of polarization ρ, which varies between 0 and 1, inclusively. It represents the proportion of the magnitude of reflected light that is linearly polarized; that magnitude is relative to the total magnitude of reflected light. At $\rho = 0$, reflected light is unpolarized. The state where we observe $\rho = 1$ requires the following conditions: reflected light is completely linearly polarized; the reflection is a purely specular reflection; the object is a dielectric surface; and the incidence angle is the Brewster angle.

The geometry of our measurement system is shown in Figure 2. We define the plane of incidence as the one that includes the direction of a light source, a viewer, and a surface normal. Since we are measuring the highlight of transparent objects, the reflection angle is equal to the incidence angle. We can obtain the direction of the surface normal at each point of the object surface by using the orientation of the plane of incidence and the reflection angle. We denote the orientation of the plane with ϕ measured around the viewer's line of sight, and denote the angle of incidence with θ measured on the plane of incidence.

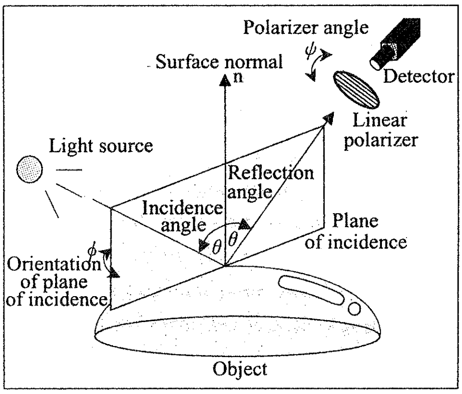

Figure 2: Geometrical location of light, object, camera

As seen in equation (1), intensity reflectance depends on a direction of a plane of oscillation; parallel or perpendicular. The linear combination of $I_{\max}$ and $I_{\min}$ is equal to the total light intensity of the surface component I_S.

$$I_{\max} = \frac{F_\perp}{F_\parallel + F_\perp} I_S, \quad I_{\min} = \frac{F_\parallel}{F_\parallel + F_\perp} I_S \tag{4}$$

Since $I_{\min}$ is the component parallel to the plane of incidence, the orientation of the plane of incidence ϕ can be determined when $I_{\min}$ appears while the polarizer is being rotated.

Because of the structure of linear polarizer which is symmetric in point and line, we unfortunately obtain two candidates of ϕ, ϕ_1 and ϕ_2; the difference of the degree of those two are 180°. The method of choosing the correct ϕ is shown in section 3.

Substituting equations (1) and (4) for (3) and considering the Snell's law, the degree of polarization ρ is given by

$$\rho = \frac{2\sin\theta\tan\theta\sqrt{n^2 - \sin^2\theta}}{n^2 - \sin^2\theta + \sin^2\theta\tan^2\theta} \tag{5}$$

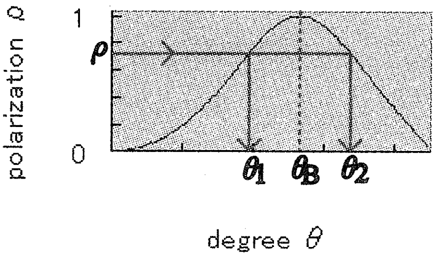

Figure 3: Graph of degree-polarization

The degree of polarization ρ is a function of the angle of incidence θ under a given refractive index n. Thus, from the measured degree of polarization, we can obtain the angle of incidence θ from equation (5) (Figure 3). Though, as seen in the graph of polarization, we obtain two θ, θ_1 and θ_2. The method of choosing the correct θ is shown in section 3.

3 Shape measurement

From the object's polarization characteristic, we can calculate the surface normal of the object. The surface normal is represented by two components — azimuth angle and elevation. Suppose that we put the object on the ground or a table or anything you like. We define the x-y plane as the ground (or something) and the z axis as the upper direction. The azimuth angle is the angle from the positive direction of x axis to the positive direction of y axis. The elevation is the angle from the positive direction of z axis to the negative direction of z axis. We will write the azimuth angle as angle ϕ and elevation as angle θ (Figure 13).

The angle of the incidence plane represents the azimuth angle and the angle of incidence represents the elevation in Figure 2 when we locate the CCD camera right above the object and observe the object downward.

From the polarization data from the taken pictures, we get two possible azimuth angles and two possible elevations. So we must choose the correct surface normal from the possible four surface normals.

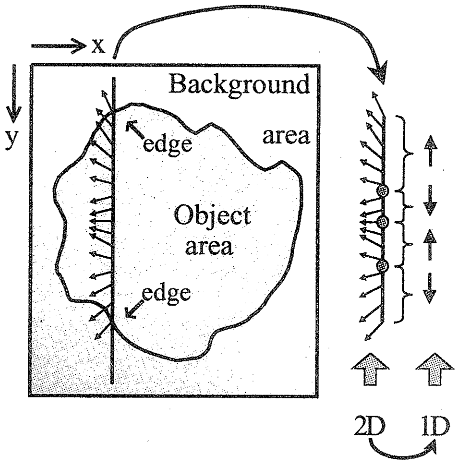

Figure 4: Consider 1D direction of 2D azimuth angle

We use geometrical and topological characteristic to choose the correct angle. We assume that the object we are measuring has a continuous and smooth surface, and also that the object must has no areas that are occluded from the observer (here, 'occlude' refers to the geometry not the visual).

We took a picture of the object from one direction, thus, we use 2 dimensional information obtained from the CCD camera. In this paper, we call this 2D information the 'angle image'. Each pixel of angle image has two possible azimuth angles, ϕ_1 and ϕ_2, two possible elevations, θ_1 and θ_2, and a boolean information whether it is an object or a background.

3.1 Choosing correct azimuth angle

Suppose we bisect the angle image with a straight line (Figure 4). And we only consider the component of the angle parallel to this line; though there is infinite possible value of 2D orientation, there is only two possible value of 1D orientation (the projection of 2D orientation to 1D line), and we express those orientations as 'upward direction' and 'downward direction'. We examine whether the angle is in an upward direction or a downward di-

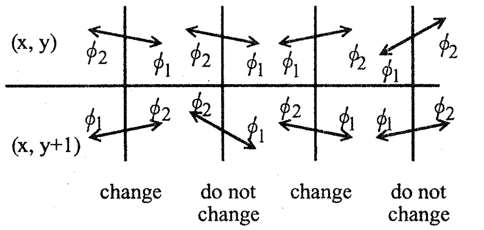

Figure 5: Detect where azimuth direction changes

rection along the line. We examine through this line and examine only the object area; we do not examine the background. If there are several object areas along this line, we examine them in turn.

The angle of an edge of an object has a direction toward the outside of an object. Thus, angles of the two edges of the object area has directly the opposite direction; this means that, concerning such 1D information, there is a singular point where the direction reverses and the numbers of these points are odd numbers.

Now we will describe a practical method for choosing the correct angle ϕ. We examine along y directional lines at each x value. At each line, we examine object areas. At each object area, we know the edge direction and we detect the singular points which reverses the direction; we detect a singular point if the angles of the adjacent two pixels $\phi_{y'}$ and $\phi_{y'+1}$ stride across the line parallel to x axis (Figure 5). As a result, we are able to determine all directions along this area.

3.2 Choosing correct elevation

Figure 3 shows the relation between polarization ρ and elevation θ (incidence angle). We can obtain the polarization from equation (3). From equation (5) and the obtained polarization ρ, we obtain two possible elevations, θ_1 and θ_2. We should choose the correct angle θ from those two angles.

Since we assume the object is smooth, if we draw a contour line of polarization ρ, the line will be a closed curve. Suppose we drew all closed contours of Brewster angle. It is obvious that the value of all of the angle in the area between two contours

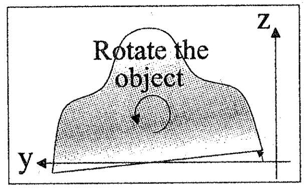

Figure 6: Rotation of object

is either more or less than Brewster angle. For an easy description, we will call the angle greater than the Brewster angle the 'upper angle' and the angle less than the Brewster angle the 'lower angle'.

To get additional constraints for choosing the correct angle, we rotate the object slightly. We rotate the object around x axis (from the negative direction of y axis to the positive direction of y axis; see Figure 6). We get two polarization images; the information about a non-rotated object and the information about a slightly rotated object. (We define the phrase 'polarization image' as which pixel has polarization and a boolean information as to whether it is an object or a background.)

By comparing those two polarization images and using the azimuth angle image, we can choose the correct angle θ. (The 'azimuth angle image' is the angle image; though the correct angle ϕ is already chosen.)

We bisect the object with the plane, thereby transforming the difficult 3D problem into an easier 2D problem. We cut off the object with some planes parallel to y-z plane. We compare two cross sections; one is of a non-rotated object and the other is of a slightly rotated object.

Section 3.2.1 shows the method of choosing the correct angle θ by using cross sections. Section 3.2.2 shows the proof that this method of manipulating 2D information works consistently even if the object is 3D. Section 3.2.3 shows the method of searching identical points of non-rotated object and slightly rotated object. Section 3.2.4 shows the summary of the method.

3.2.1 Changes of polarization

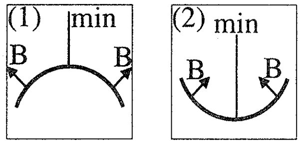

Figure 7: Pattern 1(left) and 2(right) $[\theta < \theta_B]$
direction of ϕ changes at min
B: Brewster angle
min: minimum polarizaition point

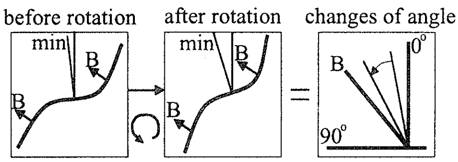

Figure 8: Pattern 3 $[\theta < \theta_B]$
Through rotation, ρ increases because θ changes from 0°-side to θ_B-side

In this section, we will show that we can determine the angle of the area enclosed with the points of the Brewster angle.

We will show all 6 patterns of surface shape permitted in the area enclosed with the points of the Brewster angle (Figure 7 - 11) (recall that we assumed the object surface is smooth).

Pattern 1 is a convex shape (Figure 7). The direction reverses at the topmost point of the convexity $(\theta < \theta_B)$.

Pattern 2 is a concave shape (Figure 8). The direction reverses in the bottommost point of the concavity $(\theta < \theta_B)$. The direction of the azimuth angle of pattern 2 is the opposite to that of pattern 1.

Pattern 3 is a curve where direction of the surface normal is the same as the positive direction of y axis $(\theta < \theta_B)$. We compare the polarization ρ of the same point through the non-rotated object and the slightly rotated object. In pattern 3, the polarization of the same point increases through rotation.

By rotating the object, the 2 dimentional appearance of the object from a certain view changes a little; however, rotation has no effect on the

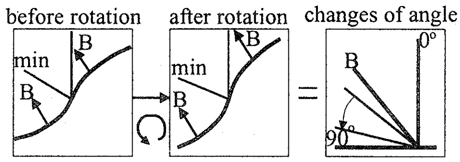

Figure 9: Pattern 4 $[\theta > \theta_B]$
Through rotation, ρ decreases because θ changes from θ_B-side to 90°-side

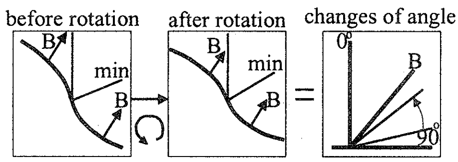

Figure 10: Pattern 5 $[\theta > \theta_B]$
Through rotation, ρ increases because θ changes from 90°-side to θ_B-side

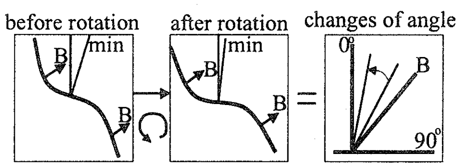

Figure 11: Pattern 6 $[\theta < \theta_B]$
Through rotation, ρ decreases because θ changes from θ_B-side to 0°-side

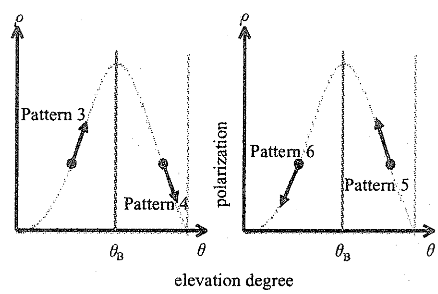

Figure 12: Pattern 3, 4, 5, 6

shape's characteristics. If we see the same characteristic between the non-rotated object and the slightly rotated object, we can say that those are the same points. We recognize that the point where we can observe a minimum polarization degree between the two Brewster angles could be identified as the same point through rotation (this applies only to pattern 3 to 6). More information are given in section 3.2.3.

Pattern 4 is a curve where direction of the surface normal is the same as that of pattern 3 ($\theta > \theta_B$) (Figure 9). The polarization degree ρ of the point of minimum polarization degree decreases through rotation.

Pattern 5 is a curve where direction of the surface normal is the same as the negative direction of y axis ($\theta > \theta_B$) (Figure 10). The polarization degree ρ of the point of minimum polarization degree increases through rotation.

Last, pattern 6 is a curve where direction of the surface normal is the same as that of pattern 5 ($\theta < \theta_B$) (Figure 11). The polarization degree ρ of the point of minimum polarization degree decreases through rotation.

3.2.2 Proof of method

Now we have to prove that we can determine the elevation of the entire object surface by examining through the y-directional cross sections at each column one by one instead of examining the whole 3 dimensional information at once. In this section, we will prove that we can use the previous section's method even if the azimuth angle is not parallel to y axis (Figure 13).

Using the azimuth angle ϕ and elevation θ, we can calculate y-directional θ (imagine the surface normal projected to y-z plane). We write this angle as θ_y. θ_y can be calculated from θ and ϕ (Figure 13).

Now we define the rotation value as $d\theta_y$. Consider the point X and write the polarization of X as ρ^X, the true elevation of X as θ^X, and the azimuth angle of X as ϕ^X. We define $d\theta^X$ as the rotation value along the direction of ϕ^X so that $\theta^X + d\theta^X$ represents the value of the elevation after the slight rotation. $d\theta^X$ is calculated from $d\theta_y$ and ϕ^X. One will notice that the signs of $d\theta_y$ and $d\theta^X$ are the same, signifying that the change of the θ_y and θ has similar behavior.

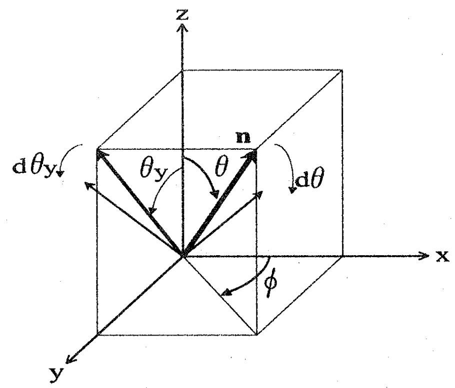

Figure 13: Projection of θ to y-z plane

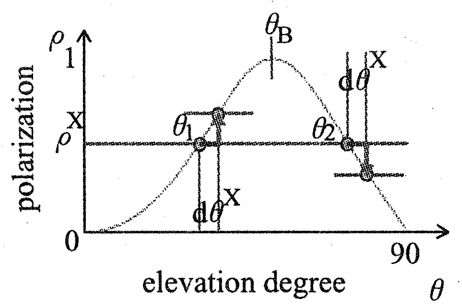

Figure 14: Changes of polarization through rotation

See Figure 14. We define that θ_1^X and θ_2^X satisfy $\theta_1^X < \theta_B < \theta_2^X$. The differences of the polarization of θ_1^X and $\theta_1^X + d\theta^X$ and the polarization of θ_2^X and $\theta_2^X + d\theta^X$ have different signs. The polarization of one increases, while that of the other decreases. As a result, we know the correct θ from the difference of the polarization through the rotation.

The arguement in this section indicates that if we only examine the changes of polarization through the rotation, we can know the angle θ as well as θ_y. We only need to acquire the value of the difference of ρ instead of the value of ρ. We do not need to worry that actual θ is not equal to θ_y.

3.2.3 Correspondent points

The previous section explained that we could determine θ by comparing two ρ of non-rotated object and slightly rotated object. The fact indictes that there is no necessity of detecting Brewster angles for just comparing the polarization degree.

But, we must compare two polarization degrees at the identical point of the object. To detect the same point through the non-rotated object and the slightly rotated object, we identify the points of those objects as the same point where we detect the minimum value of the polarization degree enclosed by two Brewster angle. The reason why we can identify these points as the smame point is that the point where we observe the minimum value of the polarization degree enclosed by two Brewster angle is an inflection point (see Figure 8 - 11). Even if we rotate the object slightly, we can observe the locally minimum value at the inflection point.

The inflection point is detected not only within two Brewster angle points but also within two locally maximum polarization points. Please read the section 3.2.1 by replacing the phrase 'the area enclosed with the points of the Brewster angle' with 'the area enclosed with the points of the locally maximum polarization'.

Though we give a proof in section 3.2.2, there exist two cases which the proof does not really work. One is the case that θ is close to the Brewster angle and the other is the case that ϕ is close to $0°$ or $180°$.

This paragraph is about the former case: the case that θ is close to the Brewster angle. We define θ_1 and θ_2 as $\theta_1 < \theta_B < \theta_2$. If ρ is not close to 1, $\theta_1 < \theta_B$ and $\theta_1 + d\theta < \theta_B$, $\theta_2 > \theta_B$ and $\theta_2 + d\theta > \theta_B$, since we rotate the object slightly ($d\theta$ is a little value). But if ρ is close to 1, the situation of $\theta_1 + d\theta > \theta_B$ or $\theta_2 + d\theta < \theta_B$ might probably occurs. We use the points of minimum polarization between the Brewster angle to avoid the point where ρ is close to 1.

This paragraph is about the latter case: the case that ϕ is close to $0°$ or $180°$. We cannot compare two ρ at this case. Reason 1: If the feature of the point is pattern 1 or 2, the minimum polarization points is not identical between non-rotated object and slightly rotated object. Watching the change of ρ makes no sense (theoritically ρ equals to 0 in both object). Reason 2: If the feature of the point is not pattern 1 or 2, the direction of azimuth angle (whether it is upward direction or downward direction) might probably changes through rotation. Thus, we cannot apply the above-mentioned method. Reason 3: Whether the feature of the point is pattern 1/2 or not, the point is not an inflection point.

Thus, we cannot compare two ρ where ϕ is close to $0°$ or $180°$. We think of the middle point within the locally maximum point and the locally minimum point. Only in this case, we assume that the slight rotation does not cause a change of the appearance and we are able to identify the points as the same point if those points are in the same position. We divide the area enclosed within two locally maximum polarization points into two areas: each enclosed within the locally maximum polarization point and the locally minimum polariztion point. We calculate the middle point of each divided area and used these points for comparison.

3.2.4 Summary of algorithm

The following summarizes the algorithm of choosing the correct angle θ from two possible angles θ_1 and θ_2.

We examine along y directional lines at each x value. At each line, we examine object areas. At each object area, we detect entire points where the polarization degree locally maximizes.

We detect the minimum polarization point between two locally maximum polarization points. We describe the detected minimum polarization point as 'comparing point' except the case where ϕ is close to $0°$ or $180°$ near the minimum polarization point. In such case, we divide the zone enclosed within two locally maximum polarization

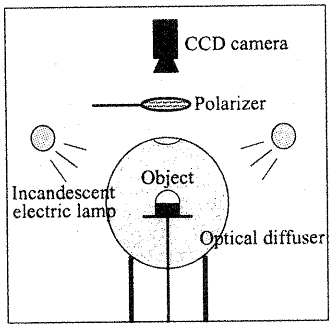

Figure 15: Setup

point into two zones by the minimum polarization point. We detect two middle points of such two zones; zone that is enclosed within locally maximum polarization points and minimum polarization point and we describe these two middle points as 'comparing points'.

We also calculate the middle point within the locally maximum polarization points and the boundary point of the area and also describe as 'comparing point'.

At each comparing point, we check the following.

If the direction of azimuth angle (upward or downward) is the same as positive direction of y axis: if polarization increases through rotation, then $\theta < \theta_B$ (pattern 3) else $\theta > \theta_B$ (pattern 4); or, if polarization increases through rotation, then $\theta > \theta_B$ (pattern 5) else $\theta < \theta_B$ (pattern 6).

4 Experimental measurement

4.1 Experimental setup

The experimental apparatus is depicted in Figure 15. An optical diffuser of a white translucent plastic sphere is used to light an object from all directions. The diffuser is illuminated using three incandescent electric lamps placed at intervals of 120 degrees. This makes a spherical extended light source, which will contribute to detecting the highlight of the entire surface of the object. An object is placed in the center of this sphere. Using a CCD camera, images of the object are taken through a hole located at the north pole of the sphere.

We rotate the polarizer from 0 degrees to 180 degrees by intervals of 5 degrees and take 36 images with a CCD camera. Since the sampling point is not continuous, we matched the light intensity of each 5 degrees to sine curve by using the least squares method (LSM) and detected the maximum and minimum of the light intensity. We obtained the polarization by using maximum and minimum of the light intensity from the equation (3).

We did the same thing with the slightly rotated object. As a result, the input data sums up to 76 image data.

4.2 Experimental result

Using all the knowledge described in all the previous sections, we can get the surface normal of the object surface. We calculate the surface shape from surface normal by using a relaxation method [9].

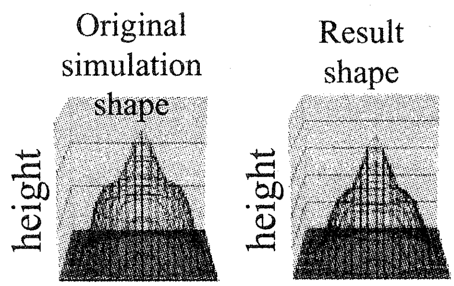

Figure 16: Simulation actual object (left) and simulation result object (right)

Before measuring actual objects, we calculate several simulation objects to check whether the method runs succesfully (Figure 16).

Transparent objects transmit and refract lights as well as reflect lights. Many kind of lights from many directions complexly transmit, refract, and reflect on any part of the object surface and also other objects nearby. Thus, we observe the light

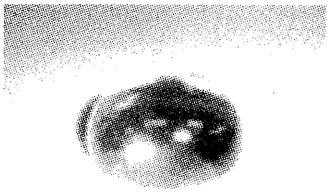

Figure 17: Actual hemisphere ornament

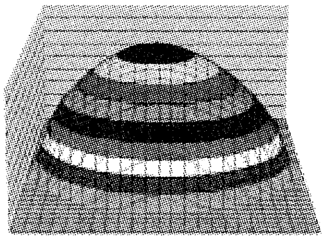

Figure 18: Result hemisphere

of the complex mixture of those lights transmitted, refractted, and reflectted here and there, in addition to the desired data. We infer those error is little enough to calculate using this method.

The result of the shape of the actual hemisphere object is shown in Figure 18. The 3D geometric shape is automatically calculated with the method's software. Our method is useful for obtaining the 3D geometric shape of transparent objects.

5 Conclusion

In this paper, we proposed a method for measuring transparent objects automatically. The experimental setup is simple, and we can measure the object without any contact with it. We showed the measured virtual 3D object and demonstrated that this method was useful for measuring transparent objects. This paper explained the theory of measuring transparent objects using the polarization characteristics and explained the algorithm of determining shape surface automatically by using the geometrical characteristics.

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