社団法人 情報処理学会 研究報告
IPSJ SIG Technical Report

2006－CVIM－153 (50)
2006／3／17

尺度空間の中での領域の階層構造

西口 遼彦† 井宮 淳†† 酒井 智弥††

† 千葉大学大学院自然科学研究科 〒263-8522 千葉市稲毛区弥生町 1-33
†† 千葉大学総合メディア基盤センター 〒263-8522 千葉市稲毛区弥生町 1-33
E-mail: †halu@graduate.chiba-u.jp, ††imiya@faculty.chiba-u.jp

あらまし 本論文では, 領域抽出の前処理となる境界抽出を尺度空間の中において, 2 階微分の非線形 2 次形式の零点集合の抽出問題として再定式化する. このことによって, 尺度空間の中の他の零点の階層構造との関係が明らかになる. そこで, 尺度変化を利用した領域の階層構造を抽出できることになる.

キーワード 線形尺度空間, 尺度空間解析, 画像の階層解析, 領域分割, 微分幾何学

Segment Heirarchy in Linear Scale Space

Haruhiko NISHIGUCHI†, Atsushi IMIYA††, and Tomoya SAKAI††

† Faculty of Engineering, Chiba University Yayoi-cho 1-33, Inage-ku, Chiba 263-8522, Chiba, Japan
†† IMIT, Chiba University, Yayoi-cho 1-33, Inage-ku, Chiba 263-8522, Chiba, Japan
E-mail: †halu@graduate.chiba-u.jp, ††imiya@faculty.chiba-u.jp

Abstract Singular points in the linear scale space provide fundamental features for the extraction of dominant parts of an image. In this paper, we develop an algorithm on edge detection for segmentation using deep structure in the linear scale space. A typical and well-established pre-smoothing is the convolution of an image with Gaussian kernel with an appropriate variance. We introduce a mathematical strategy for the selection of the variance of the Gaussian kernel using the deep structure in the linear scale space. This selection strategem derives the hierarchical structure of the segments.

Key words Linear Scale Space, Diffusion Filtering, Deep Structure Analysis, Hierarchical Analysis of Images, Segmentation, Differential Invariant

1. Introduction

In this paper, we develop an algorithm on edge detection for segmentation using configurations of singular pointd in the linear scale space. For the segmentation, usually, pre-smoothing for images are operated. A typical and awell-established pre-smoothing is the convolution of an image with Gaussian kernel with an appropriate variance. Then, a class of deferential operations are operated for the detection of steepest points as candidates of segment-edges. In these process, the variance of Gaussian kernel, which defines the band-width in Fourier domain, is heuristically selected. We introduce a mathematical strategy for the selection of the variance of the Gaussian kernel using linear scale space analysis, since the convolution of image with Gaussian kernel defines the linear scale space.

The Gaussian scale-space analysis [1]～[4], [9] is an established image analysis tool which provides multi-resolution analysis and expression of steel images and sequence of images [8], [10]. The singular-point configuration in the linear scale space yielded by Gaussian blurring of function is called deep structure in the linear scale space. Hereafter, we use DSSS for the abbreviation of deep structure in the linear scale space. DSSS describes hidden topological nature of the original functions dealing with gray values of a n-variable function in the scale space as a $(n+1)$-dimensional topographical maps [5]～[8], [11]～[13].

For applications of computer vision algorithms to medical image analysis for medical diagnosis, we are required quantitative methods for the validity evaluation of algorithms. The mathematical analysis of segmentation and edge extraction goes back to Canny [18], Torre and Poggio [16], and Krueger and Phillips [17]. These results were proposed in mid '80 to late '80. During '90, there was a progress in scale space theory for image analysis, which were also dealt with in these early works. This paper deals with segmentation and edge extraction from view point of linear scale space analysis to show mathematical meanings of parameters of Canny operations. Torre and Poggio [16] numerically showed validity of multi-scale analysis for the edge extraction. Canny [18], and Krueger and Phillips [17] numerically showed the validity of the same idea. We develop a mathematical strategy for the selection of parameters of Gaussian kernel for pre-smoothing for edge-extraction employing the results in the linear scale space analysis of images.

2. Edge Detection in Scale Space

In the 2-dimensional Euclidean space $\mathbf{R}^2$, for an orthogonal coordinate system x-y defined in $\mathbf{R}^2$, a vector in $\mathbf{R}^2$ is expressed by $\boldsymbol{x} = (x, y)^\top$ where $\cdot^\top$ is the transpose of a vector. Setting $|\boldsymbol{x}|$ to be the length of $\boldsymbol{x}$, for the Gaussian G,

$$G(\boldsymbol{x},\tau) = \frac{1}{4\pi\tau}\exp\left(-\frac{|\boldsymbol{x}|^2}{4\tau}\right) \quad (1)$$

the linear scale-space transform for function $f(\boldsymbol{x})$, such that

$$f(\boldsymbol{x},\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\boldsymbol{y})G(\boldsymbol{y}-\boldsymbol{x},\tau)d\boldsymbol{y} = f(\boldsymbol{x}) *_2 G(\boldsymbol{x},\tau), \quad (2)$$

where $G *_2 f$ expresses the two-dimensional convolution between G and f, defines the general image of function $f(\boldsymbol{x})$.

Canny shows that the collection of curves

$$E_c = \{\boldsymbol{x}|\boldsymbol{x} = argment\{\max|\nabla_{\boldsymbol{m}} G *_2 f|\} \quad (3)$$

as a candidate of segment edges, where $\nabla_{\boldsymbol{m}}$ is the directional gradient in the direction of $\boldsymbol{m} = \frac{\nabla_G f}{|\nabla_G f|}$.

As an approximation of E_c, the collection of points detected by the next operation. The operation is an approximation for the detection of the edge points which Canny originally defined [16], [18], since the gradient map of an original image approximate the zero-crossing set defined by eq. (7).

Edge Detection

(1) Define parameter $\{\tau_i\}_{i=1}^n$ and T_1 and T_2 such that $T_1 \geqq T_2$.

(2) For a pre-determined parameter $\{\tau_i\}_{i=1}^n$ compute $h = G *_2 f$.

(3) Mark $\theta(x,y) = \tan^{-1}\frac{h_x}{h_y} = \tan^{-1}\frac{G_x *_2 f}{G_y *_2 f}$ on points as the edge direction.

(4) For $|\nabla h|$ select a point $|\nabla h| \geqq T_1$ as the starting point of edge-tracking.

(5) Track peaks using $\theta(x,y)$ of $|\nabla h|$ as for as $\nabla h \geqq T_2$.

(6) Superpose edges for all $\{\tau_i\}_{i=1}^n$.

For $n = 1$, this algorithm is called Canny operation (note 1). This expression implies that for Canny operation, we are required to pre-define two sets of parameters $\{\tau_i\}_{i=1}^n$ and $\{T_1, T_2\}$ from an image. Usually, these parameters are defined from pre-assumed noise property of an image [18].

The first aim of this paper is to derive a mathematical strategy to define $\{\tau_i\}_{i=1}^n$ and $\{T_1, T_2\}$ theoretically from an input image. The second aim is to define a method for the description of hierarchy of segments derived from scale space analysis.

3. Mathematical Property of Canny Edge

3.1 Algebraic Property of Threshold Pair

We define the following operators

$$\nabla_G f = \begin{pmatrix} G_x \\ G_y \end{pmatrix} *_2 f = \begin{pmatrix} G_x *_2 f \\ G_y *_2 f \end{pmatrix}, \quad (4)$$

$$\nabla_G^\perp f = \begin{pmatrix} -G_y \\ G_x \end{pmatrix} *_2 f = \begin{pmatrix} -G_y *_2 f \\ G_x *_2 f \end{pmatrix} \quad (5)$$

$$\boldsymbol{H}_G = [\nabla_G \nabla_G^\top] *_2 f = \begin{pmatrix} G_{xx} *_2 f & G_{xy} *_2 f \\ G_{xy} *_2 tf & G_{yy} *_2 f \end{pmatrix}. \quad (6)$$

In reference [18], Canny derived that his edge sets is the zero-grossing set as

$$\frac{\partial^2}{\partial^2 \boldsymbol{m}}(G *_2 f) = 0,\ \boldsymbol{m} = \frac{\nabla_G f}{|\nabla_G f|}. \quad (7)$$

In reference [17], it was proven that eq. (7) becomes

$$\nabla_G f^\top \boldsymbol{H}_G \nabla_G f = 0, \quad (8)$$

since

$$\frac{d^2}{d^2\boldsymbol{n}} f = \frac{f_x^2 f_{xx} + 2f_x f_y f_{xy} + f_y^2 f_{yy}}{f_x^2 + f_y^2} = \frac{1}{|\nabla f|^2}\nabla f^\top \boldsymbol{H}\nabla f, \quad (9)$$

$$\boldsymbol{n} = \frac{\nabla f}{|\nabla f|}, \quad (10)$$

for the Hessian matrix

$$\boldsymbol{H} = [\nabla\nabla^\top]f = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{pmatrix}. \quad (11)$$

Therfore, the zero-crossing set

$$E_c' = \{\boldsymbol{x}|\nabla_G f^\top \boldsymbol{H}_G \nabla_G f = 0\} \quad (12)$$

is the candidate of segment-edges (note 2). Generally, for an appropriate function F, the set of zero-crossing points

$$F = \{\boldsymbol{x}|F(\boldsymbol{x}) = 0\} \quad (13)$$

is expressed as a common set of two sets

$$F = F_+ \cap F_-,\ F_+ = \{\boldsymbol{x}|F(\boldsymbol{x}) \geqq 0\},\ F_- = \{\boldsymbol{x}|F(\boldsymbol{x}) \leqq 0\}. \quad (14)$$

(note 1) Canny [18] did not define this algorithm in the original paper. However, since the gradient-map approximates the zero-crossing of eq. (12), this algorithm is named Canny edge detector. Equation (7) was derived in reference [18].

(note 2) In reference [18], eq. (12) was not explicitly derived.

Equation (14) posses the next assertion(note 3.) .

[Assertion 1] Using eq. (14), we can track points on $\nabla_G f^{\mathrm{T}} \boldsymbol{H}_G \nabla_G f = 0$ without a pair of thresholds $\{T_1, T_2\}$.

3.2 Geometrical Property of Canny Edge

Since the edge curves detected by Canny operator is the structure line [14], [15] (See Section 5.) in the linear scale space [1], employing linear scale space analysis, we develop a strategy to define $\{\tau_i\}_{i=1}^{n}$.

The top points in the linear scale space are points satisfy the conditions

$$\nabla_G f = 0, \ \ det\boldsymbol{H}_G = 0. \qquad (15)$$

Setting τ_i^* to be a scale which derives a top point, and setting

$$\tau_k^* = argment(\min_k\{\tau^* | \nabla_G f = 0, \ \ det\boldsymbol{H}_G = 0\}), \qquad (16)$$

where $\min_k$ is the k-th largest argument of the condition, the next rule is one of possibilities for selection of the scale parameter.

[Strategy 1] (1) Compute scales $\{\tau_i^*\}_{i=0}^{k}$ such that $\tau_i^* \leqq \tau_{i+1}^*$ and $\tau_0 = 0$.

(2) Select scales in the interval (τ_i^*, τ_{i+1}^*).

(3) One of possibility is to select scales from $(0, \tau_1^*)$.

Next, we investigate a topological property of segments. For the speed of trajectory of singular points in linear scale space, we adopt the following definition [11], [12]

[Definition 1] [11], [12] For $S(\boldsymbol{x}, \tau) = |\frac{d\boldsymbol{x}(\tau)}{d\tau}|$, the stationary points on the stationary-curves are the points which satisfy $S(\boldsymbol{x}, \tau, t) = 0$ or are isolated points under the conditions $\boldsymbol{S}_\tau(\tau) = 0$ and $\boldsymbol{S}_{\tau\tau}(\tau) \geqq 0$.

Denoting a stationary point on the stationary-curves as $(\boldsymbol{x}_i, \tau_i)$, we define the stable attention point and the attention field on an image $f(x, y)$ and a view-controlled image of the original image,

[Definition 2] [11], [12] A point $\boldsymbol{x}_i$ and the region of interest $\boldsymbol{R}(\boldsymbol{x}_i \tau_i)$

$$\boldsymbol{R}(\boldsymbol{x}_i, \tau_i) = \{\boldsymbol{x} || \boldsymbol{x} - \boldsymbol{x}_i| \leqq \sqrt{2\tau_i} \qquad (17)$$

define the stable attention point and the attention field on an image $f(x, y)$.

[Definition 3] [11], [12] The view-controlled image of the original image is given as

$$f(\boldsymbol{x}, \boldsymbol{x}_i, \tau_i) = G(\boldsymbol{x} - \boldsymbol{x}_i) f(\boldsymbol{x}). \qquad (18)$$

A function $f(\boldsymbol{x}, \boldsymbol{x}_i, \tau_i)$ approximates an image which is observed by a vision system with a same mechanisms to those of of human beings [1]. Geometrically, view-controlled express local dominate parts of an image, and the stable attention points are centroid of local dominate parts. Therefore, edges computed for a scale τ which defines stable view point are segment-edge of local dominate parts.

This property of the singular points in the scale space suggest the next criterion for the selection of scale τ in the interval (τ_i^*, τ_{i+1}^*).

[Strategy 2] Select τ in (τ_i^*, τ_{i+1}^*), such that $\boldsymbol{S}_\tau(\tau) = 0$ and $\boldsymbol{S}_{\tau\tau}(\tau) \geqq 0$

$$\boldsymbol{H}\frac{d}{d\tau}\boldsymbol{x}(\tau) = -\nabla\Delta f(\boldsymbol{x}(\tau), \tau).$$

Using these analysis, Canny operation without thresholds and a parameter is described as following.

Edge Detection in the Linear Scale Space

(1) Using strategies 1 and 2, select a scale τ or a collection of scales $\{\tau_k\}_{k=1}^{n}$.

(2) Compute the zero-crossing set $\nabla_G f^{\mathrm{T}} \boldsymbol{H} \nabla_G f = 0$. for a scale or a collection of scales.

(3) Detect closed curves which encircles extremal.

(4) For a collection of scales, superpose zero-crossing sets.

In the following sections, using scale space analysis and differential geometry of the surface, we describe the validity of this criterion.

4. Hierarchy in the Linear Scale Space

4.1 Linear Scale Space

Equation (2) defines the general image of function $f(\boldsymbol{x})$ defined in $\mathbf{R}^2 \times \mathbf{R}_+$ [1]. The function $f(\boldsymbol{x}, \tau)$ is the solution of the linear diffusion equation

$$\frac{\partial f(\boldsymbol{x}, \tau)}{\partial \tau} = \Delta f(\boldsymbol{x}, \tau), \ \tau > 0, \ f(\boldsymbol{x}, 0) = f(\boldsymbol{x}). \qquad (19)$$

Stationary points for the topographical maps in the scale space [1], [11] are defined as the solutions of the equation $\nabla f(\boldsymbol{x}, \tau) = 0$. The stationary-curves in the scale space are the collections of the stationary points. We denote the trajectories of the stationary points as $\boldsymbol{x}(\tau)$. Setting $\boldsymbol{H}$ to be the Hessian matrix of $f(\boldsymbol{x}, \tau)$, Zhao and Iijima [11] showed that the stationary-curves for a 2-dimensional image are the solution of,

$$\boldsymbol{H}\frac{d\boldsymbol{x}(\tau)}{d\tau} = -\nabla\Delta f(\boldsymbol{x}(\tau), \tau) \qquad (20)$$

and clarified topological properties of the stationary-curves for two-dimensional patterns Since the Hessian matrix is always singular for singular points, this equation is valid for nonsingular points. The definitions are formally valid to functions defined in $\mathbf{R}^2$. Using the second derivations of $f(\boldsymbol{x}, \tau)$, we classify the topological properties of the stationary points on the topographical maps. In the neighbourhood of the point $\boldsymbol{x}$ which satisfies the relation $\nabla f(\boldsymbol{x}, \tau) = 0$, we have the equation

(note 3) Considering computational epsilon, eq. (14), is expressed as

$$F_\epsilon = \{\boldsymbol{x} || F(\boldsymbol{x})| \leqq \epsilon\} = \{\boldsymbol{x} | F(\boldsymbol{x}) \geqq -\epsilon\} \cap \{\boldsymbol{x} | F(\boldsymbol{x}) \leqq \epsilon\}$$

$$\frac{d^2 f}{d\,\boldsymbol{n}^2} = \boldsymbol{n}\cdot\nabla(\boldsymbol{n}\cdot\nabla f) = \boldsymbol{n}^\top \boldsymbol{H}\boldsymbol{n} \tag{21}$$

Equation (21) means that the eigenvectors of Hessian matrix of $f(\boldsymbol{x},\tau)$ gives the extremal of D^2 and that the extremal are achieved by the eigenvalues of the Hessian of $f(\boldsymbol{x},\tau)$, since $\alpha_1 \geqq \boldsymbol{n}^\top\boldsymbol{H}\boldsymbol{n} \geqq \alpha_n$ for $|\boldsymbol{n}| = 1$. Furthermore, the rank of the Hessian matrix in the higher-dimensional space classifies the properties of the singular points.

[Definition 4] For 2D functions, a point is the singular point, if the rank of the Hessian matrix at the point is one.

4.2 Scale Space Zero-crossing

Denoting the signs of the eigenvalues of the Hessian matrix of a function f which is expressed as $\boldsymbol{H} = \nabla\nabla^\top f$, as MM, Mm and mm in the linear scale space, these labels of points correspond to the local maximum points, the saddle points, and the local minimum points, respectively.

Since the stationary curves consist of many curves for $\tau > 0$, we call each curve a branch curve. The point $\boldsymbol{x}_\infty$ for

$$\lim_{\tau\to\infty} \boldsymbol{x}(\tau) = \boldsymbol{x}_\infty \tag{22}$$

is uniquely determined for any image. We call a curve on which point $\boldsymbol{x}_\infty$ lies and a curve which is open to the direction of $-\tau$, the trunk and branch, respectively. At the top of each branch, a singular point exists. Therefore, for the construction of a unique hierarchical expression of stationary points, Zhao and Iijima [11] proposed the following rule.

Tree Construction

(1) *The sub-root of a branch is the singular point, such that $\nabla f = 0$, at the top of the branch curve and a sub-root.*

(2) *The sub-root is connected to the trunk by a line segment parallel to the x-y plane.*

This rule yields a monotonically branching curve from infinity to zero along the τ-axis in the linear scale space. On the monotonically branching curve,

4.3 Combinatorial Property of Singular Points

For two-dimensional positive functions with a finite number of extrema, we define labelling function such that

$$S(\boldsymbol{x},\tau) = \begin{cases} MM, & r = 2, \quad \alpha_i < 0, \\ Mm, & r = 2, \quad \alpha_1\cdot\alpha_2 < 0, \\ mm, & r = 2, \quad \alpha_i > 0, \\ sM, & r = 1, \quad \alpha_1 < 0, \\ sm, & r = 1, \quad \alpha_1 > 0, \\ m_\infty & |\boldsymbol{x}| = \infty, \end{cases} \tag{23}$$

for points $\nabla f = 0$, where r is the rank of the Hessian matrix $\boldsymbol{H}_G$ for each scale τ.

According to one-to-one mapping between function on Euclidean plane $\mathbf{R}^2$ and the unit sphere S^2, the scale space extrema, local maxima, saddle, local minima correspond to vertices, edges, and faces on a polyhedron, respectively. Using this one-to-one correspondences between a sphere and the Euclidean plane, for the numbers of the singular points, we have the next theorem.

[Theorem 1] Setting $|MM|$, $|Mm|$, $|mm|$, and $|m_\infty|$ to be the numbers of singular points with symbols MM, Mm, mm, and m_∞, for χ_2.

$$\chi_2 = |MM| - |Mm| + (|mm| + |m_\infty|) \tag{24}$$

the relation $\chi_2 = 2$ is satisfied for $0 \leqq \tau \leqq \infty$.

5. Structure Lines [14], [15], [17]

For a two-argument function $z = f(x,y)$, setting $\boldsymbol{H}$ to be the Hessian matrix of f, Enomoto and Katayama [14], Enomoto, Yonizaki, and Watanabe [15] Krueger and Phillips [17] defined three types of second order singular point sets as

$$\nabla f^\top \boldsymbol{H}\nabla f = 0,\ \nabla^\perp f^\top \boldsymbol{H}\nabla f = 0,\ \nabla^\perp f^\top \boldsymbol{H}\nabla^\perp f^\top = 0, \tag{25}$$

where $\nabla^\perp f^\top \nabla f = 0$, that is, for $\nabla f = (f_x, f_y)^\top$, $\nabla^\perp f = (-f_y, f_x)^\top$. Enomoto and Katayama [14] called the point sets

$$E = \{\boldsymbol{x}|\nabla f^\top \boldsymbol{H}\nabla f = 0\}, \tag{26}$$

$$C = \{\boldsymbol{x}|\nabla^\perp f^\top \boldsymbol{H}\nabla f = 0\}, \tag{27}$$

$$D = \{\boldsymbol{x}|\nabla^\perp f^\top \boldsymbol{H}\nabla^\perp f^\top = 0\}, \tag{28}$$

the edge, characteristic, and division lines, respectively.

Setting $\boldsymbol{n}$ and $\boldsymbol{t}$ to be the normal and tangent vectors on the iso-level contour curves of surface $f(x,y) - z = 0$, points on E, C, and D, satisfy the following geometrical properties [14], [15], [17].

- On E, $\nabla f = 0$ or the normal curvature on the surface $f(x,y) - z = 0$ for the direction of ∇f is zero. On E, $\frac{d}{d\boldsymbol{n}}|\nabla f| = 0$ since

$$\frac{d}{d\boldsymbol{n}}|\nabla f| = \frac{1}{|\nabla f|^2}\nabla f^\top \boldsymbol{H}\nabla f. \tag{29}$$

- On C, an eigenvector of $\boldsymbol{H}$ is ∇f, since $\nabla^\perp f^\top \nabla f = 0$ and $\nabla^\perp f^\top \boldsymbol{H}\nabla f$ imply that $\boldsymbol{H}\nabla f = \alpha\nabla f$ for $\alpha \in \mathbf{R}$. On C, $\frac{d}{d\boldsymbol{t}}|\nabla f| = 0$ since

$$\frac{d}{d\boldsymbol{t}}|\nabla f| = \frac{1}{|\nabla f|^2}\nabla^\perp f^\top \boldsymbol{H}\nabla f. \tag{30}$$

- On D $\nabla f = 0$ or the normal curvature on the surface $f(x,y) - z = 0$ for the direction of $f(x,y) = const.$ is zero. On D, $\frac{d}{dt}\frac{f_x}{f_y} = 0$ since

$$\frac{d}{dt}\frac{f_x}{f_y} = \frac{1}{|\nabla f|^2}\nabla^\perp f^\top \boldsymbol{H}\nabla^\perp f. \tag{31}$$

Krueger and Phillips [17] showed that C describes local symmetry and E detects edge of a segment. We define structure lines in the linear scale space.

[Definition 5] Structure lines in the linear scale space $E(\tau)$, $C(\tau)$, and $D(\tau)$ are

$$E(\tau) = \{\boldsymbol{x}|\nabla_G f^\top \boldsymbol{H}_G\nabla_G f = 0\}, \tag{32}$$

$$C(\tau) = \{\boldsymbol{x}|\nabla_G^\perp f^\top \boldsymbol{H}_G\nabla_G f = 0\}, \tag{33}$$

$$D(\tau) = \{\boldsymbol{x}|\nabla_G^\perp f^\top \boldsymbol{H}_G\nabla_G^\perp f^\top = 0\}. \tag{34}$$

These definitions correspond to the definitions of zero-crossing of Krueger and Phillips [17].

From these mathematical backgrounds, we have the next property of the edge curves define by Canny [18].

[Assertion 2] The edge of segment detected by Canny operation in reference [18] is $E(\tau)$, which is edge-line in the linear scale space for an appropriate scale parameter.

6. Topological Stability of Segments

For a fixed τ, $E(\tau)$ is the edge detected by Canny edge-detection operator [17]~[19]. The top points in linear scale space are points which satisfy the conditions

$$\nabla_G f = 0, \; det\boldsymbol{H}_G = 0. \qquad (35)$$

Let $\sharp(\tau)$ be the number of extremal for the scale τ. We have the next assertion.

[Assertion 3] • Setting τ^* to be a scale which derives a top point, the deference between $\sharp(\tau^* + \epsilon)$ and $\sharp(\tau^* - \epsilon)$ is at least one as described in section 3.

• E-line crosses at saddle points and a simple closed portion of E-line encircles at most one extremal [15].

These two geometric properties in the linear scale space and topology of E-line lead to the next assertions.

[Assertion 4] The difference between the numbers of simple closed curves for scales $\tau^* + \epsilon$ and $\tau^* - \epsilon$ is at least one.

[Assertion 5] A closed curve encircles an extrema has common region with a stable view field.

These topological properties define the local hierarchy of segments.

[Proposition 1] If a pair of branches of stationary curve is merged at a top point, a pair of simple closed curves in E-line which share a saddle point is merged to a simple closed curve

.

Therefore, for a pair of scales which are separated by a top point. the topology of E-line varies. This property implies the validity of strategy 1 for the selection of scales in edge detection as zero-closing of $\nabla_G f^\top \boldsymbol{H}_G \nabla_G f$.

Furthermore, proposition 1 and strategy 1 show that, with carful selection of the Gaussian-kernel variance, Canny edge-detection algorithm eliminates small segments which are caused by noise for the protection of over segmentation. On the other hand, over-smoothing by a large variance of the Gaussian kernel eliminates some segments and causes under segmentation. The configuration of top-points, which are second order singularity in the linear scale space, clarifies the topological change of the segments and boundary curves detected by Canny edge detection algorithm.

7. Numerical Example

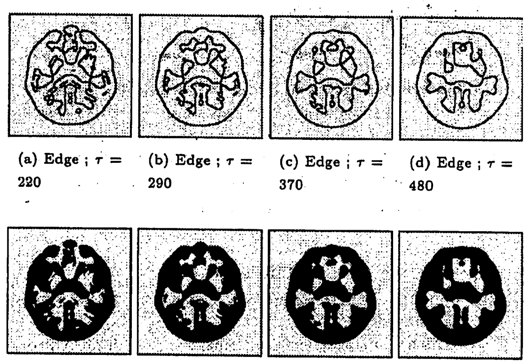
Figure 1 shows edges and segments for selected scales. Scales in Figure 1 are selected based on the number of saddle points listed in table 1. These numbers define τ^*. Since, at a top-point, at least one saddle point disappears based on the geometrical property of scale space saddle, that is, a saddle point and a local maximal point are merged to yield a top point and a top point is linked to a local maximal point.

Table 1 The number of saddle points for scales.

scale	number of saddle points
$730 > \tau \geq 400$	4
$> \tau \geq 380$	5
$> \tau \geq 360$	6
$> \tau \geq 230$	7
$> \tau \geq 220$	8

If a closed curve for a small scale encircles a collection of closed curves in a large scale, this relation defines a hierarchy of segments across the scale. Figure 2 shows the tree extracted form the singular points in the linear scale space and the tree extracted from segments. The tree constructed from segments may define a strategy for the unification of small segments to a large segments for the controlling of over-segmentation.

Figure 4 shows gradient maps for $\tau = 30, 50, 70, 90$, for $|\nabla G *_2 f \geq 0.5$. The connectivity of edges collapse if the scale increases. Therefore, edges detected by the edge detection algorithm described in section 2 does not allow to construct hierarchical tree of segments across the scales.



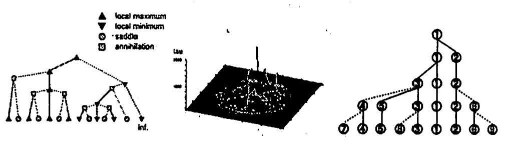
(a) Edge ; $\tau = 220$ (b) Edge ; $\tau = 290$ (c) Edge ; $\tau = 370$ (d) Edge ; $\tau = 480$

(e) Segment ; $\tau = 220$ (f) Segment ; $\tau = 290$ (g) Segment ; $\tau = 370$ (h) Segment ; $\tau = 480$

Fig. 1 Edges and segments in the linear scale space. From left to right, figures show edges and segments for selected scales.

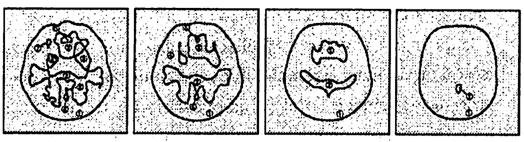
8. Conclusions

In this paper, we developed an algorithm on edge detection for segmentation using deep structure in the linear scale space. Since Canny developed an edge detection method for pre-smoothed images with Gaussian kernel. Therefore, we analysed Canny operator using Gaussina scale space framefork, and found out theoretical strategy on the determination of parameters involved in Canny operation.



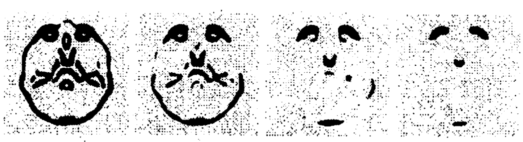
(a) Scale Space tree (b) Stationary curve (c) Segment tree

Fig. 2 The singular point tree and the segment tree. A tree in (a) is extracted based on the curves in (b). A tree in (c) is extracted from hierarchy of segments of Figure 3.



(a) Segment ; $\tau = 360$ (b) Segment ; $\tau = 560$ (c) Segment ; $\tau = 960$ (d) Segment ; $\tau = 1570$

Fig. 3 Edges and segments in the linear scale space. If a closed curve for a small scale encircles a collection of closed curves in a large scale, this relation defines a hierarchy of segments across the scale.



(a) Gradient Map ; $\tau = 30$ (b) Gradient Map ; $\tau = 50$ (c) Gradient Map ; $\tau = 70$ (d) Gradient Map ; $\tau = 90$

Fig. 4 Gradient map in the linear scale space. Gradient maps for $\tau = 30, 50, 70, 90$, for $\nabla G *_2 f \geqq 5.0$. The connectivity of edges collapse if the scale increases.

Reference

[1] Iijima, T., *Pattern Recognition*, Corona-sha, Tokyo, 1974 (in Japanese).

[2] Witkin, A.P., Scale space filtering, Pros. of 8th IJCAI, 1019-1022, 1993.

[3] Lindeberg, T., *Scale-Space Theory in Computer Vision,* Kluwer, Boston 1994.

[4] ter Haar Romeny, B.M. *Front-End Vision and Multi-Scale Image Analysis Multi-scale Computer Vision Theory and Applications, written in Mathematica*, Springer 2003.

[5] Yuille, A. L., Poggio, T., Scale space theory for zero crossings, IEEE PAMI, **8**, 15-25, 1986.

[6] Kuijper, A., Florack, L.M.J., Viergever, M.A., Scale Space Hierarchy, Journal of Mathematical Imaging and Vision **18**, 169-189, 2003.

[7] Kuijper, A., Florack, L.M.J., The hierarchical structure of images, IEEE, Trans. Image Processing **12**, 1067- 1079, 2003.

[8] Imiya, A., Katsuta, R. Extraction of a structure feature from three-dimensional objects by scale-space analysis, LNCS, **1252**, 353-356, 1997.

[9] Weickert, J., *Anisotropic Diffusion in Image Processing,* Teubner, 1998.

[10] Imiya, A., Sugiura, T., Sakai, T, Kato, Y., Temporal structure tree in digital linear scale space, LNCS, **2695**, 356-371, 2003.

[11] Zhao, N.-Y., Iijima, T., Theory on the method of determination of view-point and field of vision during observation and measurement of figure IECE Japan, Trans. D., **J68-D**, 508-514, 1985 (in Japanese).

[12] Zhao, N.-Y., Iijima, T., A theory of feature extraction by the tree of stable view-points. IECE Japan, Trans. D., **J68-D**, 1125-1135, 1985 (in Japanese).

[13] Pelillo, M., Siddiqi, K., Zucker, S.W., Matching hierarchical structures using association graphs, IEEE, Trans, PAMI **21**, 1105-1120, 1999.

[14] Enomoto, H., Katayama, T., Structure lines of images, Proc. 3ed IJCPR, 811-815, 1976.

[15] Enomoto, H., Yonezaki, N., Watanabe, Y., Application of structure lines to surface construction and 3-dimensional analysis, 106-137, in Fu., K.-S., Kunii, T.L., eds., *Picture Engineerting,* Springer, 1982.

[16] Torre, V., Poggio, T., On edge detection, IEEE, PAMI, **8**, 147-163, 1986.

[17] Krueger,W. M., Phillips, K., The geometry of differential operator with application to image processing, IEEE, PAMI, **11**, 1252-1264, 1989.

[18] Canny, J., A computational approach to edge detection, IEEE, PAMI, **8**, 679-698, 1986.

[19] Najman, L., Schmitt, M., Watershed of a continuous function, Signal Processing, **38**, 99-112, 1994.