帰納的実数値関数の帰納推論における論駁性と信頼性

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概要

本論文では，まず，帰納的実数値関数の帰納推論の新しいモデルとして，論駁推論および信頼推論を導入し，論駁推論の成功基準 RealRefExと信頼推論の成功基準 RealRelExについて考察する．そして，この二つの基準と帰納的実数値関数の既存の推論基準である極限同定の成功基準 RealEx，有限推論の成功基準 RealFin，および枚挙推論の成功基準 RealNum! を比較し，相互関係を明らかにする．

Refutability and Reliability for Inductive Inference of Recursive Real-Valued Functions

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Abstract

In this paper, we introduce new models of *refutably* and *reliably* inductive inference of recursive real-valued functions, and consider the new criteria RealRefEx for refutable inference and RealRelEx for reliable inference. Then, we compare these two criteria with RealEx for identification in the limit, RealFin for learning finitely and RealNum! for learning by enumeration that have been already introduced in the previous works, and investigate their interaction.

1 Introduction

Inductive inference gives us a theoretical model of concept learning from examples. In inductive inference, whether or not a learning process is successful is determined by a sequence of hypotheses as outputs under several criteria.

Historically, many researchers have developed inductive inference of recursive functions, by introducing the criteria Ex [6], Fin [6] and Num! [3, 4]. They correspond to identification in the limit, learning finitely and learning by enumeration, respectively.

Mukouchi and Arikawa [13] have first formulated and developed *refutably* inductive inference of formal languages and formal systems. In their framework, a learning machine will discover a hypothesis which produces examples if it is in a hypothesis space, otherwise it will refute the whole hypothesis space and stop.

Minicozzi [12] and L. and M. Blum [5] have introduced the *reliability* requiring that whenever a learning machine converges to some hypothesis from given data of a recursive function, it always identifies the function. The reliability realizes the requirement that a reliable scientist never fails to signal the inaccuracy of a previous false hypothesis. We denote the criterion for learning reliably in the limit by RelEx.

Recently, Jain *et al.* [10] have deeply studied *refutably* inductive inference of recursive func-

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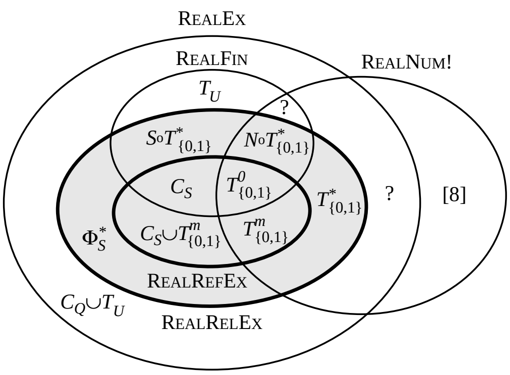
tions. They have introduced the new criterion RefEx for learning refutably and investigated the relationship between RefEx and the criteria Ex, Num! and RelEx.

On the other hand, a *recursive real-valued function* is one of the formulations for computable real numbers. Inductive inference of recursive real-valued functions has been first investigated by Hirowatari and Arikawa [7, 8] and developed by their co-authors [1, 2, 9]. In their works, the criteria RealEx, RealFin and RealNum! have been introduced as the extensions of Ex, Fin and Num!, respectively, and their interaction has been investigated.

Hence, in this paper, we investigate *refutably* and *reliably* inductive inference of *recursive real-valued functions*. First, we introduce the new criteria RealRefEx for learning refutably and RealRelEx for learning reliably of recursive real-valued functions. Then, we obtain the interaction of our criteria described in the following figure.



2 Inductive Inference of Recursive Real-Valued Functions

In this section, first we prepare some notions necessary to the later discussion. We omit the formal definition of *recursive real-valued functions*. Refer to [2, 8, 9] in more detail. We denote the sets of all natural numbers, positive natural numbers, rational numbers, real numbers and recursive real-valued functions by N, N^+, Q, R and $\mathcal{RRVF}$, respectively.

By φ_j we denote the partial recursive function from N to N computed by a program j. By $\mathcal{P}$ we denote the set $\{\varphi_0, \varphi_1, \varphi_2, \cdots\}$ of all partial recursive functions from N to N and by $\mathcal{R}$ the set of all recursive functions.

Definition 1 Let $S_0 \subseteq N$ be the domain of $\varphi_j \in \mathcal{P}$. Then, a function $h_j : S \to R$ $(S \subseteq R)$ is called the *stair function* of φ_j if h_j satisfies the following conditions:

1. $S = \bigcup_{i \in S_0} (i - \frac{1}{2}, i + \frac{1}{2})$,
2. $h_j(x) = \varphi_j(i)$ for any $x \in (i - \frac{1}{2}, i + \frac{1}{2})$ and $i \in S_0$.

For $\mathcal{S} \subseteq \mathcal{P}$, we call a stair function of a function in $\mathcal{S}$ a *stair function* in $\mathcal{S}$ simply.

Definition 2 For $\varphi_j \in \mathcal{R}$, the following function $h_j : [0, \infty) \to R$ is called the *line function* of φ_j.

$$\begin{aligned} h_j(x) &= (\varphi_j(i+1) - \varphi_j(i))x \\ &+ \varphi_j(i)(i+1) - \varphi_j(i+1)i \\ &\quad \text{for any } x \in [i, i+1] \text{ and } i \in N. \end{aligned}$$

For $\mathcal{T} \subseteq \mathcal{R}$, we call a line function of a function in $\mathcal{T}$ a *line function* in $\mathcal{T}$ simply.

In order to consider recursive real-valued functions, we deal with the approximation of a real number x, instead of the exact value of x, and capture it as a pair $\langle p, \alpha \rangle$ of rational numbers such that p is an approximate value of the number x and α is its positive error bound, i.e., $x \in [p - \alpha, p + \alpha]$. We call such a pair $\langle p, \alpha \rangle$ a *datum* of x.

An *example* of a function $h : S \to R$ $(S \subseteq R)$ is a pair $\langle \langle p, \alpha \rangle, \langle q, \beta \rangle \rangle$ satisfying that there exists a real number $x \in S$ such that $\langle p, \alpha \rangle$ and $\langle q, \beta \rangle$ are data of x and $h(x)$, respectively.

A *presentation* of a function $h : S \to R$ $(S \subseteq R)$ is an infinite sequence $\sigma = w_1, w_2, \ldots$ of examples of h in which, for any real number x in the domain of h and any $\zeta > 0$, there exists an example $w_k = \langle \langle p_k, \alpha_k \rangle, \langle q_k, \beta_k \rangle \rangle$ such that $x \in [p_k - \alpha_k, p_k + \alpha_k]$, $h(x) \in [q_k - \beta_k, q_k + \beta_k]$, $\alpha_k \leq \zeta$ and $\beta_k \leq \zeta$. By $\sigma[n]$ we denote the initial segment of n examples in σ.

An *inductive inference machine* (*IIM*) is a procedure that requests inputs from time to time and produces from time to time algorithms that compute recursive real-valued functions. These algorithms produced by an IIM while receiving examples are called *conjectures*.

For an IIM $\mathcal{M}$ and a finite sequence $\sigma[n] = \langle w_1, w_2, \cdots, w_n \rangle$, by $\mathcal{M}(\sigma[n])$ we denote the last conjecture of $\mathcal{M}$ after requesting examples $w_1, w_2, \cdots, w_n$ as inputs.

Let σ be a presentation for some function and $\{\mathcal{M}(\sigma[n])\}_{n\geq 1}$ the infinite sequence of conjectures produced by an IIM $\mathcal{M}$. The sequence $\{\mathcal{M}(\sigma[n])\}_{n\geq 1}$ *converges* to an algorithm $\mathcal{A}_h$ if there exists a number $n_0 \in N$ such that $\mathcal{M}(\sigma[m])$ equals $\mathcal{A}_h$ for any $m \geq n_0$.

The criteria RealEx, RealFin and RealNum! for inductive inference of recursive real-valued functions (see [8, 9] for formal definitions) correspond to the standard criteria Ex, Fin and Num! for inductive inference of recursive functions [3, 4, 6, 10, 11]. Now, we introduce two criteria RealRefEx for *refutably* inductive inference of recursive real-valued functions corresponding to RefEx [10], and RealRelEx for *reliably* inductive inference of recursive real-valued functions corresponding to RelEx [5, 10, 12].

Let h be a recursive real-valued function, σ a presentation of h, and $\mathcal{T}$ a class of recursive real-valued functions. Also let RealEx($\mathcal{M}$) be the set of all recursive real-valued functions inferred by an IIM $\mathcal{M}$ in the limit, and $\perp$ the *refutation symbol*.

Definition 3 We say that an IIM $\mathcal{M}$ *refutably infers* $\mathcal{T}$ *in the limit* if $\mathcal{M}$ satisfies the following conditions:

1. $\mathcal{T} \subseteq$ RealEx($\mathcal{M}$).
2. If $h \in$ RealEx($\mathcal{M}$), then $\mathcal{M}(\sigma[n]) \neq \perp$ for any σ and $n \in N$.
3. If $h \in \mathcal{RRVF} \setminus$ RealEx($\mathcal{M}$), then there exists an $n \in N$ such that $\mathcal{M}(\sigma[m]) \neq \perp$ for any σ and $m < n$, and $\mathcal{M}(\sigma[m]) = \perp$ for any σ and $m \geq n$.

Definition 4 We say that an IIM $\mathcal{M}$ *reliably infers* $\mathcal{T}$ *in the limit* if $\mathcal{M}$ satisfies the following conditions:

1. $\mathcal{T} \subseteq$ RealEx($\mathcal{M}$).
2. If $h \in \mathcal{RRVF} \setminus$ RealEx($\mathcal{M}$), then a sequence $\{\mathcal{M}(\sigma[n])\}_{n\geq 1}$ does not converge to an algorithm for any σ.

By RealRefEx (*resp.*, RealRelEx) we denote the collection of all classes of recursive real-valued functions that are *refutably* (*resp.*, *reliably*) inferable in the limit.

3 Comparison of Criteria

In this section, we compare the new criteria RealRefEx and RealRelEx with the previous criteria. Note here that the following statements hold by our previous works [8].

1. RealFin $\subsetneq$ RealEx.
2. RealFin $\cap$ RealNum! $\neq \emptyset$.
3. RealNum! $\setminus$ RealEx $\neq \emptyset$.

Theorem 5 The following statements hold.

1. RealRefEx $\subsetneq$ RealRelEx $\subsetneq$ RealEx.
2. RealRefEx$\cap$RealFin$\cap$RealNum! $\neq \emptyset$.
3. RealFin $\setminus$ (RealRelEx $\cup$ RealNum!) $\neq \emptyset$.
4. RealRelEx $\setminus$ (RealRefEx $\cup$ RealFin $\cup$ RealNum!) $\neq \emptyset$.

Theorem 6 For $I_i \in$ {RealRefEx, RealFin, RealNum!} $(i = 1, 2, 3)$ such that $I_i \neq I_j$ $(i \neq j)$, the following statements hold.

1. $(I_1 \cap I_2) \setminus I_3 \neq \emptyset$.
2. (RealRelEx $\cap I_1) \setminus (I_2 \cup I_3) \neq \emptyset$.

In the reminder of this section, we give several examples, instead of the proofs of the above theorems.

For $\varphi \in \mathcal{R}$, $\varphi^{-1}(0)$ denotes the set $\{n \in N \mid \varphi(n) = 0\}$. Then, $\mathcal{R}_{\{0,1\}}$, $\mathcal{R}^m_{\{0,1\}}$ and $\mathcal{R}^*_{\{0,1\}}$ are defined as follows.

$$\mathcal{R}_{\{0,1\}} = \{\varphi : N \to \{0,1\} \mid \varphi \in \mathcal{R}\},$$
$$\mathcal{R}^m_{\{0,1\}} = \{\varphi \in \mathcal{R}_{\{0,1\}} \mid \#\varphi^{-1}(0) \leq m\},$$
$$\mathcal{R}^*_{\{0,1\}} = \cup_{m\in N}\mathcal{R}^m_{\{0,1\}}.$$

Also let $\mathcal{T}^m_{\{0,1\}}$ and $\mathcal{T}^*_{\{0,1\}}$ be the sets of all line functions in $\mathcal{R}^m_{\{0,1\}}$ and in $\mathcal{R}^*_{\{0,1\}}$, respectively. Then:

1. $\mathcal{T}^0_{\{0,1\}} \in$ RealRefEx $\cap$ RealFin $\cap$ RealNum!.
2. $\mathcal{T}^m_{\{0,1\}} \in$ (RealRefEx $\cap$ RealNum!) $\setminus$ RealFin for $m \in N^+$.

3. $\mathcal{T}^*_{\{0,1\}} \in$ (RealRelEx $\cap$ RealNum!) $\setminus$ (RealRefEx $\cup$ RealFin).

Let U be the set of all recursive functions f from N to N such that $\varphi_{f(0)} = f$ and $\mathcal{T}_U$ the set of all stair functions in U. Then:

$\mathcal{T}_U \in$ RealFin $\setminus$ (RealRelEx $\cup$ RealNum!).

For $m \in N^+$ and $\varphi_j \in \mathcal{R}^m_{\{0,1\}} \setminus \mathcal{R}^{m-1}_{\{0,1\}}$, let $\varphi_{m,j}$ be the following function:

$$\varphi_{m,j}(n) = \begin{cases} m & \text{if } n = 0, \\ \varphi_j(n-1) & \text{otherwise.} \end{cases}$$

Let $S \subseteq N$ and $S \circ \mathcal{T}^*_{\{0,1\}}$ the set of all line functions of $\varphi_{m,j}$ for $m \in S$ and $\varphi_j \in \mathcal{R}^m_{\{0,1\}} \setminus \mathcal{R}^{m-1}_{\{0,1\}}$ ($N \circ \mathcal{T}^*_{\{0,1\}}$ if $S = N$). Assume that S is not recursively enumerable. Also let $\mathcal{C}_S$ (*resp.*, $\mathcal{C}_Q$) be the set of all constant functions $c_s : [0,1] \to S$ (*resp.*, $c_q : [0,1] \to Q$) such that $c_s(x) = s$ for $s \in S$ (*resp.*, $c_q(x) = q$ for $q \in Q$). Then:

1. $S \circ \mathcal{T}^*_{\{0,1\}} \in$ (RealRelEx $\cap$ RealFin) $\setminus$ (RealRefEx $\cup$ RealNum!).
2. $N \circ \mathcal{T}^*_{\{0,1\}} \in$ (RealFin $\cap$ RealNum!) $\setminus$ RealRefEx.
3. $\mathcal{C}_S \in$ (RealRefEx $\cap$ RealFin) $\setminus$ RealNum!.
4. $\mathcal{C}_S \cup \mathcal{T}^m_{\{0,1\}} \in$ RealRefEx $\setminus$ (RealFin $\cup$ RealNum!) for $m \in N^+$.
5. $\mathcal{C}_Q \cup \mathcal{T}_U \in$ RealEx $\setminus$ (RealRelEx $\cup$ RealFin $\cup$ RealNum!).

Finally, for any subset $F \subseteq N$, let φ_F be the following function:

$$\varphi_F(n) = \begin{cases} 0 & \text{if } n \in F, \\ 1 & \text{otherwise.} \end{cases}$$

Let $S \subsetneq N$ be an infinite subset that is not recursively enumerable and Φ^*_S the set of all line functions of φ_F such that F is a finite subset of S. Then:

$\Phi^*_S \in$ RealRelEx $\setminus$ (RealRefEx $\cup$ RealFin $\cup$ RealNum!).

4 Conclusion

In this paper, we have introduced the criteria RealRefEx and RealRelEx for *refutably* and *reliably* inductive inference of recursive real-valued functions, and compared them with RealEx, RealFin and RealNum!, as described in the figure in the last of Section 1.

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