2006－MPS－60 (2)
2006／6／26

灰区間演算に基づく業者選び問題の意思決定解析方法

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業者選び問題自体は多目標意思決定（MADM）問題である。意思決定問題の多くは選択対象属性に対する評価値がしばしば不確定であるため，良い業者の選択をより難しくしている。灰色システム理論ではこのような不確定な問題を，不確定情報を含んだシステムとしてデータ処理するのに優れた数学的な分析方法である。本論文では業者の選択問題を扱うために，灰区間演算に基づく数理アプローチ方法を提案する。はじめに全ての選択対象属性と重みに対する評価を，灰数と呼ばれる言語変数によって記述する。次に提案する灰区間演算で得られる可能度により全ての選択対象の順序を決定する。具体的な業者の選択問題の計算例を掲げ，本手法の評価を行う。

A Grey-Based Decision Making Approach to Suppliers Selection Problem

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The suppliers selection is a multiple attribute decision making (MADM) problem. Since the decision maker (DM)s like preferences on alternatives or on attributes of suppliers are often uncertain, and thus the selection of g... suppliers becomes more difficult. Grey system theory is one of the methods that are used to study uncertainty, it is superior in mathematical analysis of systems with uncertain information. In this paper, we proposed a new grey-based approach to deal with selection problem of suppliers. The work procedure is shown as follows briefly: First, the weight and rating of attribute for all alternatives are described by linguistic variables that can be expressed in grey number. Second, proposed grey possibility degree to determine the ranking order of all alternatives. Finally, an example of selection problem of suppliers was used to illustrate the proposed approach. The proposal is illustrated with several examples in detail.

1. Introduction

With the globalization of economic market and the development of information technology, suppliers selection problem become one of the most important components in supply chain management [1]. The suppliers selection is a multiple attribute decision making (MADM) problem. The decision maker (DM)s always express their preferences on alternatives or on attributes of suppliers, which can be used to help ranking the suppliers or selecting the most desirable one. The preference information on alternatives of suppliers and on attributes belongs to DMs' subjective judgments. Generally, DMs' judgment are often uncertain and can't be estimated by the exact numerical value. And thus the selection problem of suppliers has many uncertainties and becomes more difficult.

Grey system theory [2], [3] is one of the methods that are used to study uncertainty, it is superior in mathematical analysis of systems with uncertain information. Up to present, fuzzy-based approach has been proposed to deal with the suppliers selection problem under certainty [4]. The advantage of grey system theory over fuzzy theory [5] is that grey system theory considers the condition of the fuzziness. That is, grey system theory can flexibly deal with the fuzziness situation [6].

In this paper, we proposed a new grey-based approach to deal with selection problem of suppliers under uncertainty environment. The work procedure is shown as follows briefly: First, the weight and rating of attribute for all suppliers alternatives are described by linguistic variables that can be expressed in grey number. Then we can obtain the grey decision matrix of suppliers. Second, proposed grey possibility degree to determine the ranking order of all alternatives of suppliers. Finally, an example of selection problem of suppliers was used to illustrate the proposed approach.

This paper is organized as follows: Section 2 describes preliminaries of grey systen theory and

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grey number comparison. Section 3 introduces proposed grey-based approach. In Section 4 the proposed approach is applied to the suppliers selection problem. Finally, conclusions are described in Section 5.

2. Preliminaries

2.1 Grey System Theory and Grey Interval Analysis

Grey system theory [3], originally developed by Deng in 1982, has become a very effective method of solving uncertainty problems under discrete data and incomplete information. Grey system theory has now been applied to various areas such as forecasting, system control, decision making and computer graphics. Here, we give some basic definitions of grey system, grey set and grey number in grey system theory.

Definition 1. A grey system is defined as a system containing uncertain information presented by grey number and grey variables.

Definition 2. Let X be the universal set. Then a grey set G of X is defined by its two mappings $\overline{\mu}_G(x)$ and $\underline{\mu}_G(x)$.

$$\begin{cases} \overline{\mu}_G(x) : x \longrightarrow [0,1] \\ \underline{\mu}_G(x) : x \longrightarrow [0,1] \end{cases} \tag{1}$$

$\overline{\mu}_G(x) \geq \underline{\mu}_G(x)$, $x \in X, X = R$, $\overline{\mu}_G(x)$ and $\underline{\mu}_G(x)$ are the upper and lower membership functions in G respectively. When $\overline{\mu}_G(x) = \underline{\mu}_G(x)$, the grey set G becomes a fuzzy set. It shows that grey systme theory considers the condition of the fuzziness and can flexibly deal with the fuzziness situation.

Definition 3. A grey number is one of which the exact value is unknown, while the upper and/or the lower limits can be estimated. Generally grey number is written as $\otimes G$, ($\otimes G = G|_{\underline{\mu}}^{\overline{\mu}}$).

Definition 4. If the lower and upper limits of G can be estimated and G is defined as interval grey number.

$$\otimes G = [\underline{G}, \overline{G}] \tag{2}$$

Definition 5. The basic operation laws [7] of grey numbers $\otimes G_1 = [\underline{G}_1, \overline{G}_1]$ and $\otimes G_2 = [\underline{G}_2, \overline{G}_2]$ can be expressed as follows:

$$\otimes G_1 + \otimes G_2 = [\underline{G}_1 + \underline{G}_2, \overline{G}_1 + \overline{G}_2] \tag{3}$$

$$\otimes G_1 - \otimes G_2 = [\underline{G}_1 - \overline{G}_2, \overline{G}_1 - \underline{G}_2] \tag{4}$$

$$\begin{aligned} &\otimes G_1 \times \otimes G_2 = [\underline{G}_1, \overline{G}_1] \times [\underline{G}_2, \overline{G}_2] \\ &= [\min(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2), \\ &\quad \max(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2)] \end{aligned} \tag{5}$$

$$\otimes G_1 \div \otimes G_2 = [\underline{G}_1, \overline{G}_1] \times [\frac{1}{\underline{G}_2}, \frac{1}{\overline{G}_2}] \tag{6}$$

Definition 6. The lenth of grey number $\otimes G$ is defined as

$$L(\otimes G) = [\overline{G} - \underline{G}] \tag{7}$$

2.2 Comparison of Grey Number

We proposed grey possibility degree to compare the ranking of grey numbers.

Definition 7. For two grey numbers $\otimes G_1[\underline{G}_1, \overline{G}_1]$ and $\otimes G_2 = [\underline{G}_2, \overline{G}_2]$, the possibility degree of $\otimes G_1 \leq \otimes G_2$ can be expressed as follows [8] :

$$P\{\otimes G_1 \leq \otimes G_2\} = \frac{\max(0, L^* - \max(0, \overline{G}_1 - \underline{G}_2))}{L^*} \tag{8}$$

where $L^* = L(\otimes G_1) + L(\otimes G_2)$.

For the position relationship between $\otimes G_1$ and $\otimes G_2$, there exist four possible cases on the real number axis. The relationship between $\otimes G_1$ and $\otimes G_2$ are determined as follows:

1. If $\underline{G}_1 = \underline{G}_2$ and $\overline{G}_1 = \overline{G}_2$, we say that $\otimes G_1$ is equal to $\otimes G_2$, denoted as $\otimes G_1 = \otimes G_2$. Then $P\{\otimes G_1 \leq \otimes G_2\}$=0.5.
2. If $\underline{G}_2 > \overline{G}_1$, we say that $\otimes G_2$ is larger than $\otimes G_1$, denoted as $\otimes G_2 > \otimes G_1$. Then $P\{\otimes G_1 \leq \otimes G_2\}$=1.
3. If $\overline{G}_2 < \underline{G}_1$, we say that $\otimes G_2$ is smaller than $\otimes G_1$, denoted as $\otimes G_2 < \otimes G_1$. Then $P\{\otimes G_1 \leq \otimes G_2\}$=0.
4. If there is an intercrossing part in them, when $P\{\otimes G_1 \leq \otimes G_2\}$ >0.5, we say that $\otimes G_2$ is larger than $\otimes G_1$, denoted as $\otimes G_2 > \otimes G_1$. When $P\{\otimes G_1 \leq \otimes G_2\}$ <0,5, we say that $\otimes G_2$ is samller than $\otimes G_1$, denoted as $\otimes G_2 < \otimes G_1$.

3. Proposed Approach

A new approach based on grey possibility degree is proposed to make the order preference of suppliers. This method is very suitable for solving the group decision-making problem under uncertainty environment. Assume that $S = \{S_1, S_2, \cdots, S_m\}$ is a discrete set of m possible suppliers alternatives. $Q = \{Q_1, Q_2, \cdots, Q_n\}$ is a set of n attributes of suppliers. The attributes are additively independent. $\otimes w = \{\otimes w_1, \otimes w_2, \cdots, \otimes w_n\}$ is the vector of attribute weights. In this paper, the attribute weights and ratings of suppliers are considered as linguistic variables. Here these linguistic variables can be expressed in grey numbers by 1-7 scale shown in Table 1. The attribute ratings $\otimes G$ can be also expressed in grey numbers by 1-7 scale shown in Table 2. The procedures are summarized as follows:

Step 1:

Table 3 Grey weighted normalized decision table

S_i	Q_1	Q_2	Q_3	Q_4
S_1	[0.470, 0.925]	[0.550, 0.950]	[0.367, 0.682]	[0.295, 0.525]
S_2	[0.409, 0.729]	[0.550, 0.950]	[0.383, 0.750]	[0.326, 0.550]
S_3	[0.389, 0.701]	[0.325, 0.570]	[0.350, 0.659]	[0.249, 0.385]
S_4	[0.368, 0.617]	[0.200, 0.443]	[0.367, 0.682]	[0.286, 0.502]
S_5	[0.368, 0.617]	[0.250, 0.475]	[0.300, 0.500]	[0.350, 0.550]
S_6	[0.430, 0.757]	[0.200, 0.443]	[0.283, 0.477]	[0.249, 0.385]

Table 1 The scale of attribute weights $\otimes w$

Scale	$\otimes w$
Very low (VL)	[0.0,0.1]
Low (VL)	[0.1,0.3]
Medium low (ML)	[0.3,0.4]
Medium (M)	[0.4,0.5]
Medium high (MH)	[0.5,0.6]
High (H)	[0.6,0.9]
Very high (VH)	[0.9,1.0]

Table 2 The scale of attribute ratings $\otimes G$

Scale	$\otimes G$
Very poor (VP)	[0,1]
Poor (P)	[1,3]
Medium poor (MP)	[3,4]
Fair (F)	[4,5]
Medium good (MG)	[5,6]
Good (G)	[6,9]
Very good (VG)	[9,10]

Form a committee of decision-maker and identify attribute weights of suppliers. Assume that a decision group has K persons, then the attribute weight of attribute Q_j can be calculated as

$$\otimes w_j = \frac{1}{K}[\otimes w_j^1 + \otimes w_j^2 + \cdots + \otimes w_j^K] \quad (9)$$

where $\otimes w_j^K (j = 1, 2, \cdots, n)$ is the attribute weight of Kth DM and can be described by grey number $\otimes w_j^K = [\underline{w}_j^K, \overline{w}_j^K]$.

Step 2:

Using linguistic variables for the ratings to make attribute rating value. Then the rating value can be calculated as

$$\otimes G_{ij} = \frac{1}{K}[\otimes G_{ij}^1 + \otimes G_{ij}^2 + \cdots + \otimes G_{ij}^K] \quad (10)$$

where $\otimes G_{ij}^K (i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ is the attribute rating value of Kth DM and can be described by grey number $\otimes G_{ij}^K = [\underline{G}_{ij}^K, \overline{G}_{ij}^K]$.

Step 3:

Establishment of grey decision matrix.

$$D = \begin{bmatrix} \otimes G_{11} & \otimes G_{12} & \cdots & \otimes G_{1n} \\ \otimes G_{21} & \otimes G_{22} & \cdots & \otimes G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes G_{m1} & \otimes G_{m2} & \cdots & \otimes G_{mn} \end{bmatrix} \quad (11)$$

where $\otimes G_{ij}$ are linguistic variables based on grey number.

Step 4:

Normalization grey decision matrix.

$$D^* = \begin{bmatrix} \otimes G_{11}^* & \otimes G_{12}^* & \cdots & \otimes G_{1n}^* \\ \otimes G_{21}^* & \otimes G_{22}^* & \cdots & \otimes G_{2n}^* \\ \vdots & \vdots & \ddots & \vdots \\ \otimes G_{m1}^* & \otimes G_{m2}^* & \cdots & \otimes G_{mn}^* \end{bmatrix} \quad (12)$$

where

For benefit attribute, $\otimes G_{ij}^*$ is expressed as

$$\otimes G_{ij}^* = \left[\frac{\underline{G}_{ij}}{G_j^{\max}}, \frac{\overline{G}_{ij}}{G_j^{\max}}\right] \quad (13)$$

$G_j^{\max} = \max_{1\le i\le m}\{\overline{G}_{ij}\}$

For cost attribute, $\otimes G_{ij}^*$ is expressed as

$$\otimes G_{ij}^* = \left[\frac{G_j^{\min}}{\overline{G}_{ij}}, \frac{G_j^{\min}}{\underline{G}_{ij}}\right] \quad (14)$$

$G_j^{\min} = \min_{1\le i\le m}\{\underline{G}_{ij}\}$.

The normalization method mentioned above is to preserve the property that the ranges of normalized grey number belong to [0, 1].

Step 5:

Establishment of weighted normalized grey decision matrix. Considering the different important of each attribute, the weighted normalized grey decision matrix can be established as

$$D^* = \begin{bmatrix} \otimes V_{11} & \otimes V_{12} & \cdots & \otimes V_{1n} \\ \otimes V_{21} & \otimes V_{22} & \cdots & \otimes V_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes V_{m1} & \otimes V_{m2} & \cdots & \otimes V_{mn} \end{bmatrix} \quad (15)$$

where $\otimes V_{ij} = \otimes G_{ij}^* \times \otimes w_j$.

Step 6:

Making the ideal alternative as referential alternative. For m possible suppliers alternatives set $S = \{S_1, S_2, \cdots, S_m\}$, the ideal referential supplier alternative $S^{\max} = \{\otimes G_1^{\max}, \otimes G_2^{\max}, \cdots, \otimes G_n^{\max}\}$ can be obtained by

$$S^{\max} = \{[\max_{1\leq i\leq m} \underline{V}_{i1}, \max_{1\leq i\leq m} \overline{V}_{i1}], [\max_{1\leq i\leq m} \underline{V}_{i2}, \max_{1\leq i\leq m} \overline{V}_{i2}], \cdots, [\max_{1\leq i\leq m} \underline{V}_{in}, \max_{1\leq i\leq m} \overline{V}_{in}]\} \quad (16)$$

Step 7:

Calculation grey possibility degree between compared suppliers alternatives set $S = \{S_1, S_2, \cdots, S_m\}$ and ideal referential supplier alternative $S^{\max}$.

$$P\{S_i \leq S^{\max}\} = \frac{1}{n}\sum_{j=1}^{n} P\{\otimes V_{ij} \leq \otimes G_j^{\max}\} \quad (17)$$

Step 8:

Ranking order of suppliers alternatives. When $P\{S_i \leq S^{\max}\}$ is smaller, the ranking order of S_i is better. Otherwise, the ranking order is worse.

According to above procedures, we can determine the ranking order of all suppliers alternatives and select the best one form among a set of feasible suppliers.

4. Application

There are six suppliers $S_i (i = 1, 2, \cdots, 6)$ are as selected alternatives against four attributes $Q_j (j = 1, 2, \cdots, 4)$. The four attributes are product quality, service quality, delivery time and price respectively [9]. Q_1, Q_2 and Q_3 are benefit attributes, the larger values are better. Q_4 is cost attributes, the smaller values are better. A committee of four DMs, D_1, D_2, D_3 and D_4 has been formed to express their preferences and to select the most best suppliers.

According to the proposed approach shown in Section 3, the grey normalized decision table for six suppliers is shown in Table 3. Then, we make the ideal supplier $S^{\max}$ as referential alternative. According to Eq. (16), the the ideal supplier $S^{\max}$ is shown as follows:

$$S^{\max} = \{[0.470, 0.925], [0.550, 0.950], [0.383, 0.750], [0.350, 0.550].\}$$

According to Eq. (17), we can obtain the result of ranking order as $S_1 > S_2 > S_4 > S_5 > S_3 > S_6$. We can say that the supplier S_1 is the best supplier in six suppliers. S_1 should be as an important alternative for company. The next important alternative is S_2. Because of the grey possibility degrees of S_1 and S_2 against ideal $S^{\max}$ are almost equal. S_4, S_5 and S_3 are better suppliers and S_6 is the worse supplier.

The selection problem of suppliers is a MADM problem. In conventional MADM methods, the ratings and the weights of the attribute must be known precisely [10]. But, DMs' judgment are often uncertain and cannot be estimated by the exact numerical value. Thus, conventional MADM methods is limited. The selection problem of suppliers has many certainties, we view it as a grey process and can be resolved by grey system theory. As the same time, we introduced grey possibility degree to compare the ranking of grey numbers. Through a verify example, we obtained the effectiveness of proposed approach.

5. Conclusions

In this paper, we proposed a new grey-based approach to deal with selection problem of suppliers under uncertainty environment. An example of selection problem of suppliers was used to illustrate the proposed approach. The experimental result shows that proposed approach is reliable and reasonable.

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